



Cardinality

$$|\{1, 2, 4\}| = 3$$

$$\{1, 2, 3\} = \{3, 2, 1\}$$

$$\{1\} \neq 1$$

$$\{1, \{1\}\}$$

$\in$

$\cup$

$\cap$

# Set-builder notation

 $\mathbb{Z}$ 

$$\left\{ \underbrace{x^2}_{\text{expression}} \mid \underbrace{x \in \mathbb{Z}}_{\text{predicate}} \right\} = \{0, 1, 4, 9, 16, 25, \dots\}$$

expression  
variable  
True

$$\left\{ \frac{x}{|x|} \mid x \in \mathbb{R} \text{ and } x \neq 0 \right\} = \{1, -1\}$$

$$A \cup B = \{x \mid x \in A \vee_{\text{or}} x \in B\} \quad \text{Union}$$

$$A \cap B = \{x \mid x \in A \wedge_{\text{and}} x \in B\} \quad \text{Intersection}$$

$$A \setminus B = \{x \mid x \in A \wedge \neg(x \in B)\} \quad \text{Set Difference}$$

$$\{1, 2, 4\} \setminus \{1, 3\} = \{2, 4\}$$

$A \subseteq B$   
 - is the same as -

$A \cup B = B$

$A < B$   $\begin{cases} A \subseteq B \\ A \neq B \end{cases}$   
 proper subset

$\subseteq \leq \quad \neg (x \in B)$   
 $\subset < \quad x \notin B$

Class  $\supseteq$  friends / Class = friend  
 or  
 (Class  $\supset$  friends)

$A \supseteq B$   
 Superset of

$A \cup B = A$

dorm room in class  
 $\{you, friends, \dots\} \subseteq \{you, friends, teacher, others \dots\}$   
 Subset      Subset of      Superset

$A \subseteq B$

all  $x \in A$  are also  $\in B$

$4 \neq 3$   
 $\ddots$

$4 \geq 3$   
 $\cup$

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 $\ddots$

$4 \geq 3$   
 $\cup$

$$|\mathcal{P}(A)| = 2^{|A|}$$

$$|A \cup B| \leq |A| + |B|$$

$$|A \cap B| \leq \min(|A|, |B|)$$

$$x \in \mathcal{P}(A) \quad \equiv \quad x \subseteq A$$

pow(A)

$\mathcal{P}(A)$

$$= \{S \mid S \subseteq A\}$$

Power set

$$A = \{2, 3, 5, 7\}$$

$$|A| = 4$$

$$2^4 = 16$$

$$\mathcal{P}(A) = \{ \{ \},$$

$$\{2\}, \{3\}, \{5\}, \{7\},$$

$$\{2, 3\}, \{2, 5\}, \{2, 7\}, \{3, 5\}, \{3, 7\}, \{5, 7\},$$

$$\{2, 3, 5\}, \{2, 3, 7\}, \{2, 5, 7\}, \{3, 5, 7\},$$

$$\{2, 3, 5, 7\},$$

}

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$$\{x \mid x \in \mathcal{P}(\text{class}) \wedge |x| = 3\}$$

$$\mathcal{P}(\{1\}) = \{ \{ \}, \{1\} \}$$

$$\mathcal{P}(\{ \}) = \{ \{ \} \}$$

$$\mathcal{P}(\mathcal{P}(\{ \})) = \{ \{ \}, \{ \{ \} \} \}$$

$$A \in \mathcal{P}(A)$$

$$A \subseteq A$$

$$\{ \} \in \mathcal{P}(A)$$

$$\{ \} \subseteq A$$

$$|\{ \}| = 0$$

$$|\mathcal{P}(\{ \})| = 2^{|\{ \}|} = 2^0 = 1$$



Mutually exclusive

$x \in B$  and  $x \in A$

rare in computer logic vocab

A and B are disjoint

subset  
subset

$$\text{if } A \cap B = \{\}$$

code words

Set of possible answers

set of correct answers

are disjoint

$$A \cap B \neq \{\}$$

$A \subseteq B$  is false

$B \subseteq A$  is false

$$A = \{1, 2, 4\}$$

$$B = \{1, 3\}$$