Boolean Algebra

CS 2130: Computer Systems and Organization 1

January 20, 2023

Announcements

If you need to switch labs:

- Form will be coming soon
- Must be justified (i.e. class conflicts)
- **Very** limited space to make swaps

Quiz 0 opens tonight, due Sunday 11:59pm

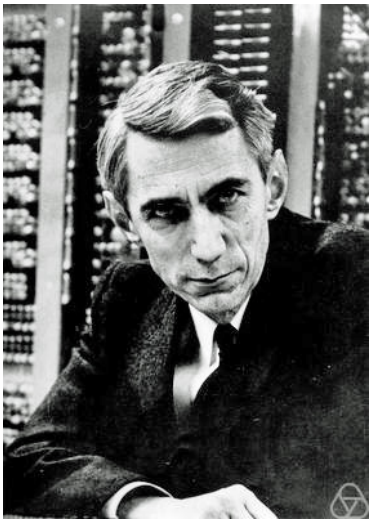
Where to start?

Where to start?

0 and 1

Why only 0 and 1?

Claude Shannon



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Why only 0 and 1?

Vocabulary

- **bit** - either a 0 or 1
- **binary** - a system that has only two positions
- **trinary** - a system that has only three positions
- **quadrinary** - a system that has only four positions
- ...

Vocabulary

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- ...
- **decinary** - ...

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- ...
- decinary - ...
- **decimal** - system that has ten positions

Boolean Algebra

George Boole

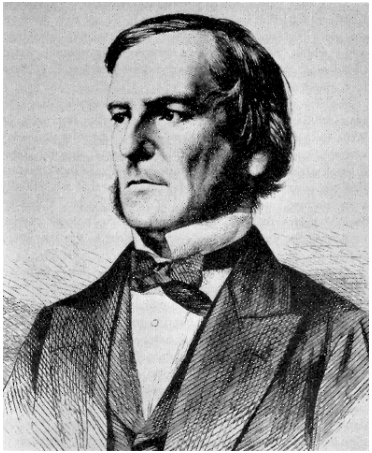

Photo Public Domain

Putting it together

Overall idea:

- Only need two things (Shannon)
- We can do math with two things (Boole)

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Now we need a physical device that deals in two levels

Transistors

More Vocabulary

Electricity (conceptually) - involves flow of electrons or other charged carriers through a conductive material

- **current** - rate of flow
- **voltage** - pressure of flow

Examples in water

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- High pressure, low flow - squirt gun

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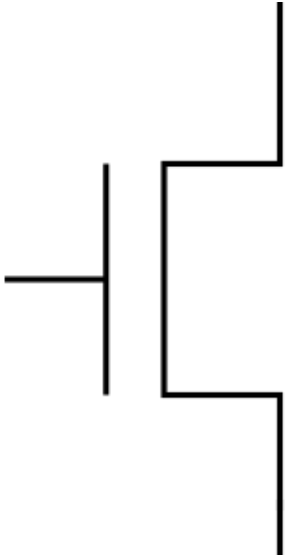
Examples in water

- High pressure, low flow - squirt gun
- Low pressure, high flow - bucket of water

Transistors



Transistors

Transistors act like an electrically-triggered switch

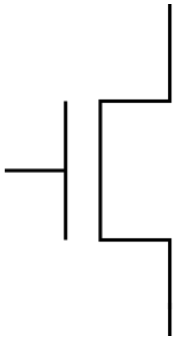
- No voltage, no current
- Apply voltage to allow current to flow

Transistors

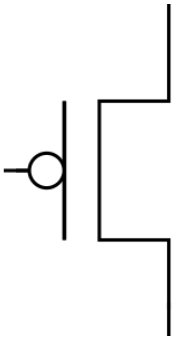
Transistors act like an electrically-triggered switch

- No voltage, no current
- Apply voltage to allow current to flow
- The amount of voltage needed to open the gate is boundary between 0 and 1
- Central technique for how we are going to build binary computers

