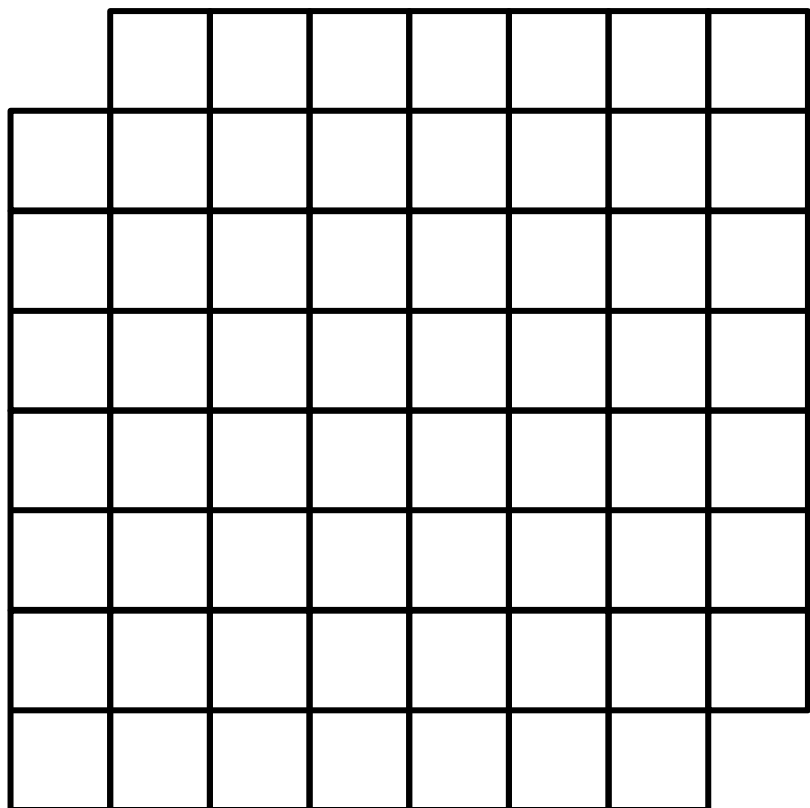


CS4102 Algorithms

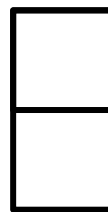
Fall 2019

Can you fill a 8×8 board with the corners missing using dominoes?

Can you tile this?



With these?

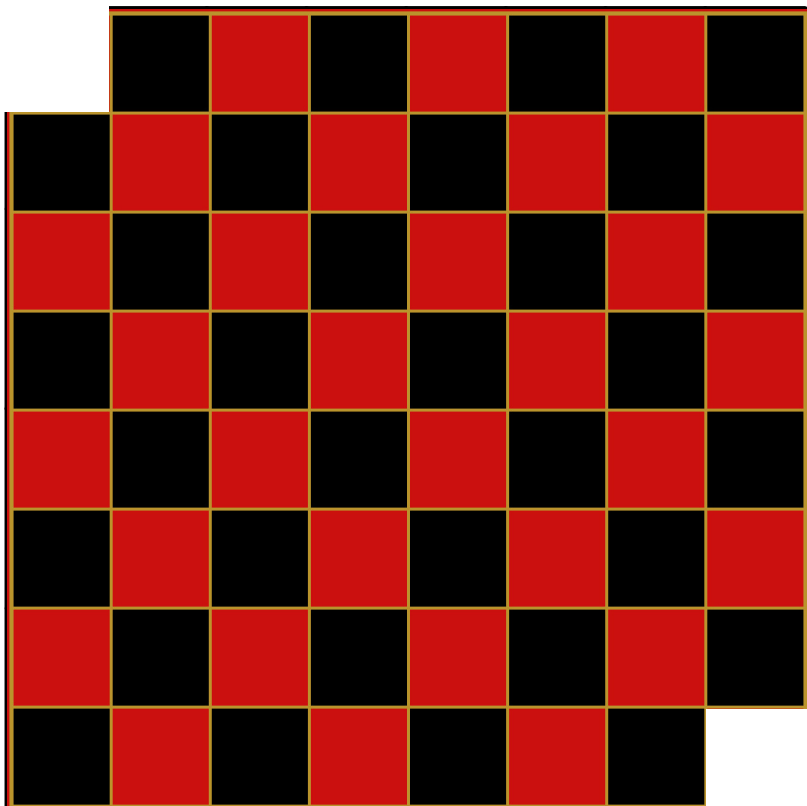


CS4102 Algorithms

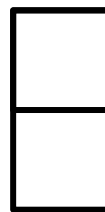
Fall 2019

Can you fill a 8×8 board with the corners missing using dominoes?

Can you tile this?



With these?



Today's Keywords

- Dynamic Programming
- Gerrymandering

CLRS Readings

- Chapter 15
- Chapter 16

Administrativa

- Midterm take-home due tonight at 11pm
 - Individual work!
- Homework 5 out tonight!
 - Due Thursday, October 24 at 11pm
 - Seam Carving!
 - Dynamic Programming (implementation)
 - Java or Python

Dynamic Programming

- Requires **Optimal Substructure**
 - Solution to larger problem contains the solutions to smaller ones
- Idea:
 1. Identify the recursive structure of the problem
 - What is the “last thing” done?
 2. Save the solution to each subproblem in memory
 3. Select a good order for solving subproblems
 - “Top Down”: Solve each recursively
 - “Bottom Up”: Iteratively solve smallest to largest

Generic Top-Down Dynamic Programming Soln

```
mem = {}  
def myDPalgo(problem):  
    if mem[problem] not blank:  
        return mem[problem]  
    if baseCase(problem):  
        solution = solve(problem)  
        mem[problem] = solution  
        return solution  
    for subproblem of problem:  
        subsolutions.append(myDPalgo(subproblem))  
    solution = OptimalSubstructure(subsolutions)  
    mem[problem] = solution  
    return solution
```

DP Algorithms so far

- $2 \times n$ domino tiling (Fibonacci)
- Log cutting
- Matrix Chaining
- Longest Common Subsequence
- $4 \times n$ domino tiling (Homework 4 Problem)
- Seam Carving (Homework 5 Problem)

Domino Tiling

Tile(n):

