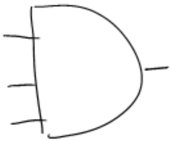
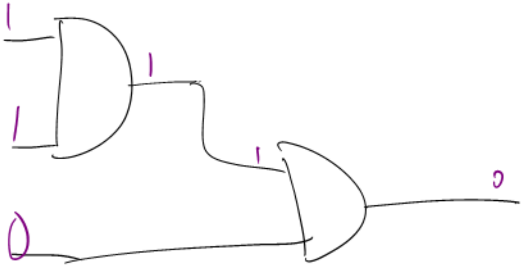
Warm up!

Can I make an *n*-input AND from 2-input AND gates?





Warm up!

What about XOR gates?

parity

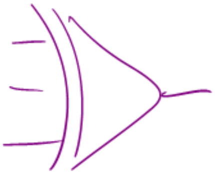
0

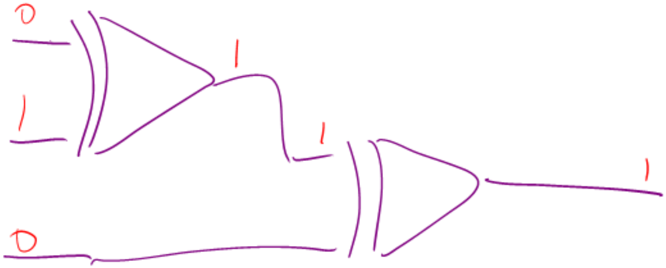
1

110 → 0

010 → 1

000 → 0





More bits, circuits, adders

CS 2130: Computer Systems and Organization 1

February 1, 2023

Announcements

- TA Office Hours start tonight!
  - Wednesdays, Rice 011
  - Thurs-Sun, Olsson 001
- Please join our Discord server
- Homework 1 due Monday

Quiz Review

$0x6 = 6$

$0x16 > 16$

16^1 10^1

22

$0xCA$ 1100 1010

$>> 3$ 11111001 = -7

~, +1 111 = 7

$<< 3$ 11001000

^ 0xCA ^11001010

00000010

$-a = \sim a + 1$

$a - a = 0$

$a + -a = 0$

$a + \sim a + 1 = 0$

$a + \sim a = -1$

Operations

So far, we have discussed:

- Addition: `x + y`
  - Can get multiplication
- Subtraction: `x - y`
  - Can get division, but more difficult
- Unary minus (negative): `-x`
  - Flip the bits and add 1

Operations (on Integers)

Bit vector: fixed-length sequence of bits (ex: bits in an integer)

- Manipulated by bitwise operations

Bitwise operations: operate over the bits in a bit vector

- Bitwise not: `~x` - flips all bits (unary)
- Bitwise and: `x & y` - set bit to 1 if *x*, *y* have 1 in same bit
- Bitwise or: `x | y` - set bit to 1 if either *x* or *y* have 1
- Bitwise xor: `x ^ y` - set bit to 1 if *x*, *y* bit differs

Operations (on Integers)

- Logical not: `!x`
  - $!0 = 1$ and $!x = 0, \forall x \neq 0$
  - Useful in C, no booleans
  - Some languages name this one differently
- Left shift: `x << y` - move bits to the left
  - Effectively multiply by powers of 2
- Right shift: `x >> y` - move bits to the right
  - Effectively divide by powers of 2
  - Signed (extend sign bit) vs unsigned (extend 0)

Floating Point Numbers

Four cases:

- **Normalized**: What we saw last time

$$s\ eeee\ ffff = \pm 1.ffff \times 2^{eeee-\mathrm{bias}}$$

- **Denormalized**: Exponent bits all 0

$$s\ eeee\ ffff = \pm 0.ffff \times 2^{1-\mathrm{bias}}$$

- **Infinity**: Exponent bits all 1, fraction bits all 0 (i.e., $\pm\infty$)
- **Not a Number (NaN)**: Exponent bits all 1, fraction bits not all 0

Our story so far

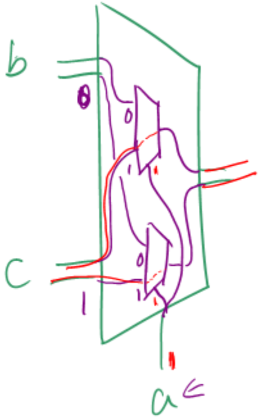
- Transistors
- Information modeled by voltage through wires (1 vs 0)
- Gates: & | ~ ^

- Multi-bit values: representing integers
  - Signed and unsigned
  - Bitwise operators on bit vectors
- Floating point

How to do the *work* of multi-bit?

