# Binary Arithmetic

CS 2130: Computer Systems and Organization 1

August 31, 2022

# Announcements

- Quiz 0 due Friday at 5pm (when Quiz 1 opens)
- TA office hours start tonight!
  - **In-person**: Olsson 001, Wed-Sun, 5-7pm
  - **Online**: Discord, Wed-Sun, varies
  - Office hour page has been updated
- My office hours start Thursday!
  - Tuesday, 4-5pm, Discord/Zoom
  - Wednesday, 4:30-6pm, Rice 210 (masks requested)
  - Thursday, 11am-12pm, Discord/Zoom
- Lab 1 late check-off through Monday
- Covid-19 make-up policies: stay home, check-off lab later

# Numbers

From our oldest cultures, how do we mark numbers?

- Arabic numerals
  - Positional numbering system

1000s 100s 10s 1s

2130

$2 \cdot 1000 + 1 \cdot 100 + 3 \cdot 10 + 0 \cdot 1$

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- Arabic numerals
    - Positional numbering system
    - The `10` is significant:
        - 10 symbols, using 10 as base of exponent
    - The `10` is *arbitrary*
    - We can use other bases! $\pi$, 2130, 2, ...

# Base-8 Example

$64 \quad 8 \quad 8$

$8^2 \quad 8^1 \quad 8^0$

Try to turn $134_8$ into base-10:

$1 \cdot 8^2 + 3 \cdot 8^1 + 4 \cdot 8^0$

$1 \cdot 64 + 3 \cdot 8 + 4 = 92_{10}$

$24$

0
1
2
3
4
5
6
7

# Bases

We will discuss a few in this class

- Base-10 (decimal) - talking to humans
- Base-8 (octal) - shows up occasionally
- Base-2 (binary) - most important! (we've been discussing 2 things!)
- Base-16 (hexadecimal) - nice grouping of bits

# Binary

2 digits: 0, 1

64 32 16 8 4 2 1

$2^6$ $2^5$ $2^4$ $2^3$ $2^2$ $2^1$ $2^0$

Try to turn $1100101_2$ into base-10:

$64 + 32 + 4 + 1 = 101$

# Binary

Any downsides to binary?

Turn $2130_{10}$ into base-2:

*hint: find largest power of 2 and subtract*

$$2^{11}$$

$$100001010010_2$$

$$\begin{array}{r} 2130 \\ -\,2048 \\ \hline 0082 \end{array} \quad = 2^{11}$$

$$\begin{array}{r} 82 \\ -\,64 \\ \hline 18 \\ -\,16 \\ \hline 2 \\ -\,2 \\ \hline 0 \end{array}$$

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How do we deal with numbers too long to read?

- Group them by 3 (right to left)
- In decimal, use commas: **,**
- Numbers between commas: 000 - 999
- Effectively base-1000

# Long Numbers in Binary

Making binary more readable

- Typical to group by 3 or 4 bits
- No need for commas *Why?*

100001010010

# Long Numbers in Binary

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- Typical to group by 3 or 4 bits
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- We can use a separate symbol per group
- How many do we need for groups of 3?

$2^2 \; 2^1 \; 2^0$

$010 = 2$

$4 = 2^2 \; 2^1 \; 2^0$

$100 = 4$

4 1 2 2

1000001010010

$2^3 = 8$

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Making binary more readable

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- How many do we need for groups of 3?
- Turn each group into decimal representation

100001010010

# Long Numbers in Binary

Making binary more readable

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- No need for commas *Why?*
- We can use a separate symbol per group
- How many do we need for groups of 3?
- Turn each group into decimal representation
- Converts binary to **octal**

100001010010

$4\ 1\ 2\ 2_8$

# Long Numbers in Binary

0000 → 1111    0 → 15

Making binary more readable

- Groups of 4 more common
- How many symbols do we need for groups of 4? 16

100001010010

# Long Numbers in Binary

Making binary more readable

- Groups of 4 more common
- How many symbols do we need for groups of 4? 16
- Converts binary to **hexadecimal**
- Base-16 is very common in computing

0 – 9

8 4 2 1 8 4 2 1

100001010010

8 5 $2_{16}$

# Hexadecimal

Need more than 10 digits. What next?

8+4+2=14

1110 = $e_{16}$

0
1
2
3
4
5
6
7
8
9
a = 10
b = 11
c = 12
d = 13
e = 14
f = 15

# Hexadecimal Exercise

Consider the following hexadecimal number:

852dab1e

Is it even or odd?

$11 \cdot 16^2 + 1 \cdot 16^1 + 14 \cdot 16^0$

# Using Different Bases in Code

| | Old Languages | New Languages |
|---|---|---|
| binary | no way | 0b1010001 |
| octal | 0273 | 0o273 |
| decimal | 2130 | 2130 |
| hexadecimal | 0x42a | 0x42a |

~~08~~

# Negative Integers

Representing negative integers

−21

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Representing negative integers

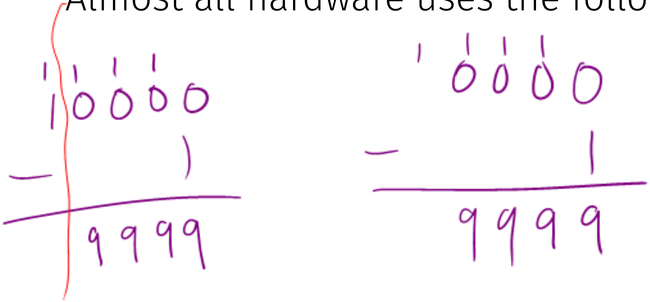
- Can we use the minus sign?

−100101

# Negative Integers

Representing negative integers

- Can we use the minus sign?
- In binary we only have 2 symbols, must do something else!
- Almost all hardware uses the following observation:



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- Computers store numbers in fixed number of wires
- Ex: consider 4-digit decimal numbers

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- Throw away the last borrow:
  - 0000 - 0001 = 9999
  - 9999 - 0001 = 9998
  - Normal subtraction/addition still works

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- Computers store numbers in fixed number of wires
- Ex: consider 4-digit decimal numbers
- Throw away the last borrow:
  - 0000 - 0001 = 9999
  - 9999 - 0001 = 9998
  - Normal subtraction/addition still works
- This works the same in binary

# Two's Complement

This scheme is called **Two's Complement**

- More generically, a *signed* integer
- There is a break as far away from 0 as possible
- First bit acts vaguely like a minus sign
- Works as long as we do not pass number too large to represent