# Logic Gates, Mux, Binary Arithmetic

CS 2130: Computer Systems and Organization 1

January 23, 2023

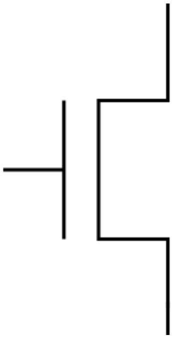
# Announcements

If you need to switch labs:

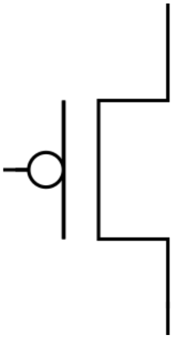
- Please fill out the form today!
- Must be justified (i.e. class conflicts)
- **Very** limited space to make swaps

Lab 1 tomorrow!

# Transistors

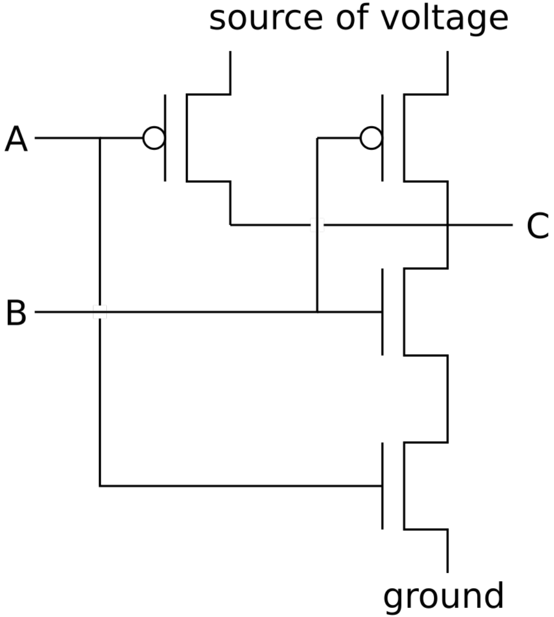


push to open



push to close

# Wiring Diagram



# So far...

Last time, we built up to logic gates:

- and, or, not
- nand, nor, xor

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Last time, we built up to logic gates:

- `and`, `or`, `not`
- `nand`, `nor`, `xor`

Now let's build something powerful

# Trinary Operator

Trinary operator

*this will be key when we are constructing our computer out of these gates and circuits*

# Trinary Operator

Trinary operator

- General idea:

```
if ( ... ) {
    ...
} else {
    ...
}
```

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- Python: `x = b if a else c`

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# Trinary Operator

## Trinary operator

- General idea:

```
if ( ... ) {
    ...
} else {
    ...
}
```

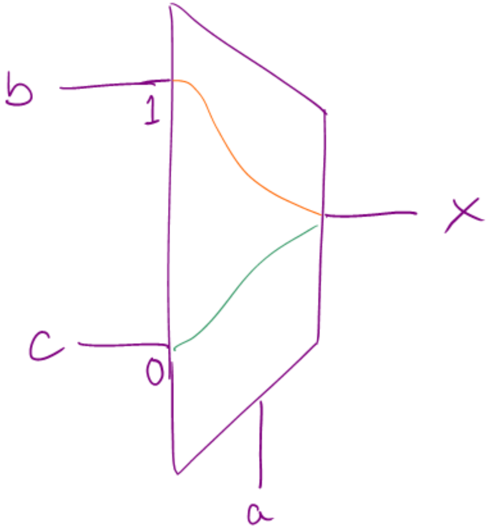
- Python: `x = b if a else c`
- Java: `x = a ? b : c`

| a | b | c | x |
|---|---|---|---|
| 0 | | | c |
| 1 | | | b |

*this will be key when we are constructing our computer out of these gates and circuits*

# Multiplexer (mux)

```
x = a ? b : c
```



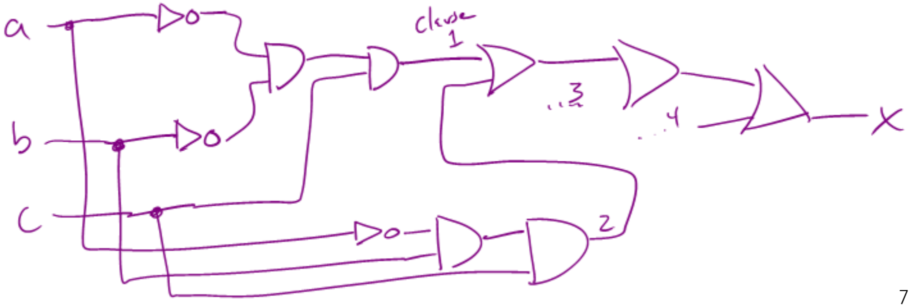
# Multiplexer (mux)