Multi-bit Mux

Our first multi-bit example: mux



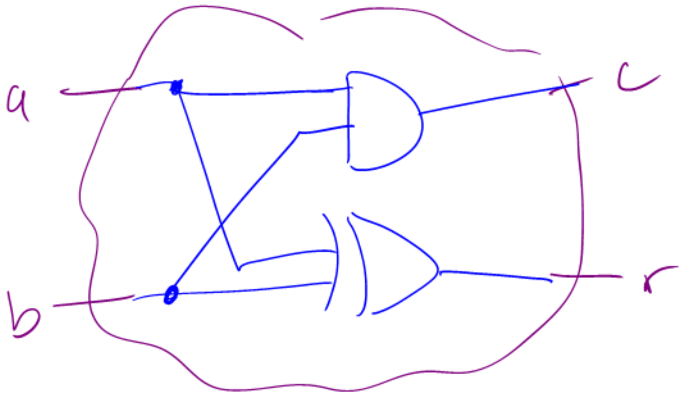
Adder

$1 + 1 = 10_2$

Add 2 1-bit numbers: a, b

$$\begin{array}{r} a \\ + \, b \\ \hline \overline{c}\ \overline{r} \end{array}$$

a	b	c	r
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



Adder

Can we use this in parallel to add multi-bit numbers?

Adder

Can we use this in parallel to add multi-bit numbers? What is missing?

Consider:

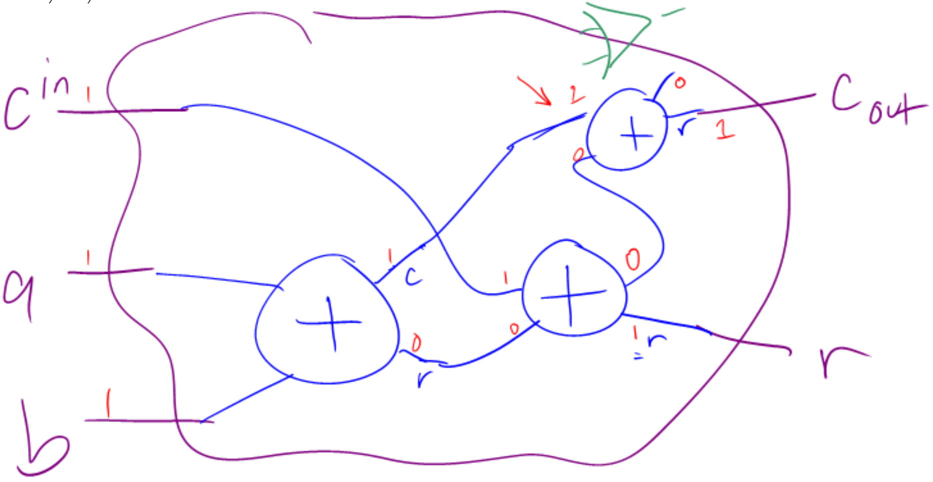
$$
\begin{array}{r}
1 \\
11 \\
+\,01 \\
\hline
0
\end{array}
$$

