Floating Point Numbers

CS 2130: Computer Systems and Organization 1

January 30, 2023

Announcements

- TA Office Hours starting Wednesday
  - Wednesdays, Rice 011
  - Thurs-Sun, Olsson 001
- Please join our Discord server
- Lab tomorrow: hex editor
- Homework 1 due Feb 6 (Mon)

Operations

So far, we have discussed:

- Addition: `x + y`
  - Can get multiplication
- Subtraction: `x - y`
  - Can get division, but more difficult
- Unary minus (negative): `-x`
  - Flip the bits and add 1

Operations (on Integers)

Bit vector: fixed-length sequence of bits (ex: bits in an integer)

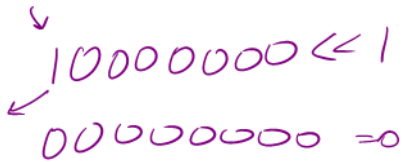
- Manipulated by bitwise operations

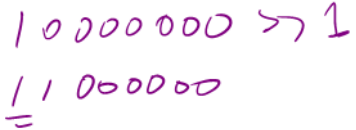
Bitwise operations: operate over the bits in a bit vector

- Bitwise not: `~x` - flips all bits (unary)
- Bitwise and: `x & y` - set bit to 1 if *x*, *y* have 1 in same bit
- Bitwise or: `x | y` - set bit to 1 if either *x* or *y* have 1
- Bitwise xor: `x ^ y` - set bit to 1 if *x*, *y* bit differs

Operations (on Integers)

- Logical not: `!x`
  - $!0 = 1$ and $!x = 0, \forall x \neq 0$
  - Useful in C, no booleans
  - Some languages name this one differently
- Left shift: `x << y` - move bits to the left
  - Effectively multiply by powers of 2
- Right shift: `x >> y` - move bits to the right
  - Effectively divide by powers of 2
  - Signed (extend sign bit) vs unsigned (extend 0)





What about other kinds of numbers?

Non-Integer Numbers

Floating point numbers

- Decimal: 3.14159

↑ decimal point

Non-Integer Numbers

Floating point numbers

- Decimal: 3.14159
- Binary: 11.10110

↑ binary point

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Challenge! only 2 symbols in binary

Scientific Notation

Convert the following decimal to scientific notation:

2130.

2.130×10^{3}

1000

Scientific Notation

Convert the following binary to scientific notation:

$$101101.$$

$$1.01101 \times 2^{5}$$

Something to Notice

An interesting phenomenon:

- Decimal: first digit can be any number *except* 0

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- Binary: first digit can be any number *except* 0 Wait!

$$-1.01101 \times 2^5$$

  - First digit can only be 1

Floating Point in Binary

We must store 3 components

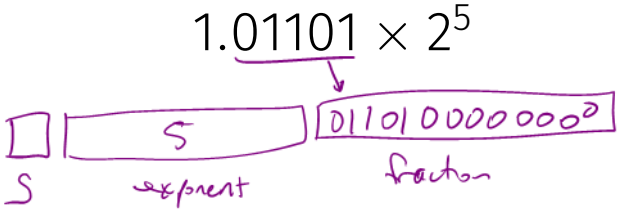
- **sign** (1-bit): 1 if negative, 0 if positive
- **fraction** or **mantissa**: (?-bits): bits after binary point
- **exponent** (?-bits): how far to move binary point

We do not need to store the value before the binary point. Why?

Floating Point in Binary

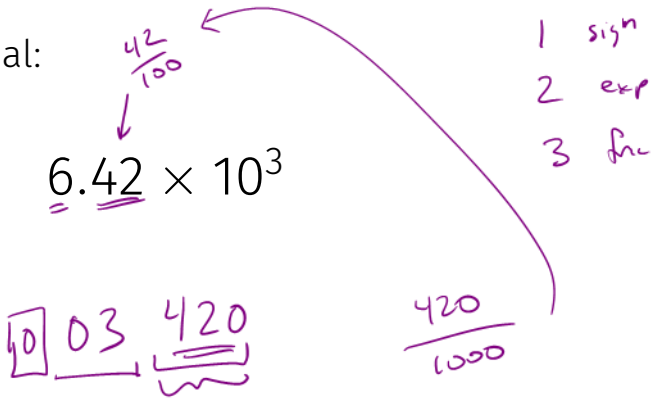
How do we store them?

- Originally many different systems
- IEEE standardized system (IEEE 754 and IEEE 854)
- Agreed-upon order, format, and number of bits for each



Example

A rough example in Decimal:



Exponent

How do we store the exponent?