Transistors

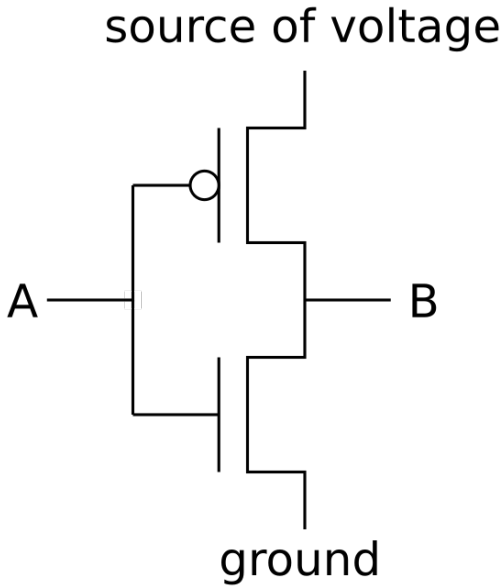


push to open



push to close

Circuit Diagram



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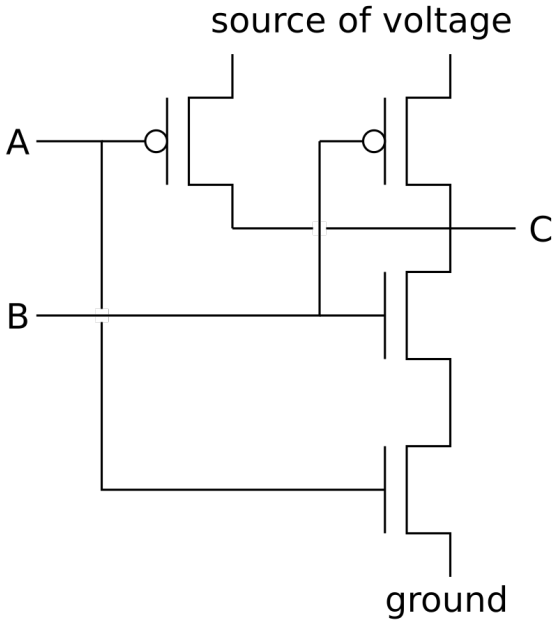

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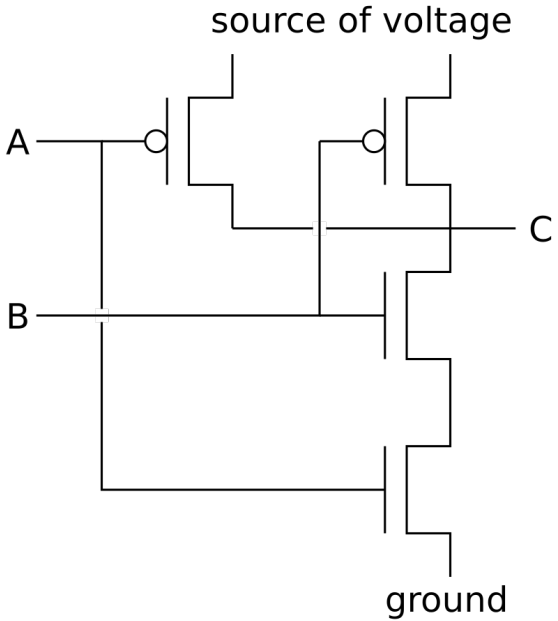

Other Gates (reading)

Building Up

Where we are now

- World with only 2 states: 0 and 1
- Re-developed Boolean logic: and, or, not

Gives us everything Boole talked about

- We can do a lot of interesting things!
- Next: build higher level ideas: the **trinary operator**

Trinary Operator

General idea

```
if ( ... ) {
    ...
} else {
    ...
}
```

Trinary operator (expression if)

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Trinary operator (expression if)

- Python: `x = b if a else c`

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if ( ... ) {
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} else {
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Trinary operator (expression if)

- Python: `x = b if a else c`
- Java: `x = a ? b : c`

Multiplexer (mux)

```
x = a ? b : c
```

Multiplexer (mux)

How can we build a mux out of what we have learned so far?

```
x = a ? b : c
```

Multiplexer (mux)

Can be built from `and`, `or`, and `not`

- Can be built using transistors
- Can physically put it in silicon!

Mux will be the key when constructing a computer out of gates and circuits!

Questions?

More bits!

2-bit Multiplexer (mux)

2-bit values instead of 1-bit values