Initialize Memory M

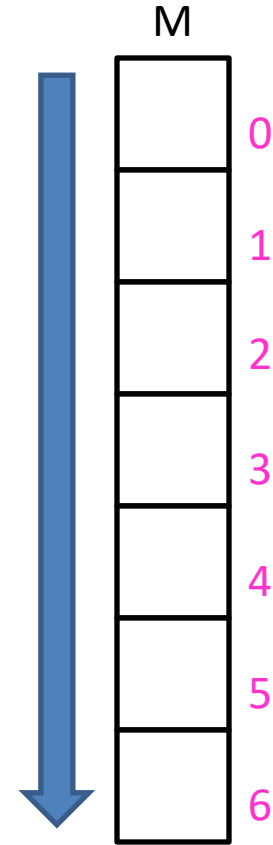
$M[0] = 0$

$M[1] = 0$

for $i = 0$ to n :

$M[i] = M[i-1] + M[i-2]$

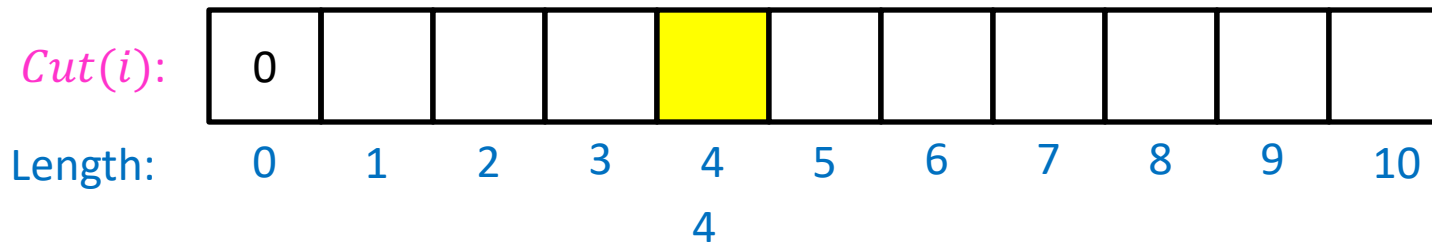
return $M[n]$



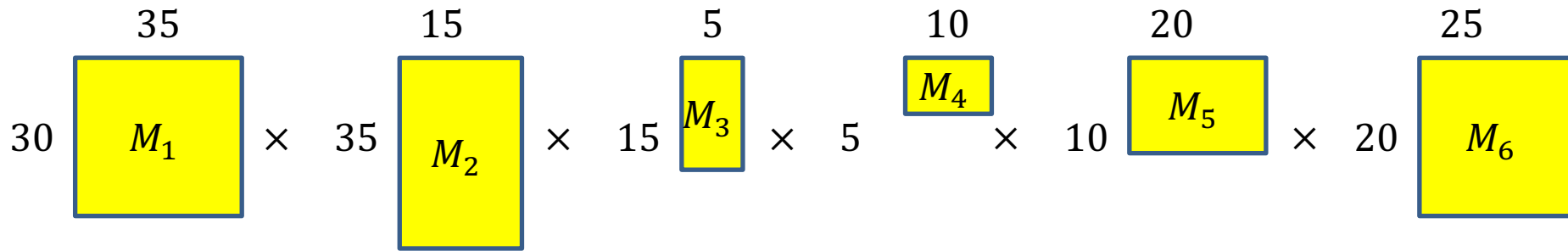
Log Cutting

Solve Smallest subproblem first

$$Cut(4) = \max \begin{cases} Cut(3) + P[1] \\ Cut(2) + P[2] \\ Cut(1) + P[3] \\ Cut(0) + P[4] \end{cases}$$



Matrix Chaining



$$Best(i, j) = \min_{k=1}^{j-1} (Best(i, k) + Best(k+1, j) + r_i r_{k+1} c_j)$$

$$Best(i, i) = 0$$

	$j = 1$	2	3	4	5	6	i
1	0	15750	7875	9375	11875	15125	1
2		0	2625	4375	7125	10500	2
3			0	750	2500	5375	3
4				0	1000	3500	4
5					0	5000	5
6						0	6

$Best(1,6) = \min$ {

- $Best(1,1) + Best(2,6) + r_1 r_2 c_6$
- $Best(1,2) + Best(3,6) + r_1 r_3 c_6$
- $Best(1,3) + Best(4,6) + r_1 r_4 c_6$
- $Best(1,4) + Best(5,6) + r_1 r_5 c_6$
- $Best(1,5) + Best(6,6) + r_1 r_6 c_6$

Generic Top-Down Dynamic Programming Soln

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def myDPalgo(problem):  
    if mem[problem] not blank:  
        return mem[problem]  
    if baseCase(problem):  
        solution = solve(problem)  
        mem[problem] = solution  
        return solution  
    for subproblem of problem:  
        subsolutions.append(myDPalgo(subproblem))  
    solution = OptimalSubstructure(subsolutions)  
    mem[problem] = solution  
    return solution
```


Generic Top-Down Dynamic Programming Soln