3-input Adder

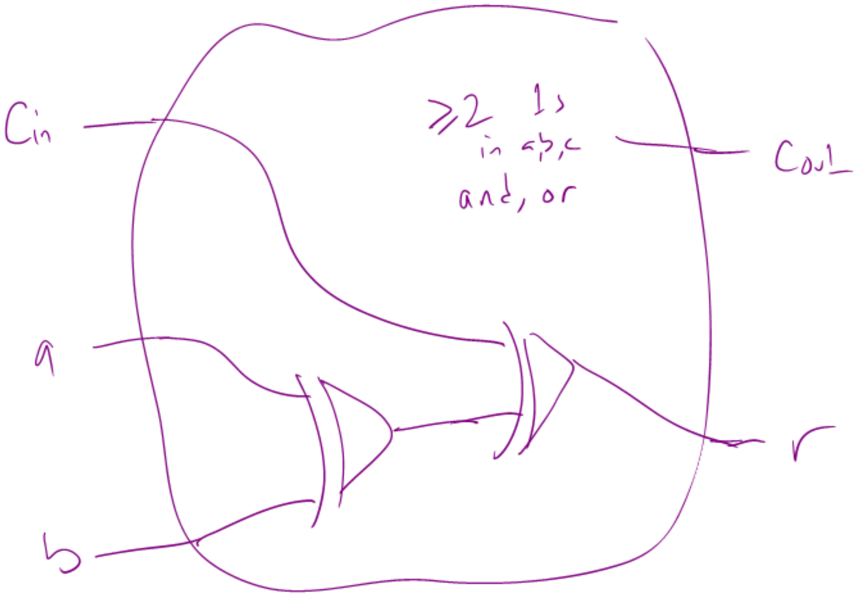
$1+1+1 = 11_2$

Add 3 1-bit numbers: a, b, c

a	b	c_{in}	C_{out}	r
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

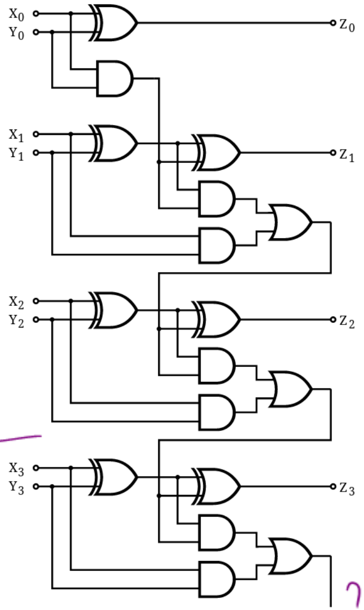


$$\begin{array}{r} 1 \\ 1 \\ 0 \\ \hline 10 \end{array}$$



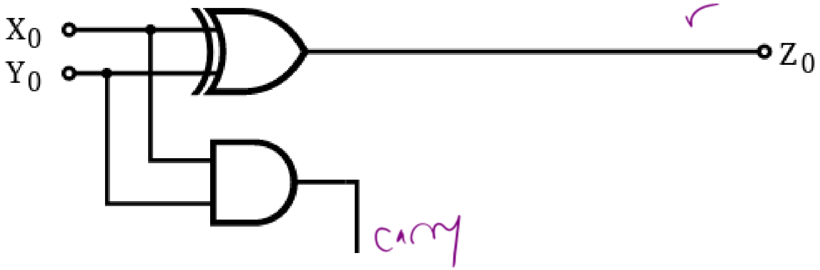
Ripple-Carry Adder

$$
\begin{array}{r}
x_3\ x_2\ x_1\ x_0 \\
+\ y_3\ y_2\ y_1\ y_0 \\
\hline
z_3\ z_2\ z_1\ z_0
\end{array}
$$

