# Floating Point Numbers

CS 2130: Computer Systems and Organization 1

January 30, 2023

# Announcements

- TA Office Hours starting Wednesday
    - Wednesdays, Rice 011
    - Thurs-Sun, Olsson 001
- Please join our Discord server
- Lab tomorrow: hex editor
- Homework 1 due Feb 6 (Mon)

# Operations

So far, we have discussed:

- Addition: `x + y`
  - Can get multiplication
- Subtraction: `x - y`
  - Can get division, but more difficult
- Unary minus (negative): `-x`
  - Flip the bits and add 1

# Operations (on Integers)

Bit vector: fixed-length sequence of bits (ex: bits in an integer)

- Manipulated by bitwise operations

Bitwise operations: operate over the bits in a bit vector

- Bitwise not: `~x` - flips all bits (unary)
- Bitwise and: `x & y` - set bit to 1 if *x*, *y* have 1 in same bit
- Bitwise or: `x | y` - set bit to 1 if either *x* or *y* have 1
- Bitwise xor: `x ^ y` - set bit to 1 if *x*, *y* bit differs

# Operations (on Integers)

- Logical not: `!x`
  - $!0 = 1$ and $!x = 0, \forall x \neq 0$
  - Useful in C, no booleans
  - Some languages name this one differently
- Left shift: `x << y` - move bits to the left
  - Effectively multiply by powers of 2
- Right shift: `x >> y` - move bits to the right
  - Effectively divide by powers of 2
  - Signed (extend sign bit) vs unsigned (extend 0)

What about other kinds of numbers?

# Non-Integer Numbers

Floating point numbers

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Challenge! only 2 symbols in binary

# Scientific Notation

Convert the following decimal to scientific notation:

$$2130$$

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Convert the following binary to scientific notation:

101101

# Something to Notice

An interesting phenomenon:

- Decimal: first digit can be any number *except* 0

$$2.13 \times 10^3$$

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  - First digit can only be 1

# Floating Point in Binary

We must store 3 components

- **sign** (1-bit): 1 if negative, 0 if positive
- **fraction** or **mantissa**: (?-bits): bits after binary point
- **exponent** (?-bits): how far to move binary point

*We do not need to store the value before the binary point. Why?*

# Floating Point in Binary

How do we store them?

- Originally many different systems
- IEEE standardized system (IEEE 754 and IEEE 854)
- Agreed-upon order, format, and number of bits for each

$$1.01101 \times 2^5$$

# Example

A rough example in Decimal:

$$6.42 \times 10^3$$

# Exponent

How do we store the exponent?

- Exponents *can* be negative

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

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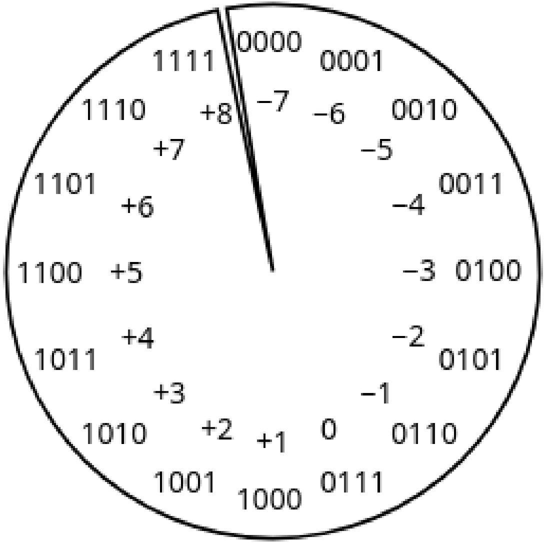
- Biased integers
  - Make comparison operations run more smoothly
  - Hardware more efficient to build
  - Other valid reasons

# Biased Integers

Similar to Two's Complement, but add **bias**

- **Two's Complement**: Define 0 as 00...0
- **Biased**: Define 0 as 0111...1
- Biased wraps from 000...0 to 111...1



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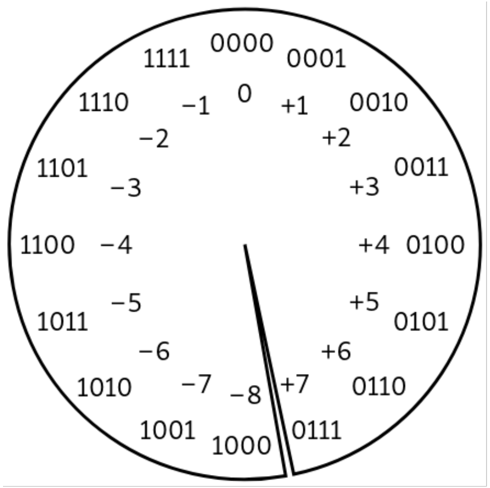
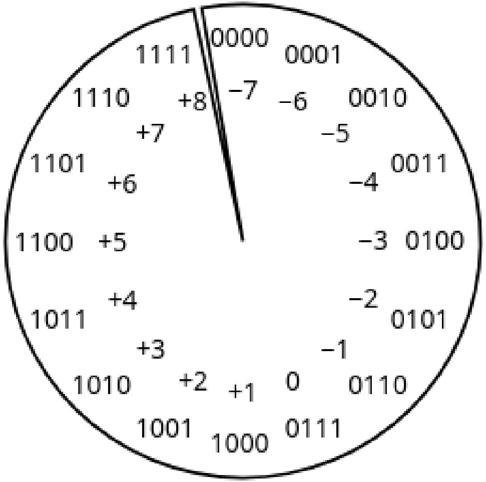



Two's Complement




Biased

# Biased Integers Example

Calculate value of biased integers (4-bit example)

0010

# Biased Integers

# Floating Point Example

$$101.011_2$$

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# Floating Point Example

What does the following encode?

| | | |
|---|---|---|
| 1 | 001110 | 1010101 |

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# What about 0?

# Floating Point Numbers

Four cases:

- **Normalized**: What we have seen today

$$s\ eeee\ ffff = \pm 1.ffff \times 2^{eeee-\mathrm{bias}}$$

- **Denormalized**: Exponent bits all 0

$$s\ eeee\ ffff = \pm 0.ffff \times 2^{1-\mathrm{bias}}$$

- **Infinity**: Exponent bits all 1, fraction bits all 0 (i.e., $\pm\infty$)
- **Not a Number (NaN)**: Exponent bits all 1, fraction bits not all 0