```
mem = []  
Matrix = [] #1-D array of all 2-D matrices  
def matrixChain(i,j):  
    if mem[i][j] not blank:  
        return mem[i][j]  
    if (j-i) == 0:  
        mem[i][j] = 0  
        return 0  
    if (j-i) == 1:  
        solution = size(Matrix[i]) * size(Matrix[j]) * size(Matrix[j][0]) #cost to multiply  
        mem[i][j] = solution  
        return solution  
    for k in i+1 .. j-1:  
        subsolutions.append(matrixChain(i,k) + matrixChain(k+1,j) + cost(i,k,j))  
    solution = min(subsolutions) # optimal substructure  
    mem[i][j] = solution  
    return solution
```

Longest Common Subsequence

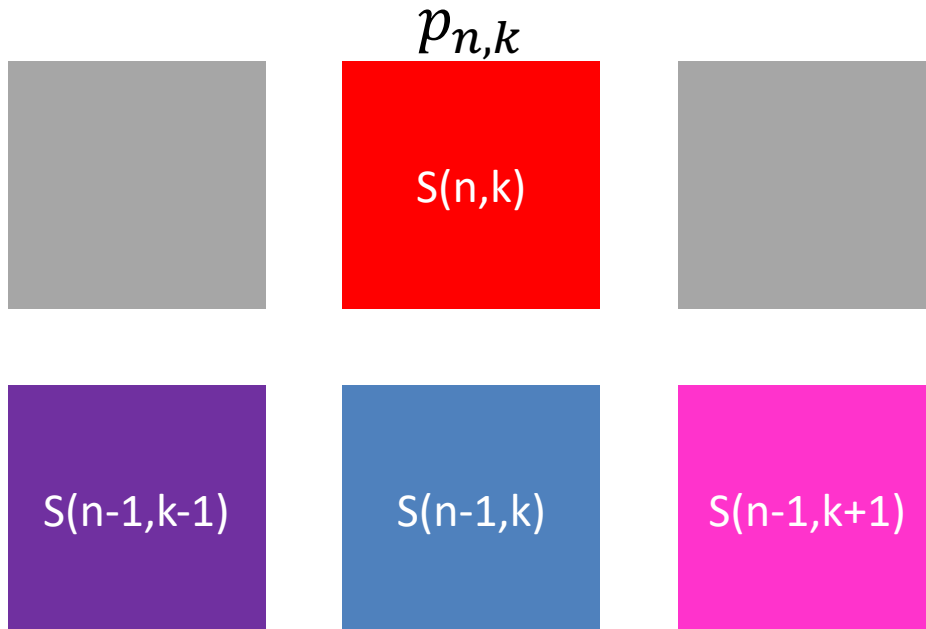
$$LCS(i, j) = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ LCS(i - 1, j - 1) + 1 & \text{if } X[i] = Y[j] \\ \max(LCS(i, j - 1), LCS(i - 1, j)) & \text{otherwise} \end{cases}$$

X =									
Y =		0	A	T	C	T	G	A	T
	0	1	2	3	4	5	6	7	
	0	0	0	0	0	0	0	0	0
T	1	0	0	1	1	1	1	1	1
G	2	0	0	1	1	1	2	2	2
C	3	0	0	1	2	2	2	2	2
A	4	0	1	1	2	2	2	3	3
T	5	0	1	2	2	3	3	3	4
A	6	0	1	2	2	3	3	4	4

To fill in cell (i, j) we need cells $(i - 1, j - 1)$, $(i - 1, j)$, $(i, j - 1)$
 Fill from Top->Bottom, Left->Right (with any preference)

Seam Carving

$$S(n, k) = \min \begin{cases} S(n-1, k-1) + e(p_{n,k}) \\ S(n-1, k) + e(p_{n,k}) \\ S(n-1, k+1) + e(p_{n,k}) \end{cases}$$





Supreme Court Associate Justice Anthony Kennedy gave no sign that he has abandoned his view that extreme partisan gerrymandering might violate the Constitution. | Eric Thayer/Getty Images

Supreme Court eyes partisan gerrymandering

Anthony Kennedy is seen as the swing vote that could blunt GOP's map-drawing successes.

SUPREME COURT OF THE UNITED STATES

Syllabus

**VIRGINIA HOUSE OF DELEGATES ET AL. v.
BETHUNE-HILL ET AL.**

APPEAL FROM THE UNITED STATES DISTRICT COURT FOR THE
EASTERN DISTRICT OF VIRGINIA

No. 18–281. Ar

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SUPREME COURT OF THE UNITED STATES

Syllabus

RUCHO ET AL. v. COMMON CAUS

APPEAL FROM THE UNITED STATES DISTRICT COURT
MIDDLE DISTRICT OF NORTH CAROLINA

No. 18–422. Argued March 26, 2019—Decided