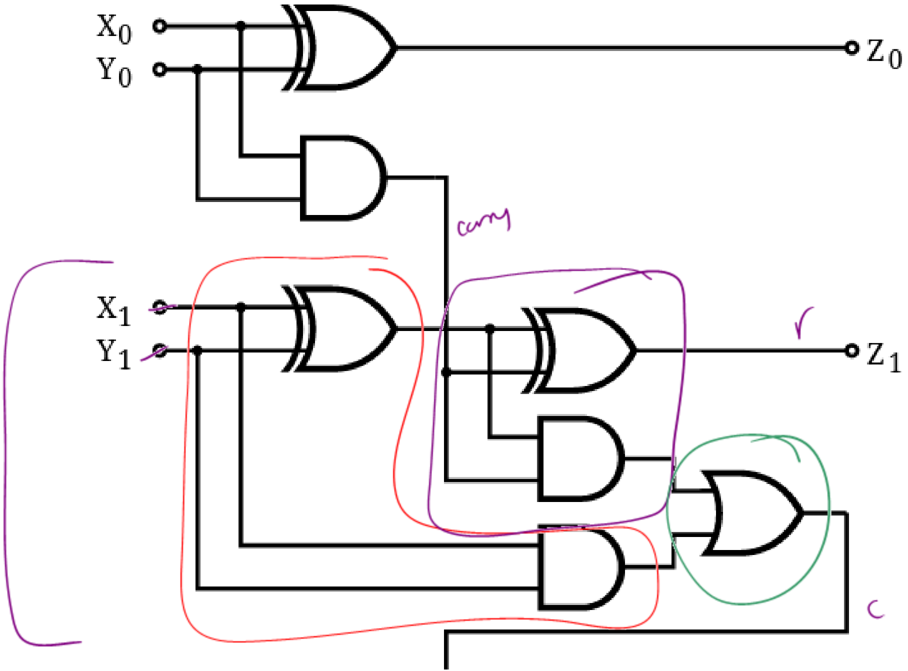


?

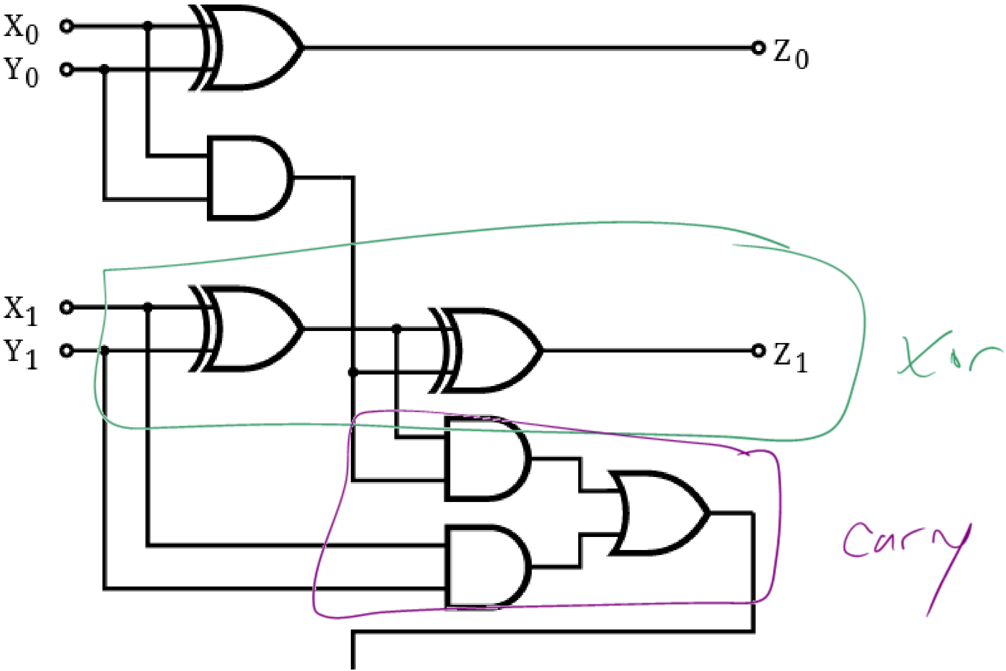
Ripple-Carry Adder: Lowest-order Bit



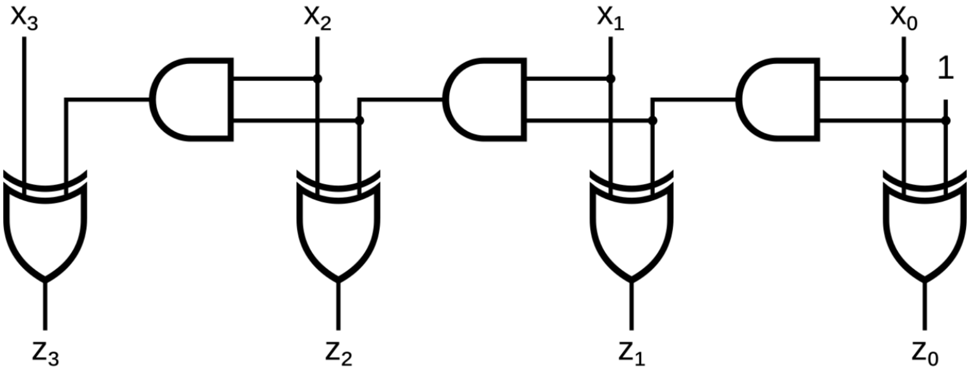
Ripple-Carry Adder: In General



Ripple-Carry Adder: In General



What does this circuit do?