How can we build a mux out of what we have learned so far?

```
x = a ? b : c
```

| a | b | c | x |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$$x = (!a \& !b \& c) \mid (!a \& b \& c) \mid (a \& b \& !c) \mid (a \& b \& c)$$



# Multiplexer (mux)

Can be built from `and`, `or`, and `not`

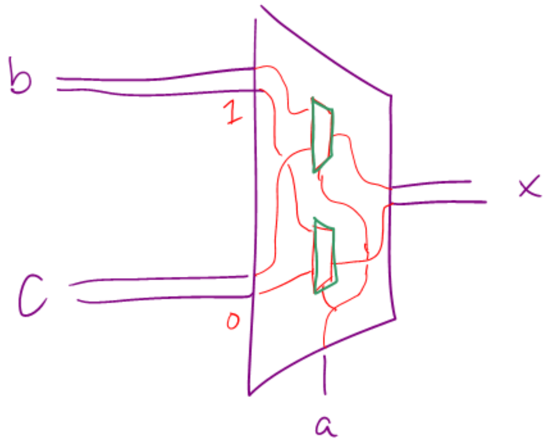
- Can be built using transistors
- Can physically put it in silicon!

# Questions?

# More bits!

# 2-bit Multiplexer (mux)

2-bit values instead of 1-bit values



# Multi-bit Values

- So far, only talking about 2 things
- Numbers, strings, objects, …

# Numbers

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- **unary** representation: make marks, one per ”thing”
  - Awkward for large numbers, ex: CS 2130?
  - Hard to tell how many marks there are
- Update: group them! 
- Romans used new symbols: V L C D M

# Numbers

From our oldest cultures, how do we mark numbers?

- Arabic numerals
  - Positional numbering system

0 1 2 3 4 5 6 7 8 9

| 10000 | 1000 | 100 | 10 | 1 |
|---|---|---|---|---|
| | 2 | 1 | 3 | 0 |
| $10^4$ | $10^3$ | $10^2$ | $10^1$ | $10^0$ |

$$2 \times 1000 + 1 \times 100 + 3 \times 10 + 0 \times 1$$

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- Arabic numerals
  - Positional numbering system
  - The `10` is significant:
    - 10 symbols, using 10 as base of exponent
  - The `10` is *arbitrary*
  - We can use other bases! $\pi$, 2130, 2, ...

# Base-8 Example

0 ... 7

Try to turn $134_8$ into base-10:

| ... | | $8^4$ | $8^3$ | $8^2$ | $8^1$ | $8^0$ |
|---|---|---|---|---|---|---|
| | | | | 1 | 3 | 4 |
| | | | | 64 | 8 | 1 |

$$1 \times 64 + 3 \times 8 + 4 \times 1 = 92_{10}$$

# Bases

We will discuss a few in this class

- Base-10 (decimal) - talking to humans
- Base-8 (octal) - shows up occasionally
- Base-2 (binary) - most important! (we've been discussing 2 things!)
- Base-16 (hexadecimal) - nice grouping of bits

# Binary

2 digits: 0, 1

Try to turn $1100101_2$ into base-10:

4096

2048

$2^6$ $2^5$ $2^4$ $2^3$ $2^2$ $2^1$ $2^0$

1024 512 256 128 64 32 16 8 4 2 1

$101_{10}$

64 + 32 + 4 + 1

# Binary

Any downsides to binary?

Turn $2130_{10}$ into base-2:

*hint: find largest power of 2 and subtract*

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- Group them by 3 (right to left)
- In decimal, use commas: ,
- Numbers between commas: 000 - 999
- Effectively base-1000

# Long Numbers in Binary

Making binary more readable

- Typical to group by 3 or 4 bits
- No need for commas *Why?*

100001010010

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- How many do we need for groups of 3?

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- Turn each group into decimal representation

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Making binary more readable

- Typical to group by 3 or 4 bits
- No need for commas *Why?*
- We can use a separate symbol per group
- How many do we need for groups of 3?
- Turn each group into decimal representation
- Converts binary to **octal**

100001010010

# Long Numbers in Binary

Making binary more readable

- Groups of 4 more common
- How many symbols do we need for groups of 4?

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# Long Numbers in Binary

Making binary more readable

- Groups of 4 more common
- How many symbols do we need for groups of 4?
- Converts binary to **hexadecimal**
- Base-16 is very common in computing

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# Hexadecimal

Need more than 10 digits. What next?

1110

# Hexadecimal Exercise

Consider the following hexadecimal number:

852dab1e

Is it even or odd?

# Using Different Bases in Code

| | Old Languages | New Languages |
|---|---|---|
| binary | | |
| octal | | |
| decimal | | |
| hexadecimal | | |