Voters and other plaintiffs in North Carolina and
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SUPREME COURT OF THE UNITED STATES Syllabus RUCHO ET AL. v. COMMON CAUSE ET AL. APPEAL FROM THE UNITED STATES DISTRICT COURT FOR THE MIDDLE DISTRICT OF NORTH CAROLINA No. 18–422. Argued March 26, 2019—Decided June 27, 2019* Voters and other plaintiffs in North Carolina and Maryland filed suits challenging their constitutional partisan gerrymandering claims that the state discriminated against voters of the First Amendment. The United States District Courts in both states granted the defendants’ appeals. Held: Partisan gerrymandering is beyond the reach of the Constitution. (a) In these cases, the question is whether the Constitution’s limitations on the Judiciary Nature of the Government, Art. III, § 1, cl. 1, extend to the state’s congressional districting process. While it is	<i>Next Gerrymandering Battle in North Carolina: Congress</i> A North Carolina court threw out the state’s congressional districting plan as an illegal gerrymander. Now the same court has asked Congress to tell the state to redraw the state’s congressional districts. 
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SUPREME COURT OF THE UNITED STATES

Syllabus

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Next Gerrymandering Battle in North Carolina: Congress

A North Carolina court threw out the state's legislative map as an illegal gerrymander. Now the same court could force the state to redraw the state's congressional districts as well.

A photograph showing two individuals at a voting station. The station has a red frame and a white panel with an American flag. A sign on the left panel reads 'NOTICE' and includes icons for 'no cameras' and 'no cell phones'. The person on the left is a man in a blue and white patterned shirt, and the person on the right is a woman with long dark hair wearing a black top. They are both looking down at the ballot.

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Gerrymandering

- Manipulating electoral district boundaries to favor one political party over others
