CS4102 Day 3 Recurrence Proofs - Spring 2019

Karatsuba Recurrence: Tree Method

Karatsuba Recurrence:

$$T(n) = 3T(\frac{n}{2}) + 8n$$

Using the tree method for solving the recurrence, we obtained the sum:

$$\begin{aligned}
T(n) &= 8n \sum_{i=0}^{\log_2 n} \left(\frac{3}{2}\right)^i \\
&= 8n \frac{(\frac{3}{2})^{\log_2 n+1} - 1}{\frac{3}{2} - 1} \\
&= 8n \frac{(\frac{3}{2})^{\log_2 n+1} - 1}{\frac{1}{2}} \\
&= 16n \left(\left(\frac{3}{2}\right)^{\log_2 n+1} - 1 \right) \\
&= 16n(2^{\log_2 3-1})^{\log_2 n+1} - 16n \\
&= 16n(2^{\log_2 3 \cdot \log_2 n - \log_2 n + \log_2 3 - 1}) - 16n \\
&= 16n((2^{\log_2 n})^{\log_2 3} \cdot 2^{-\log_2 n} \cdot 2^{\log_2 3} \cdot 2^{-1}) - 16n \\
&= 16n \left(n^{\log_2 3} \cdot \frac{1}{n} \cdot 3 \cdot \frac{1}{2} \right) - 16n \\
&= 24n^{\log_2 3} - 16n \\
&= \Theta(n^{\log_2 3}) \\
&\approx \Theta(n^{1.585})
\end{aligned}$$