Bitwise Operations, Floating Point Numbers

CS 2130: Computer Systems and Organization 1

January 27, 2023

Announcements

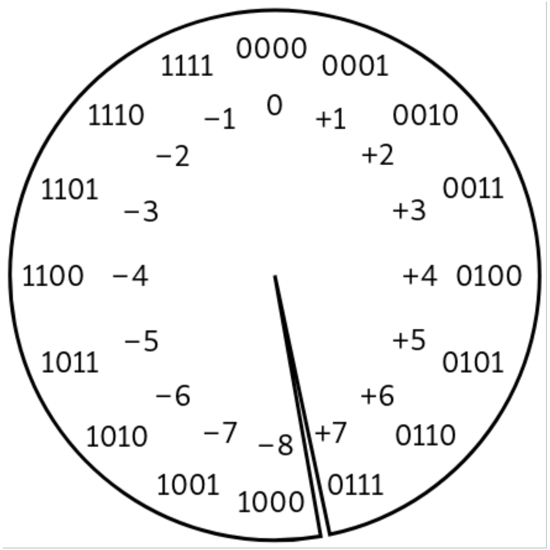
- TA Office Hours starting very soon
- Discord link coming this afternoon
- Quiz 1 opens this afternoon, due Sunday night
- Homework 1 due Feb 6 (Mon)

Two’s Complement

Two's Complement

This scheme is called **Two's Complement**

- More generically, a *signed* integer
- There is a break as far away from 0 as possible
- First bit acts vaguely like a minus sign
- Works as long as we do not pass number too large to represent



Two’s Complement

Values of Two's Complement Numbers

Consider the following 8-bit binary number in Two's Complement:

$$11010011$$

What is its value in decimal?

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1. Flip all bits
2. Add 1

Operations

So far, we have discussed:

- Addition: `x + y`
  - Can get multiplication
- Subtraction: `x - y`
  - Can get division, but more difficult
- Unary minus (negative): `-x`
  - Flip the bits and add 1

Operations (on Integers)

Bit vector: fixed-length sequence of bits (ex: bits in an integer)

- Manipulated by bitwise operations

Bitwise operations: operate over the bits in a bit vector

- Bitwise not: `~x` - flips all bits (unary)
- Bitwise and: `x & y` - set bit to 1 if *x*, *y* have 1 in same bit
- Bitwise or: `x | y` - set bit to 1 if either *x* or *y* have 1
- Bitwise xor: `x ^ y` - set bit to 1 if *x*, *y* bit differs

Example: Bitwise AND

```
  11001010
& 01111100
----------
```

Example: Bitwise OR

```
  11001010
| 01111100
```

Example: Bitwise XOR

```
  11001010
^ 01111100
----------
```

Your Turn!

What is: `0x1a ^ 0x72`

Operations (on Integers)

- Logical not: `!x`
  - $!0 = 1$ and $!x = 0, \forall x \neq 0$
  - Useful in C, no booleans
  - Some languages name this one differently
- Left shift: `x << y` - move bits to the left
  - Effectively multiply by powers of 2
- Right shift: `x >> y` - move bits to the right
  - Effectively divide by powers of 2
  - Signed (extend sign bit) vs unsigned (extend 0)

Left Bit-shift Example

`01011010 << 2`

Right Bit-shift Example

```
01011010 >> 3
```

Bit-shift

Computing bit-shift effectively multiplies/divides by powers of 2

Consider decimal:

$$\texttt{2130} \ll_{10} \texttt{2} = \texttt{213000} = \texttt{2130} \times \texttt{100}$$

$$\texttt{2130} \gg_{10} \texttt{1} = \texttt{213} = \texttt{2130} \ / \ \texttt{10}$$

Right Bit-shift Example 2

```
11001010 >> 1
```

Right Bit-shift Example 2

For **signed** integers, extend the sign bit (1)

- Keeps negative value (if applicable)
- Approximates divide by powers of 2

```
11001010 >> 1
```

Bit fiddling example

What about other kinds of numbers?

Non-Integer Numbers

Floating point numbers

- Decimal: 3.14159

Non-Integer Numbers

Floating point numbers

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- Binary: 11.10110

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- With floating point numbers, the point can move!

Non-Integer Numbers

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- With integers, the point is always fixed after all digits
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Challenge! only 2 symbols in binary

Scientific Notation

Convert the following decimal to scientific notation:

$$2130$$

Scientific Notation

Convert the following binary to scientific notation:

101101

Something to Notice

An interesting phenomenon:

- Decimal: first digit can be any number *except* 0

$$2.13 \times 10^3$$

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$$1.01101 \times 2^5$$

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  - First digit can only be 1

Floating Point in Binary

We must store 3 components

- **sign** (1-bit): 1 if negative, 0 if positive
- **fraction** or **mantissa**: (?-bits): bits after binary point
- **exponent** (?-bits): how far to move binary point

We do not need to store the value before the binary point. Why?

