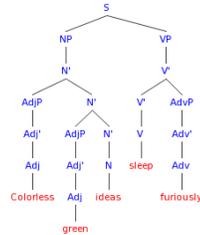


Lecture 8: Context-Free Languages

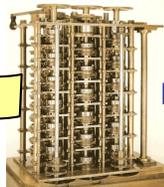


Menu

- Review Machine Models of Computing
- Linguistic Model of Computing
- Challenge Problem Near-Solution - Liuyi (Eric) Zhang
- Context-Free Grammars

Deterministic Machine Models

010110101101010101



→ "Yes"
or
"No"

Memory

Finite States
+ Stack
+ Tape

Machine

DFA
DPDA
TM

Languages

Regular
[this week]
[later]

Nondeterministic Machine Models

0101101011010101



→ "Yes"
or
"No"

Memory

Finite States
+ Stack
+ Tape

Machine

NFA
NDPDA
NDTM

Languages

Regular
[this week]
[later]

Where did these models
come from?

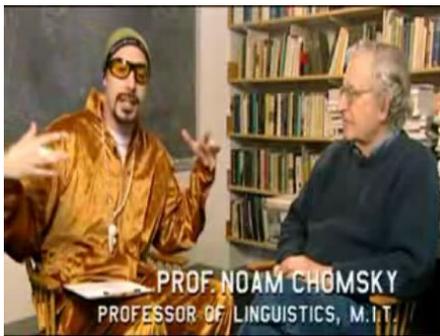
Modeling Human Intellect

- Turing Machine (Alan Turing, 1936)
 - Modeling Human Computers
- DFAs
 - McCulloch and Pitts, "A logical calculus of the ideas immanent in nervous activity", 1943
 - S. C. Kleene, *Representation of Events in Nerve Nets and Finite Automata*, 1956
 - Claude Shannon and John McCarthy, *Automata Studies*, 1956

Out theoretical objective is not dependent on the assumptions fitting exactly. It is a familiar strategem of science, when faced with a body of data too complex to be mastered as a whole, to select some limited domain of experiences, some simple situations, and to undertake to construct a model to fit these at least approximately. Having set up such a model, the next step is to seek a thorough understanding of the model itself.

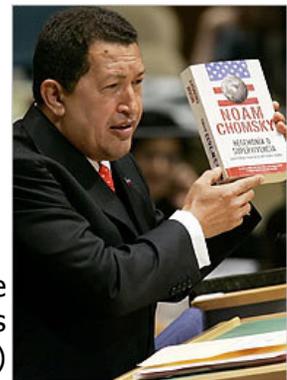
S. C. Kleene, *Representation of Events in Nerve Nets and Finite Automata*, 1956

Language-Based Models of Computation



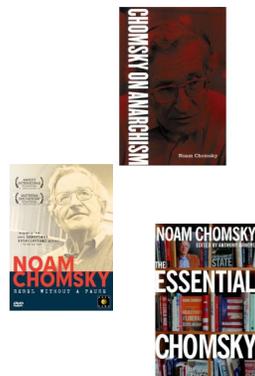
Noam Chomsky (born 1928), MIT Linguistics Professor and Leftist Political Activist

Hugo Chávez at the United Nations (20 Sept 2006)



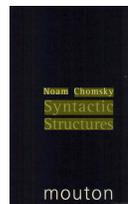
I don't know anybody who's ever read a Chomsky book, He does not write page turners, he writes page stoppers. There are a lot of bent pages in Noam Chomsky's books, and they are usually at about Page 16.

Alan Dershowitz



"I must admit to taking a copy of Noam Chomsky's *Syntactic Structures* along with me on my honeymoon in 1961. During odd moments, while crossing the Atlantic in an ocean liner and while camping in Europe, I read that book rather thoroughly and tried to answer some basic theoretical questions. Here was a marvelous thing: a mathematical theory of language in which I could use a computer programmer's intuition! The mathematical, linguistic, and algorithmic parts of my life had previously been totally separate. During the ensuing years those three aspects became steadily more intertwined; and by the end of the 1960s I found myself a Professor of Computer Science at Stanford University, primarily because of work that I had done with respect to languages for computer programming."

Donald Knuth



Replacement Grammars

Left \rightarrow *Right*

Anything that matches the left side can be replaced by what is on the right side.

Left and *Right* can be any sequence of variables (nonterminals) and symbols (terminals)

Restricted Replacement Grammars

Unrestricted: $\alpha \rightarrow \beta$
(left and right sides can be any sequence of symbols and variables)

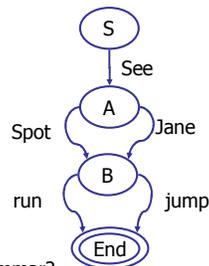
Context-Sensitive: $\alpha A \beta \rightarrow \alpha \gamma \beta$
(*A* is a variable)

Context-Free: $A \rightarrow \alpha \gamma \beta$

Regular: $A \rightarrow aB ; A \rightarrow a$

Example Regular Grammar

$S \rightarrow \text{See } A$
 $A \rightarrow \text{Spot } B$
 $A \rightarrow \text{Jane } B$
 $B \rightarrow \text{run}$
 $B \rightarrow \text{jump}$



How many possible sentences?