- Coined in an 1812 Political cartoon
- Governor **Elbridge Gerry** signed a bill that redistricted Massachusetts to benefit his Democratic-Republican Party



The Gerry-mander

According to the Supreme Court

- Gerrymandering cannot be used to:
 - Disadvantage racial/ethnic/religious groups
- It can be used to:
 - Disadvantage political parties

SUPREME COURT OF THE UNITED STATES

Syllabus

VIRGINIA HOUSE OF DELEGATES ET AL. v.
BETHUNE-HILL ET AL.

APPEAL FROM THE UNITED STATES DISTRICT COURT FOR THE
EASTERN DISTRICT OF VIRGINIA

No. 18–281. Argued March 18, 2019—Decided June 17, 2019

After the 2010 census, Virginia redrew legislative districts for the State’s Senate and House of Delegates. Voters in 12 impacted House districts sued two state agencies and four election officials (collectively, State Defendants), charging that the redrawn districts were racially gerrymandered in violation of the Fourteenth Amendment’s Equal Protection Clause. The House of Delegates and its Speaker (collectively, the House) intervened as defendants, participating in the bench trial, on appeal to this Court, and at a second bench trial, where a three-judge District Court held that 11 of the districts were unconstitutionally drawn, enjoined Virginia from conducting elections for those districts before adoption of a new plan, and gave the General Assembly several months to adopt that plan. Virginia’s Attorney General announced that the State would not pursue an appeal to this Court. The House, however, did file an appeal.

Held: The House lacks standing, either to represent the State’s interests or in its own right. Pp. 3–12.

SUPREME COURT OF THE UNITED STATES

Syllabus

RUCHO ET AL. v. COMMON CAUSE ET AL.

APPEAL FROM THE UNITED STATES DISTRICT COURT FOR THE
MIDDLE DISTRICT OF NORTH CAROLINA

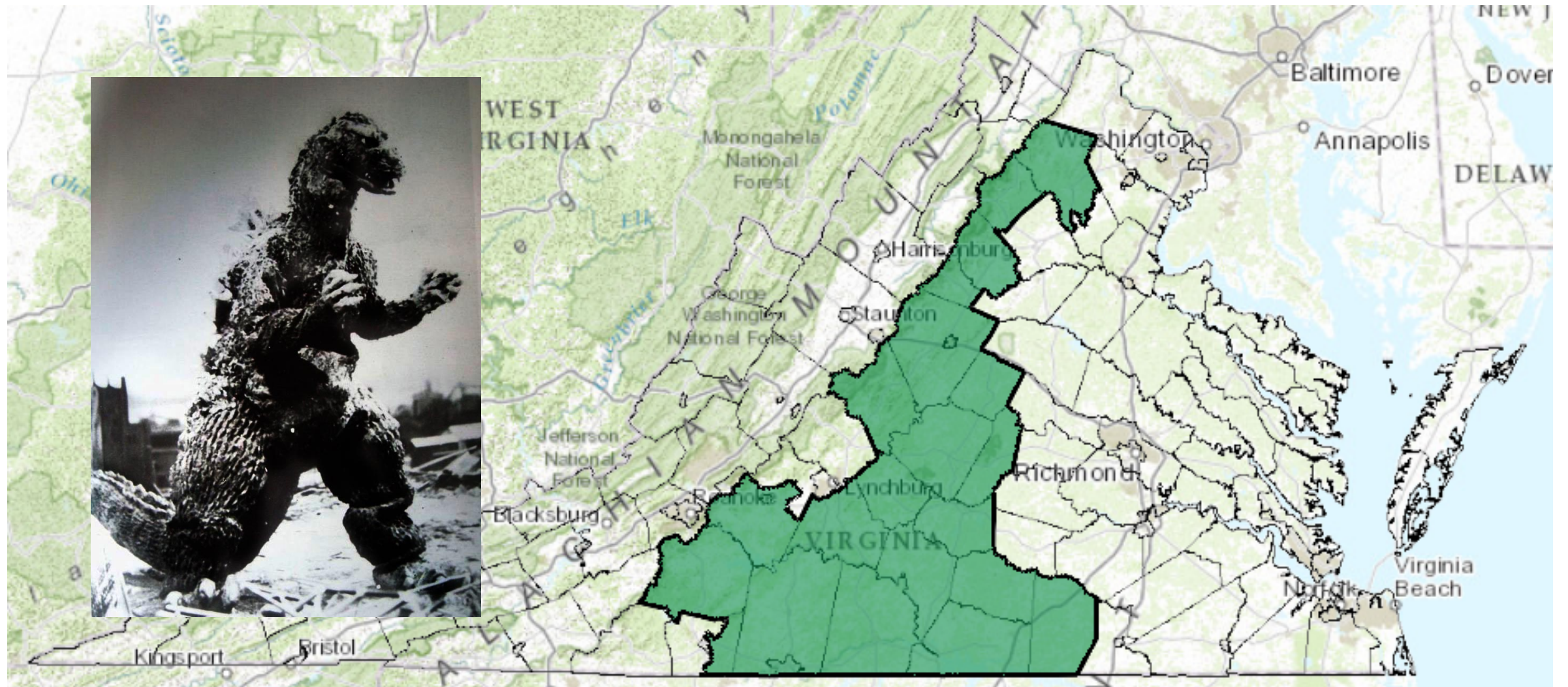
No. 18–422. Argued March 26, 2019—Decided June 27, 2019*