Floating Point in Binary

How do we store them?

- Originally many different systems
- IEEE standardized system (IEEE 754 and IEEE 854)
- Agreed-upon order, format, and number of bits for each

$$1.01101 \times 2^5$$

Example

A rough example in Decimal:

$$6.42 \times 10^3$$

Exponent

How do we store the exponent?

- Exponents *can* be negative

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

- Need positive and negative ints (but no minus sign)

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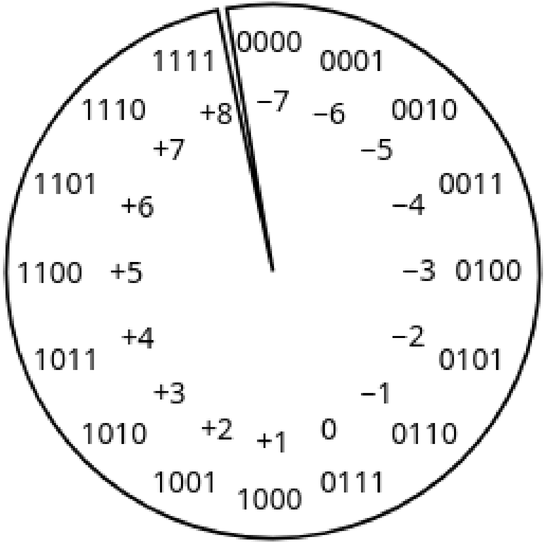
- Biased integers
    - Make comparison operations run more smoothly
    - Hardware more efficient to build
    - Other valid reasons

Biased Integers

Similar to Two's Complement, but add **bias**

- **Two's Complement**: Define 0 as 00...0
- **Biased**: Define 0 as 0111...1
- Biased wraps from 000...0 to 111...1



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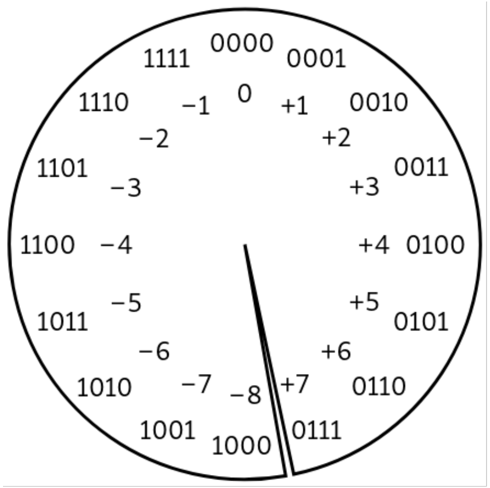
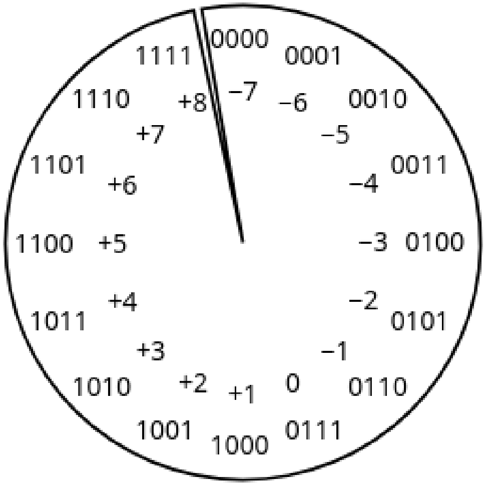


Two’s Complement



Biased

Biased Integers Example

Calculate value of biased integers (4-bit example)

0010

Biased Integers

Floating Point Example

$$101.011_2$$

Floating Point Example

$$101.011_2$$

Floating Point Example

What does the following encode?

1	001110	1010101

Floating Point Example

What does the following encode?

$\boxed{1}\ \boxed{001110}\ \boxed{1010101}$

What about 0?

Floating Point Numbers

Four cases:

- **Normalized**: What we have seen today

$$s\ eeee\ ffff = \pm 1.ffff \times 2^{eeee - \mathrm{bias}}$$

- **Denormalized**: Exponent bits all 0

$$s\ eeee\ ffff = \pm 0.ffff \times 2^{1 - \mathrm{bias}}$$

- **Infinity**: Exponent bits all 1, fraction bits all 0 (i.e., $\pm\infty$)
- **Not a Number (NaN)**: Exponent bits all 1, fraction bits not all 0