Logic Gates, Mux, Binary Arithmetic

CS 2130: Computer Systems and Organization 1

January 23, 2023

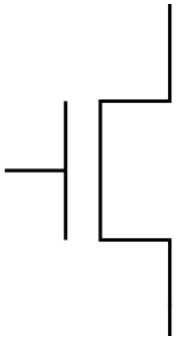
Announcements

If you need to switch labs:

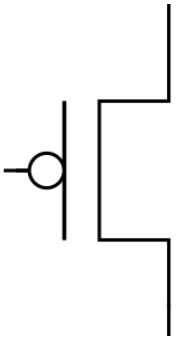
- Please fill out the form today!
- Must be justified (i.e. class conflicts)
- **Very** limited space to make swaps

Lab 1 tomorrow!

Transistors

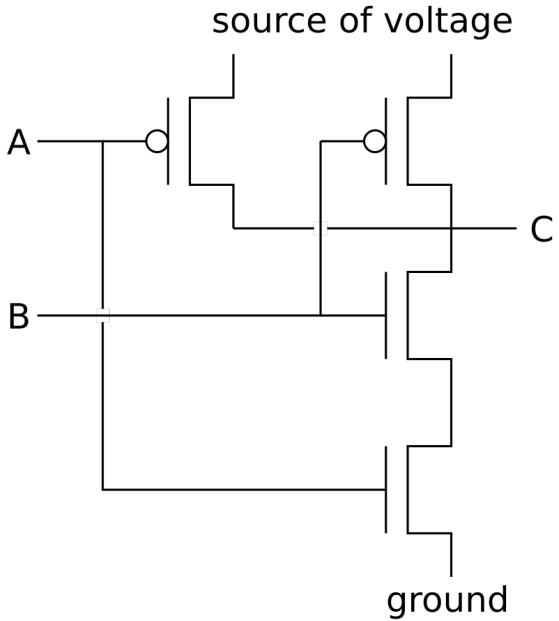


push to open



push to close

Wiring Diagram



So far...

Last time, we built up to logic gates:

- `and`, `or`, `not`
- `nand`, `nor`, `xor`

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Now let's build something powerful

Trinary Operator

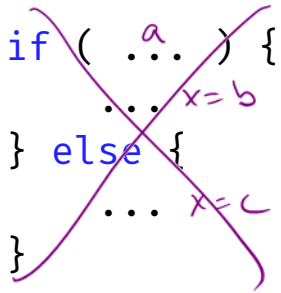
Trinary operator

this will be key when we are constructing our computer out of these gates and circuits

Trinary Operator

Trinary operator

- General idea:



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Trinary Operator

Trinary operator

- General idea:

```
if ( ... ) {
    ...
} else {
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}
```

- Python: `x = b if a else c`

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Trinary Operator

Trinary operator

- General idea:

```
if ( ... ) {
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} else {
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```
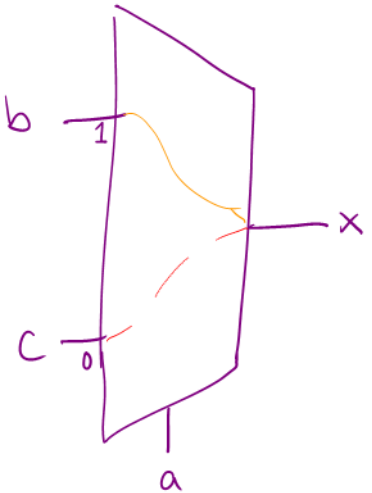

- Python: `x = b if a else c`
- Java: `x = a ? b : c`

this will be key when we are constructing our computer out of these gates and circuits

Multiplexer (mux)

```
x = a ? b : c
```



a	b	c	x
0			c
1			b

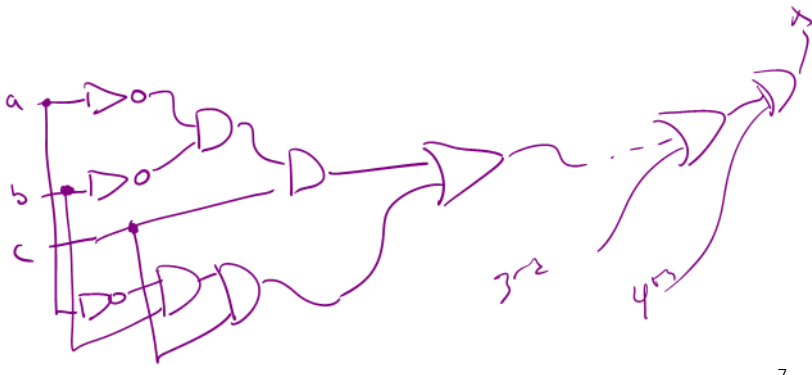
Multiplexer (mux)

How can we build a mux out of what we have learned so far?