- Exponents *can* be negative

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

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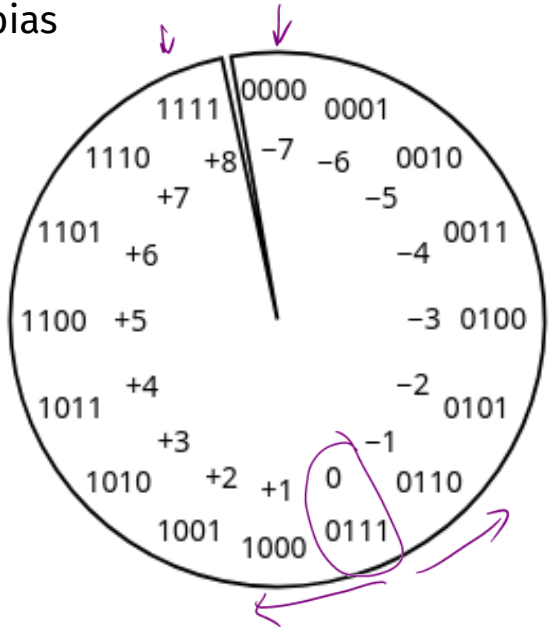
- Biased integers
  - Make comparison operations run more smoothly
  - Hardware more efficient to build
  - Other valid reasons

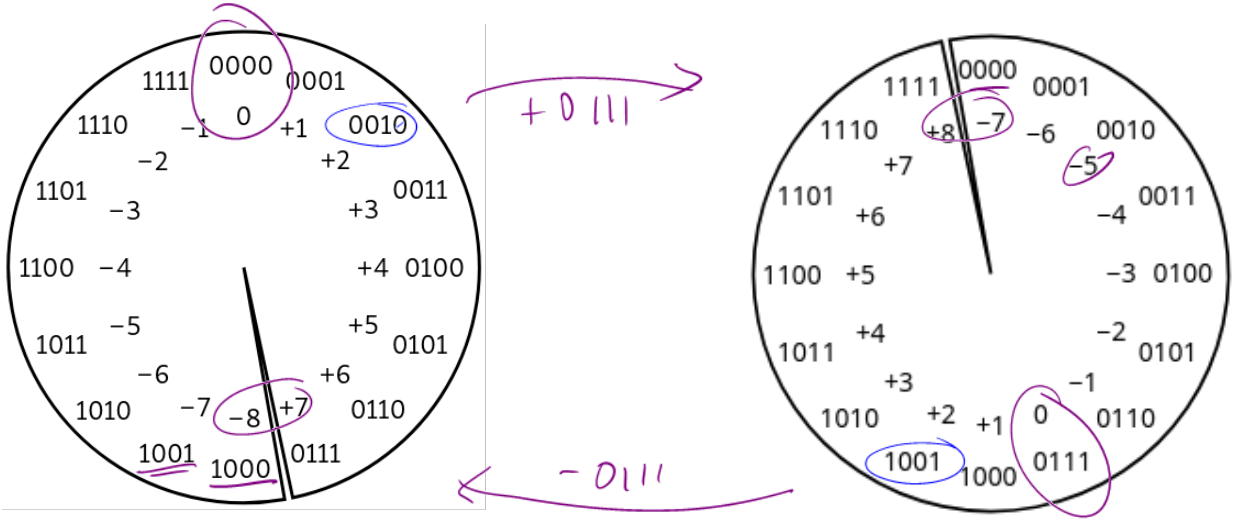
Biased Integers

Similar to Two's Complement, but add **bias**

- **Two's Complement**: Define 0 as 00...0
- **Biased**: Define 0 as 0111...1
- Biased wraps from 000...0 to 111...1



Biased Integers

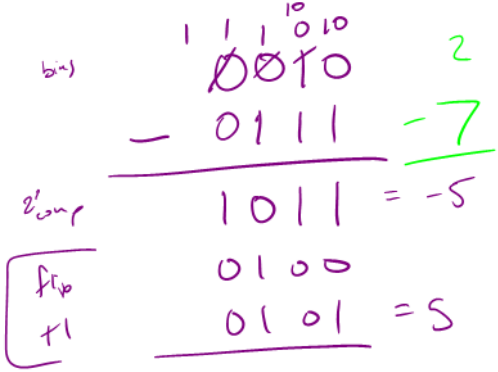


Two's Complement

Biased

Biased Integers Example

Calculate value of biased integers (4-bit example)



0010 = -5

Biased Integers

Floating Point Example

101.011

5 3/8 $2^3 = 8$

101.011

+ 101.011_2

1.01011×2^2

1 sign

4 exponent

3 fraction

exp 0010

+ 0111

1001

1	0	1	.	0	1	1
2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}
4	2	1		$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

$4 + 1 + \frac{1}{4} + \frac{1}{8} = 5\frac{3}{8}$

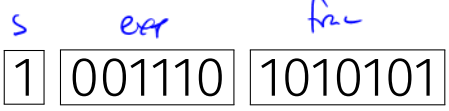
0	1001	011
sign	exponent	fraction

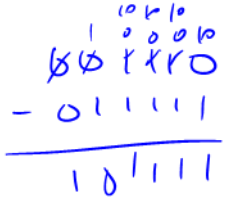
Floating Point Example

101.011_2

Floating Point Example

What does the following encode?





$$-1.1010101 \times 2^{-17}$$

$$-.00000000000000000111010101$$

Floating Point Example

What does the following encode?

What about 0?

Floating Point Numbers

Four cases:

- **Normalized**: What we have seen today

$$s\ eeee\ ffff = \pm 1.ffff \times 2^{eeee - \text{bias}}$$

- **Denormalized**: Exponent bits all 0

$$s\ eeee\ ffff = \pm 0.ffff \times 2^{1 - \text{bias}}$$

- **Infinity**: Exponent bits all 1, fraction bits all 0 (i.e., $\pm\infty$)
- **Not a Number (NaN)**: Exponent bits all 1, fraction bits not all 0