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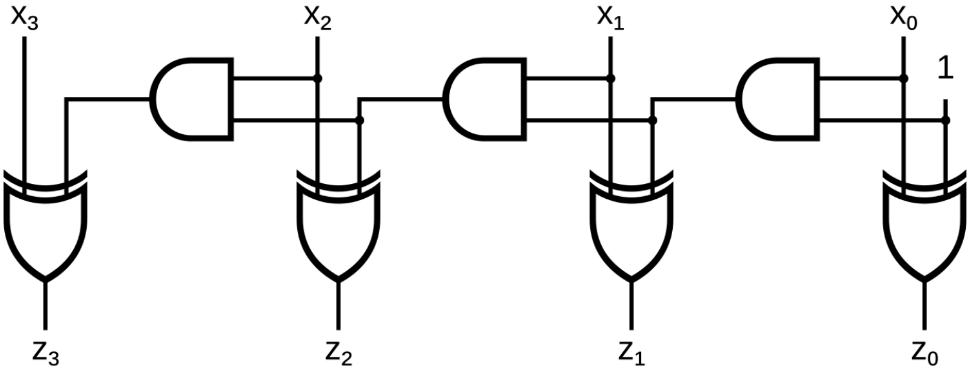
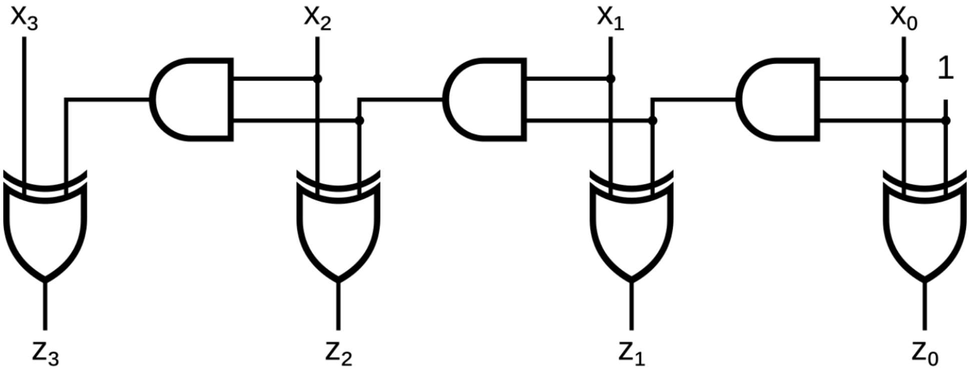
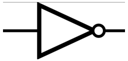


Increment Circuit



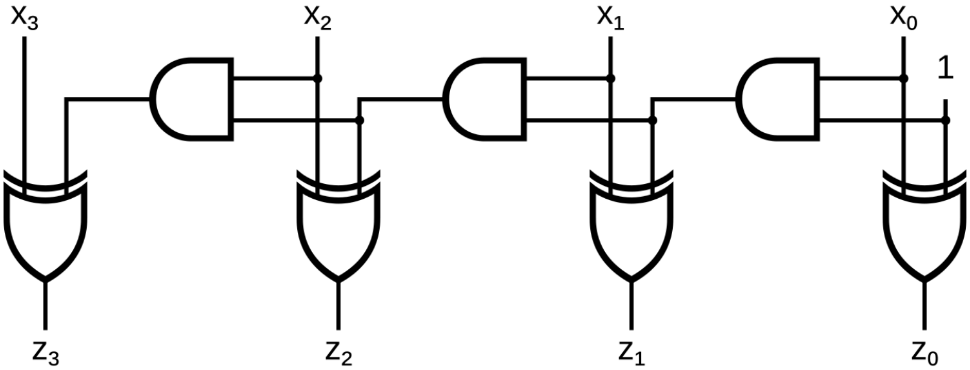
Gate Delay

What happens when I change my input?