Voters and other plaintiffs in North Carolina and Maryland filed suits challenging their States’ congressional districting maps as unconstitutional partisan gerrymanders. The North Carolina plaintiffs claimed that the State’s districting plan discriminated against Democrats, while the Maryland plaintiffs claimed that their State’s plan discriminated against Republicans. The plaintiffs alleged violations of the First Amendment, the Equal Protection Clause of the Fourteenth Amendment, the Elections Clause, and Article I, §2. The District Courts in both cases ruled in favor of the plaintiffs, and the defendants appealed directly to this Court.

Held: Partisan gerrymandering claims present political questions beyond the reach of the federal courts. Pp. 6–34.

(a) In these cases, the Court is asked to decide an important question of constitutional law. Before it does so, the Court “must find that the question is presented in a ‘case’ or ‘controversy’ that is . . . ‘of a Judiciary Nature.’” *DaimlerChrysler Corp. v. Cuno*, 547 U. S. 332, 342. While it is “the province and duty of the judicial department to

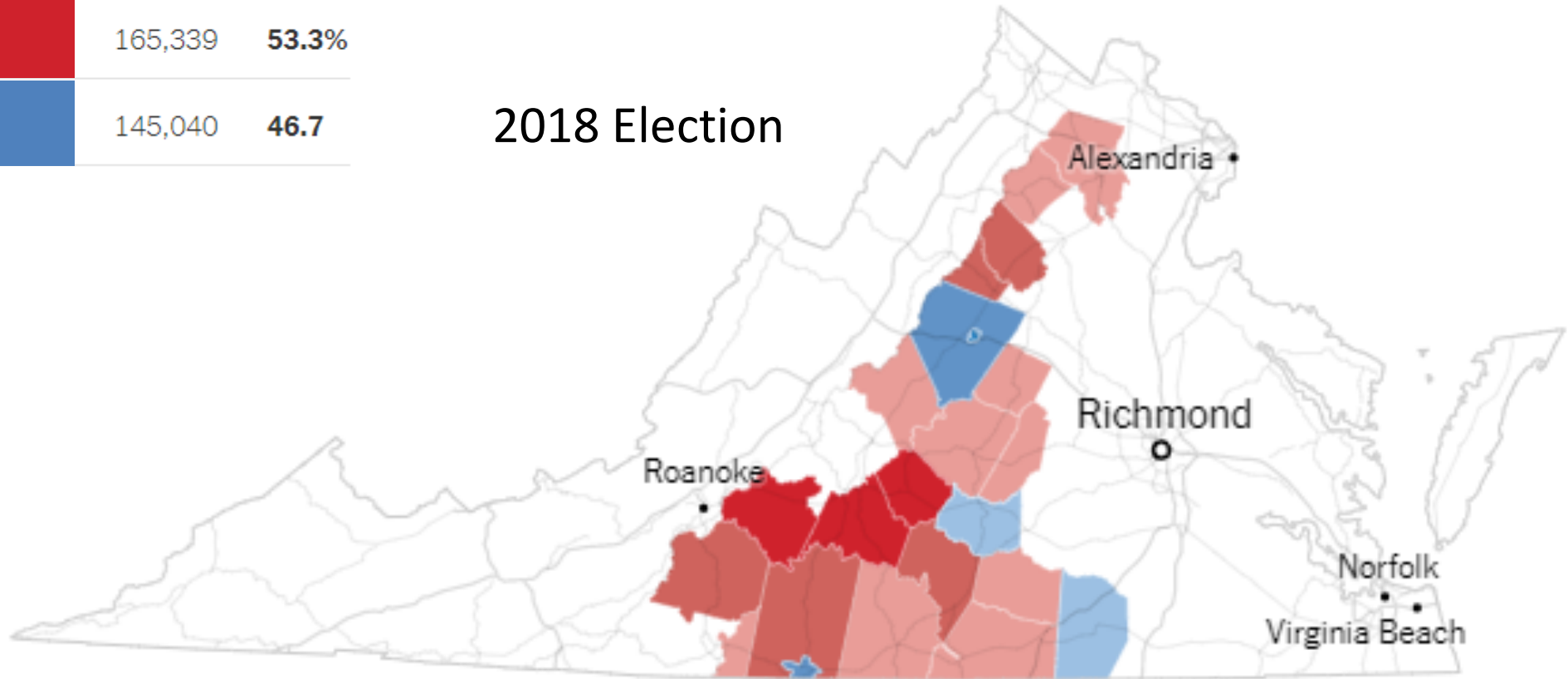
VA 5th District



VA 5th District

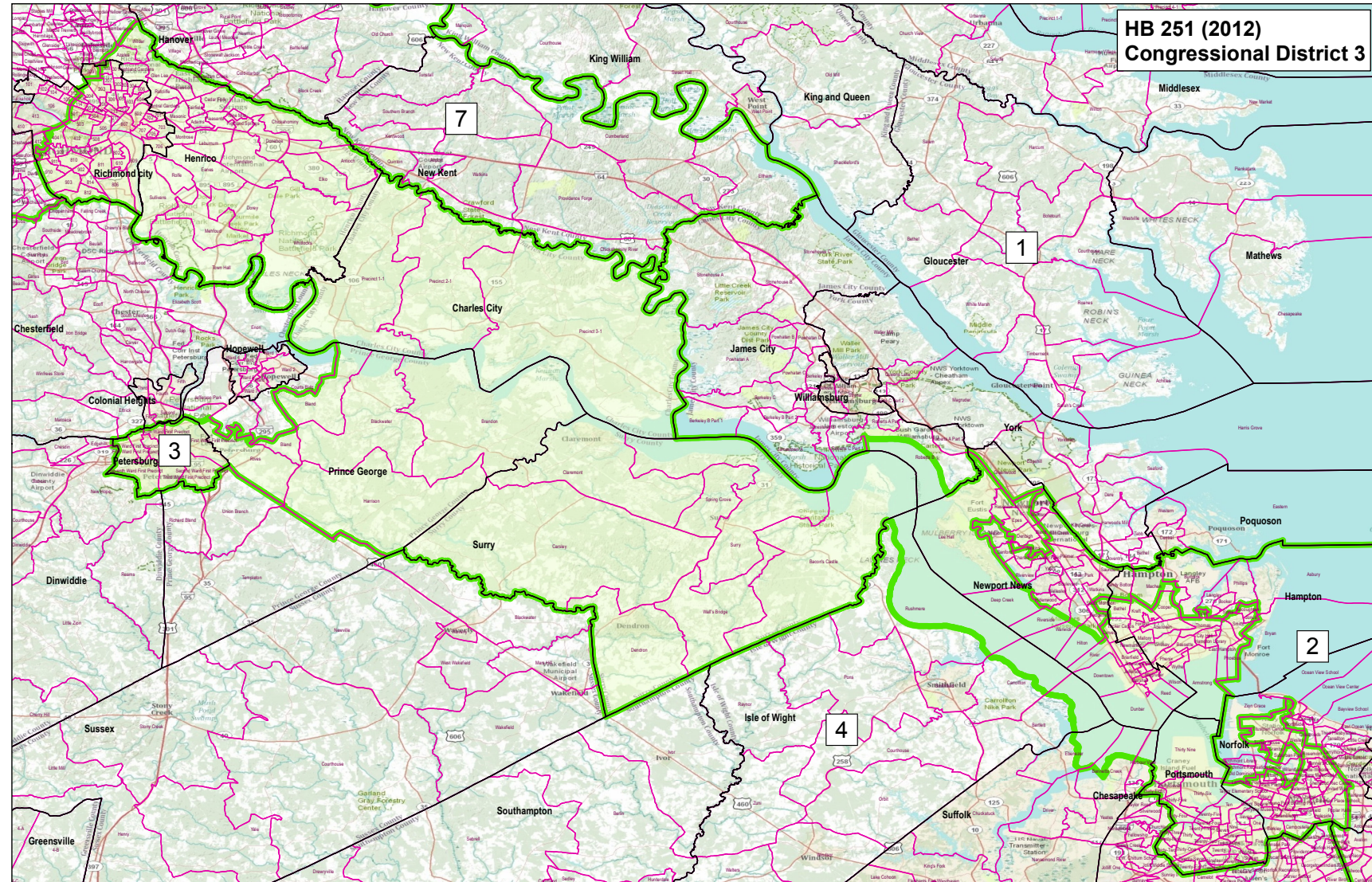
	Votes	Pct.
	165,339	53.3%
	145,040	46.7

2018 Election



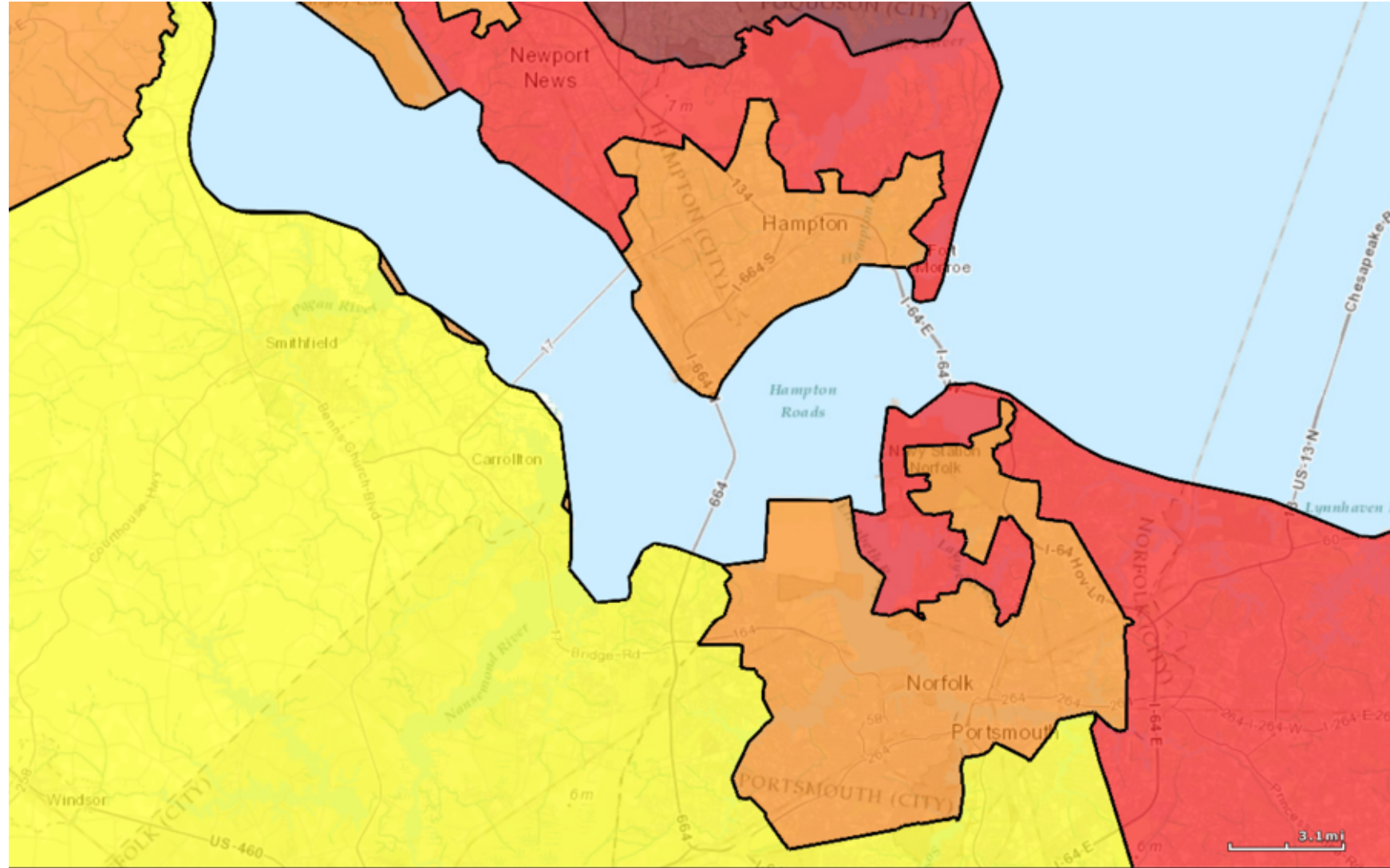
Gerrymandering Today

- Computers make it really effective



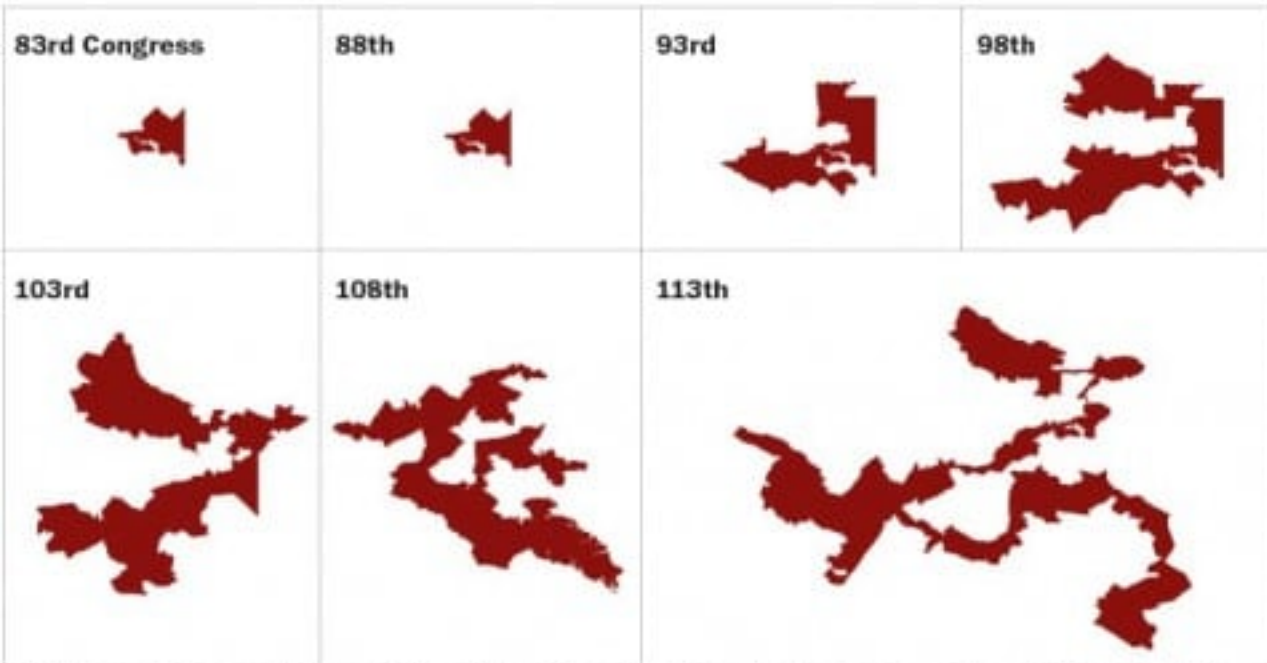
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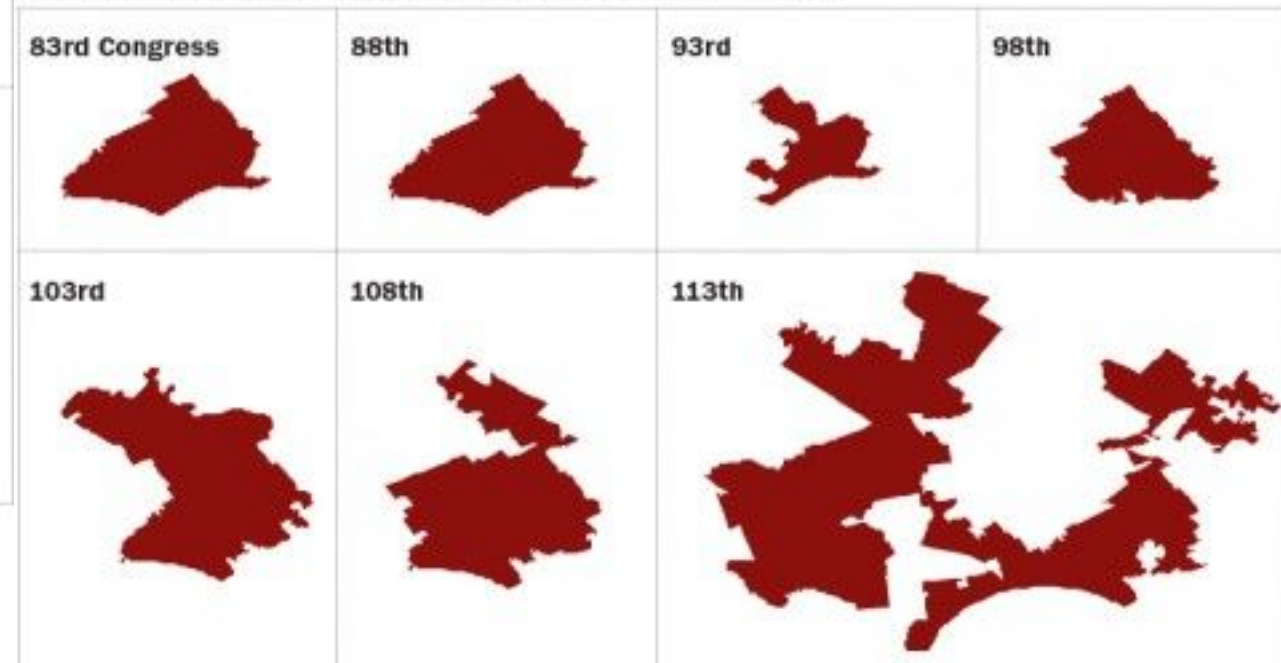
Gerrymandering Today

THE EVOLUTION OF MARYLAND'S THIRD DISTRICT



SOURCE: Shapefiles maintained by Jeffrey B. Lewis, Brandon DeVine, Lincoln Pritcher and Kenneth C. Martis, UCLA.
Drawn to scale.
GRAPHIC: The Washington Post. Published May 20, 2014

THE EVOLUTION OF PENNSYLVANIA'S SEVENTH DISTRICT



SOURCE: Shapefiles maintained by Jeffrey B. Lewis, Brandon DeVine, Lincoln Pritcher and Kenneth C. Martis, UCLA.
Drawn to scale.
GRAPHIC: The Washington Post. Published May 20, 2014

How does it work?

- States are broken into precincts
- All precincts have the same size
- We know voting preferences of each precinct
- Group precincts into districts to maximize the number of districts won by my party

Overall: R:217 D:183

R:65 D:35	R:45 D:55