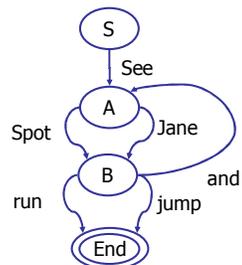
Why do we call it a *Regular* Grammar?



Kanzi and Sue Savage-Rumbaugh

Example Regular Grammar

$S \rightarrow \text{See } A$
 $A \rightarrow \text{Spot } B$
 $A \rightarrow \text{Jane } B$
 $B \rightarrow \text{run}$
 $B \rightarrow \text{jump}$
 $B \rightarrow \text{and } A$



THE INFINITE GIFT

How Children Learn and
Unlearn the Languages of the World



Charles Yang

Recursion = Human ?

We hypothesize that faculty of language in the narrow sense (FLN) only includes recursion and is the only uniquely human component of the faculty of language. We further argue that FLN may have evolved for reasons other than language, hence comparative studies might look for evidence of such computations outside of the domain of communication (for example, number, navigation, and social relations).

Marc Hauser, Noam Chomsky, Tecumseh Fitch,
The Faculty of Language: What Is It, Who Has It, and How Did It Evolve?, *Science*, Nov 2002

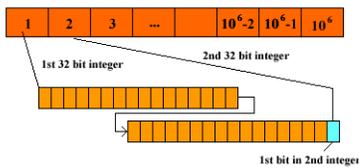
Steven Pinker and Ray Jackendoff (2004): its not just recursion...

Challenge Problem

Liuyi (Eric) Zhang

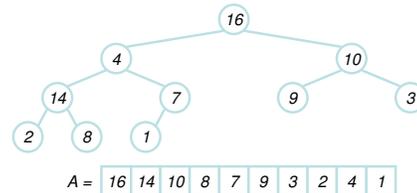
Problem: Sort 1 million 32-bit integers

Memory layout



Review: Heaps

A *heap* is a “complete” binary tree, usually represented as an array:



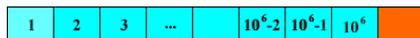
From David Luebke CS 432 Spring 2002 Slides

Sorting

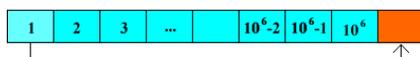
- Heap starts 32 bit before the 1st integer



- Max - Ordered Heap



- Building Sorted List

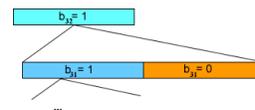


Bit Sort

- Sort bit by compare 8 bits with 01111111_2

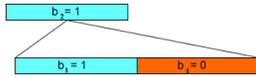


- Upper-to-Bottom bit sort

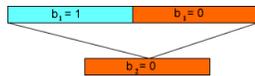


Continue

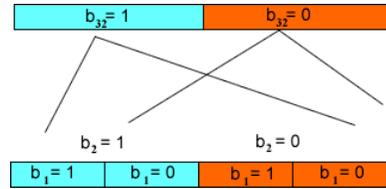
- Sort till the last bit



- Bottom-up



Memory

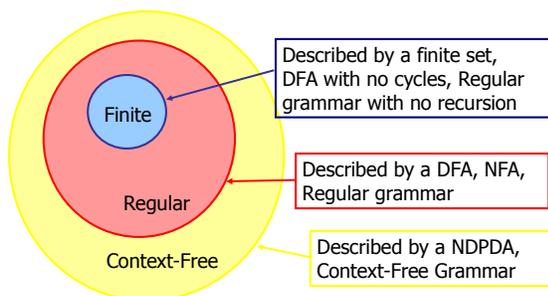


Compare

- Heap Sort
 - Requires $\sim n \log_2 n$ comparisons
 - 4 bytes to store integer
- Bit Sort:
 - Requires $\sim 32n$ comparisons
 - Memory to store (up to 33) memory addresses that separate the 1s and 0s

Challenge Recap

Language Classes



Context-Free Grammar

$$A \rightarrow \alpha\beta$$

One variable Any sequence of variables and terminals

Why is it called "Context-Free"?

Example 1

$$\begin{aligned} S &\rightarrow 0S0 \\ S &\rightarrow 1S1 \\ S &\rightarrow \varepsilon \end{aligned}$$

Example 2

Define a CFG that generates the language:

$$\{ w \mid w \in \{0, 1\}^* \text{ and } w \text{ has an equal number of 0s and 1s} \}$$

Review Questions

- How do you prove a *grammar* is context-free?
- How do you prove a *grammar* is not context-free?
- How do you prove a *language* is context-free?
- How do you prove a *language* is not context-free?

Charge

- Return PS2 and PS2 Comments now
- PS3 is posted on the course site (Due in 1 week, Feb 19)
- Thursday:
 - Equivalence of NPDAs and CFGs (sketch)
 - Non-Context-Free Languages