```
x = a ? b : c
```

a	b	c	x
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$x = (!a \,\&\, !b \,\&\, c) \mid (!a \,\&\, b \,\&\, c) \mid (a \,\&\, b \,\&\, !c) \mid (a \,\&\, b \,\&\, c)$



Multiplexer (mux)

Can be built from `and`, `or`, and `not`

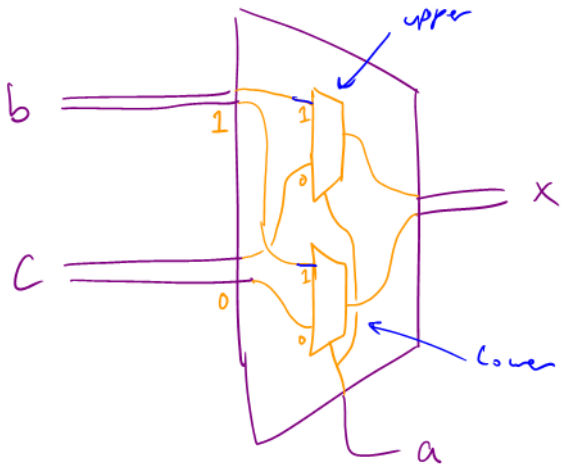
- Can be built using transistors
- Can physically put it in silicon!

Questions?

More bits!

2-bit Multiplexer (mux)

2-bit values instead of 1-bit values



Multi-bit Values

- So far, only talking about 2 things
- Numbers, strings, objects, ...

Numbers

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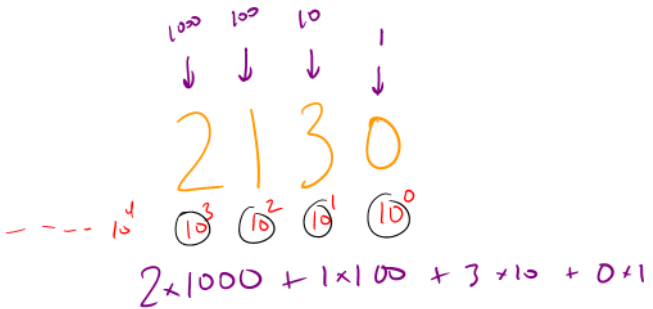
- **unary** representation: make marks, one per ”thing”
  - Awkward for large numbers, ex: CS 2130?
  - Hard to tell how many marks there are
- Update: group them!
- Romans used new symbols: V L C D M

Numbers

From our oldest cultures, how do we mark numbers?

- Arabic numerals
  - Positional numbering system



0
1
2
3
4
5
6
7
8
9

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Numbers

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- Arabic numerals
  - Positional numbering system
  - The `10` is significant:
    - 10 symbols, using 10 as base of exponent
  - The `10` is *arbitrary*
  - We can use other bases! $\pi, 2130, \underline{2}, \ldots$

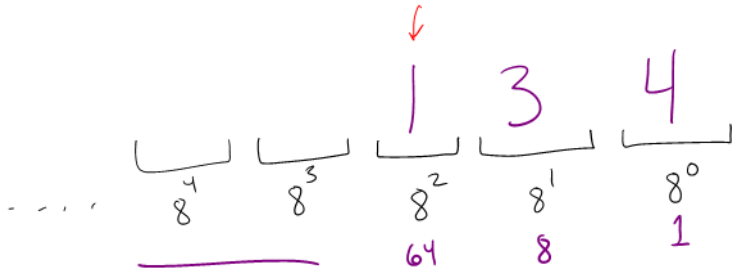
Base-8 Example

0 ... 7

$92_{10} \rightarrow$ 1 _ 8

-64

Try to turn 134_8 into base-10:



$1 \times 64_{10} + 3 \times 8 + 4 \times 1 = 92_{10}$

Bases

We will discuss a few in this class

- Base-10 (decimal) - talking to humans
- Base-8 (octal) - shows up occasionally
- Base-2 (binary) - most important! (we've been discussing 2 things!)
- Base-16 (hexadecimal) - nice grouping of bits

Binary

2 digits: 0, 1

Try to turn 1100101_2 into base-10:

2^9 2^8 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

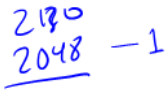
512 256 128 64 32 16 8 4 2 1

$64 + 32 + 4 + 1 = 101_{10}$

Binary

Any downsides to binary?

Turn 2130_{10} into base-2:
hint: find largest power of 2 and subtract



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- Group them by 3 (right to left)
- In decimal, use commas: **,**
- Numbers between commas: 000 - 999
- Effectively base-1000

Long Numbers in Binary

Making binary more readable

- Typical to group by 3 or 4 bits
- No need for commas *Why?*

100001010010

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- Turn each group into decimal representation

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- Typical to group by 3 or 4 bits
- No need for commas *Why?*
- We can use a separate symbol per group
- How many do we need for groups of 3?
- Turn each group into decimal representation
- Converts binary to **octal**

100001010010

Long Numbers in Binary

Making binary more readable

- Groups of 4 more common
- How many symbols do we need for groups of 4?

100001010010

Long Numbers in Binary

Making binary more readable

- Groups of 4 more common
- How many symbols do we need for groups of 4?
- Converts binary to **hexadecimal**
- Base-16 is very common in computing

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Hexadecimal

Need more than 10 digits. What next?

1110

Hexadecimal Exercise

Consider the following hexadecimal number:

852dab1e

Is it even or odd?

Using Different Bases in Code

	Old Languages	New Languages
binary		
octal		
decimal		
hexadecimal		