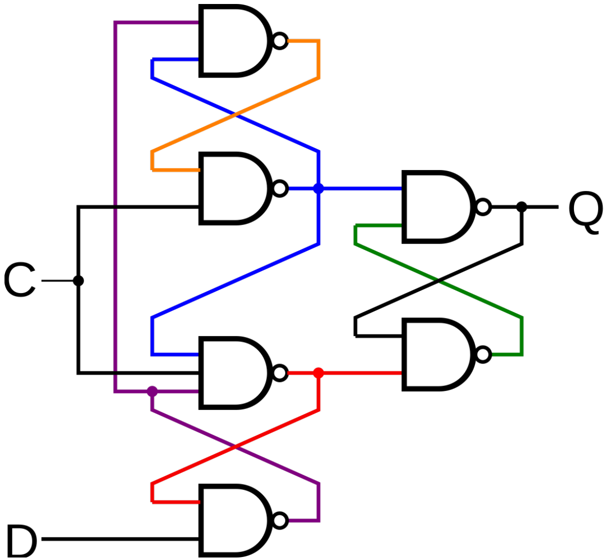
Building a Counter

Building a Counter

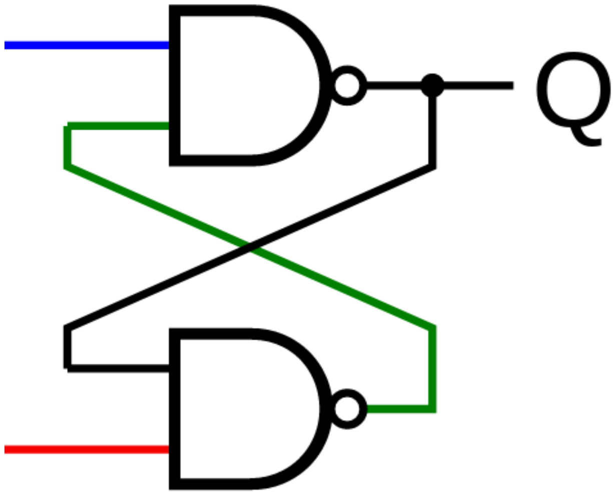


Building a Counter - Waiting

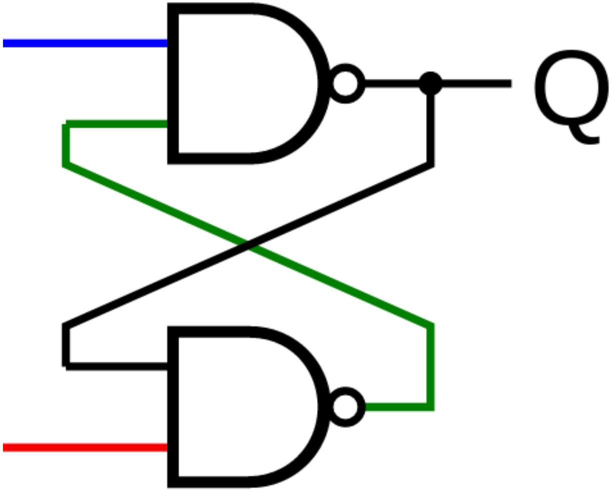
1-bit Register Circuit



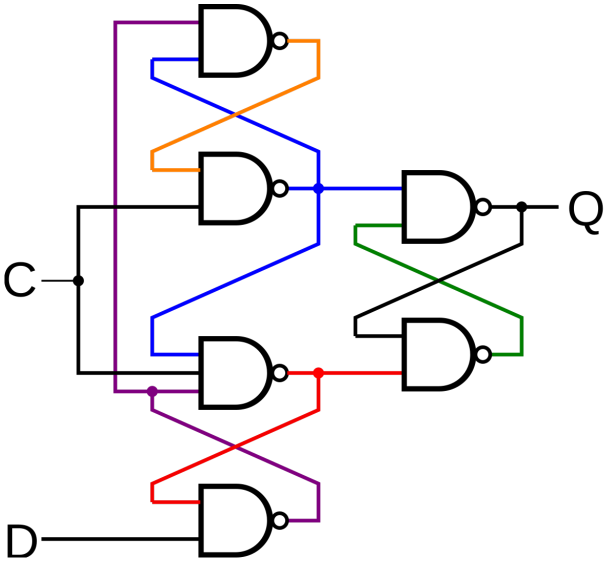
1-bit Register Circuit



1-bit Register Circuit



1-bit Register Circuit



Building a Counter