R:60 D:40	R:47 D:53



VS



How does it work?

- States are broken into precincts
- All precincts have the same size
- We know voting preferences of each precinct
- Group precincts into districts to maximize the number of districts won by my party

Overall: R:217 D:183

R:65 D:35	R:45 D:55
R:60 D:40	R:47 D:53

R:125

R:92

R:65 D:35	R:45 D:55
R:60 D:40	R:47 D:53

R:112

R:105

R:65 D:35	R:45 D:55
R:60 D:40	R:47 D:53

Gerrymandering Problem Statement

- Given:
 - A list of precincts: $p_1, p_2, \dots, p_n$
 - Each containing m voters
- Output:
 - Districts $D_1, D_2 \subset \{p_1, p_2, \dots, p_n\}$
 - Where $|D_1| = |D_2|$
 - $R(D_1) > \frac{mn}{4}$ and $R(D_2) > \frac{mn}{4}$
 - $R(D_i)$ gives number of “Regular Party” voters in D_i
 - $R(D_i) > \frac{mn}{4}$ means D_i is majority “Regular Party”
 - “failure” if no such solution is possible

Valid Gerrymandering!

$$m \cdot \frac{n}{2} \cdot \frac{1}{2}$$

Dynamic Programming

- Requires **Optimal Substructure**
 - Solution to larger problem contains the solutions to smaller ones
- Idea:
 1. Identify the recursive structure of the problem
 - What is the “last thing” done?
 2. Save the solution to each subproblem in memory
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Dynamic Programming

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 1. Identify the recursive structure of the problem
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Consider the last precinct

After assigning the
first $n - 1$ precincts

$p_1, p_2, \dots, p_{n-1}$

D_1
 k precincts
 x voters for R

D_2
 $n - k - 1$ precincts
 y voters for R

If we assign
 p_n to D_1

p_n

D_1
 $k + 1$ precincts
 $x + R(p_n)$ voters for R

Valid gerrymandering if:

$$k + 1 = \frac{n}{2},$$
$$x + R(p_n), y > \frac{mn}{4}$$

If we assign
 p_n to D_2

D_2
 $n - k$ precincts
 $y + R(p_n)$ voters for R

Valid gerrymandering if:

$$n - k = \frac{n}{2},$$
$$x, y + R(p_n) > \frac{mn}{4}$$

D_1
 $k + 1$ precincts
 $x + R(p_n)$ voters for R

D_2
 $n - k - 1$ precincts
 y voters for R

World One

D_1
 k precincts
 x voters for R

D_2
 $n - k$ precincts
 $y + R(p_n)$ voters for R

World Two

Define Recursive Structure

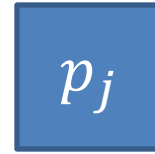
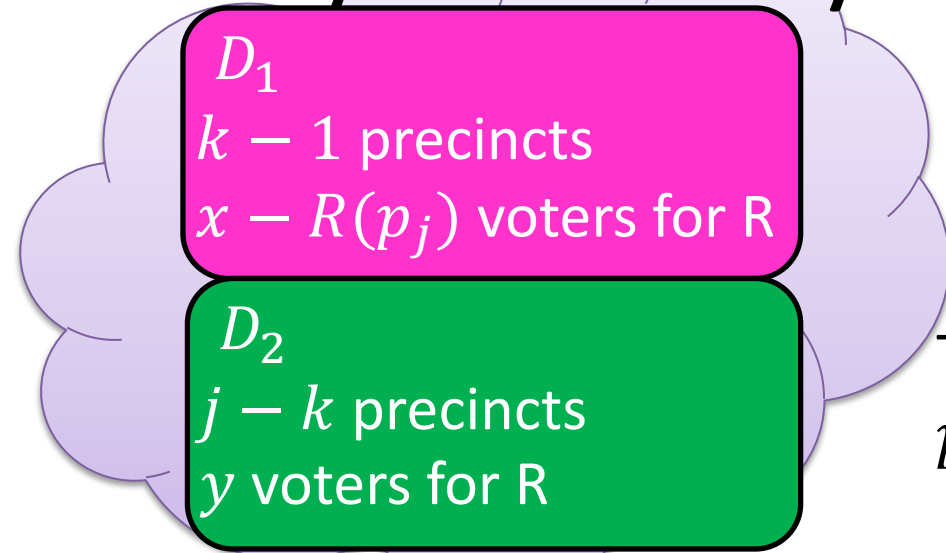
$S(j, k, x, y) = \text{True}$ if from among the first j precincts:
 k are assigned to D_1
 $n \times n \times m \times m$ exactly x vote for R in D_1
exactly y vote for R in D_2

4D Dynamic Programming!!!

Two ways to satisfy $S(j, k, x, y)$:

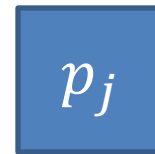
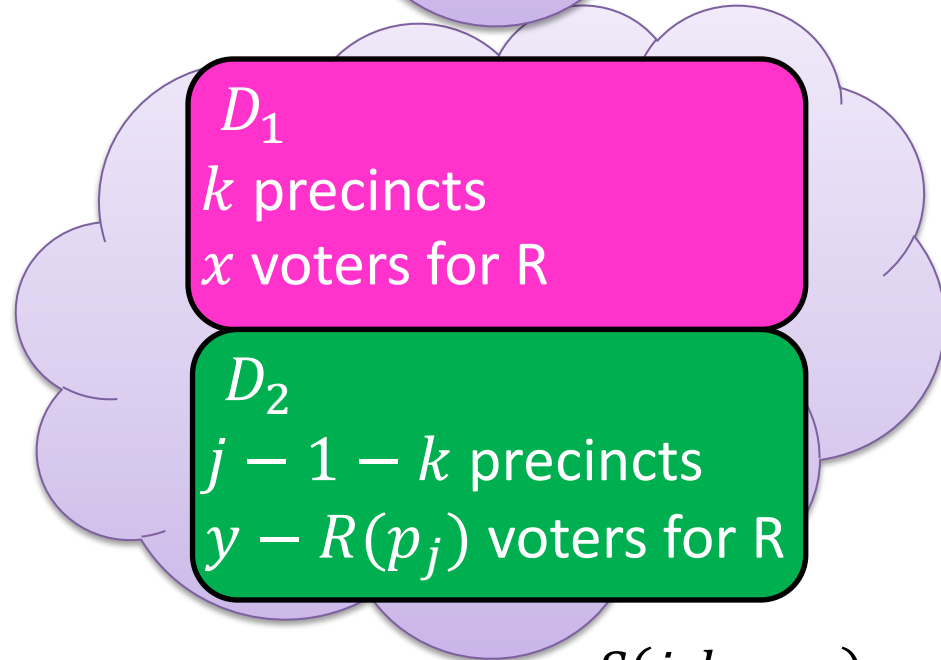
$S(j, k, x, y) = \text{True}$ if:

- from among the first j precincts k are assigned to D_1
- exactly x vote for R in D_1
- exactly y vote for R in D_2

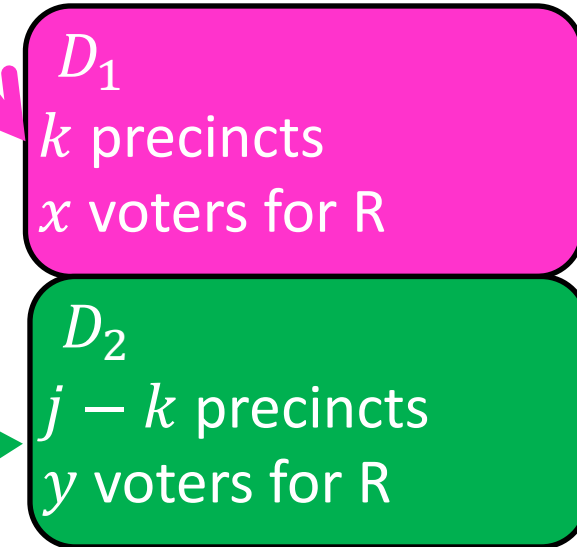


Then assign
 p_j to D_1

OR



Then assign
 p_j to D_2



$$S(j, k, x, y) = S(j - 1, k - 1, x - R(p_j), y) \vee S(j - 1, k, x, y - R(p_j))$$

Final Algorithm

$$S(j, k, x, y) = S(j-1, k-1, x-R(p_j), y) \vee S(j-1, k, x, y-R(p_j))$$

Initialize $S(0,0,0,0) = \text{True}$

for $j = 1, \dots, n$:

for $k = 1, \dots, \min(j, \frac{n}{2})$:

for $x = 0, \dots, jm$:

for $y = 0, \dots, jm$:

$S(j, k, x, y) =$

$$S(j-1, k-1, x-R(p_j), y) \vee S(j-1, k, x, y-R(p_j))$$

Search for True entry at $S(n, \frac{n}{2}, > \frac{mn}{4}, > \frac{mn}{4})$

$S(j, k, x, y) = \text{True}$ if:

from among the first j precincts

k are assigned to D_1

exactly x vote for R in D_1

exactly y vote for R in D_2

Run Time

$$S(j, k, x, y) = S(j-1, k-1, x-R(p_j), y) \vee S(j-1, k, x, y-R(p_j))$$

Initialize $S(0,0,0,0) = \text{True}$

n for $j = 1, \dots, n$:

$\frac{n}{2}$ for $k = 1, \dots, \min(j, \frac{n}{2})$:

nm for $x = 0, \dots, jm$:

nm for $y = 0, \dots, jm$:

$S(j, k, x, y) =$

$$S(j-1, k-1, x-R(p_j), y) \vee S(j-1, k, x, y-R(p_j))$$

Search for True entry at $S(n, \frac{n}{2}, > \frac{mn}{4}, > \frac{mn}{4})$

$$\Theta(n^4 m^2)$$

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- Input: list of precincts (size n), number of voters (integer m)
- Runtime depends on the *value of m* , not *size of m*
 - Run time is exponential in *size* of input
 - Input size is $n + |m| = n + \log m$
- Note: Gerrymandering is NP-Complete