



Proof by contradiction

$$- A \rightarrow L \models \neg A$$

$$\begin{array}{c} \text{assum} \\ - \boxed{A} \vdash B \\ \downarrow \\ \therefore A \rightarrow B \end{array}$$

want to prove A .

assume $\neg A$

prove L

Because assume $\neg A$ led to a contradiction, A .

$$R(x, y) : |x| = |y|$$

Theorem

is not symmetric

\mathbb{Z}

Counterex: $R(1, -1)$ but not $R(-1, 1)$

Theorem: $R(x, y) : |x| = |y|$ is not symmetric over \mathbb{Z} .

that is, $\neg \forall x, y \in \mathbb{Z}. R(x, y) \rightarrow R(y, x)$

Proof. We proceed by contradiction.

Assume that R is symmetric. That is, assume $\forall x, y \in \mathbb{Z}. R(x, y) \rightarrow R(y, x)$. Consider $R(1, -1)$. Because $|1| = |-1|$, $R(1, -1)$ is true. Because $R(x, y) \rightarrow R(y, x)$, it must be the case that $R(-1, 1)$. But $-1 \neq 1$, so $R(-1, 1)$ is false. This is a contradiction.

Because assuming $R(x, y)$ was symmetric led to a contradiction, R must not be symmetric. \square

$R(x,y) : x = |y|$ is not symmetric

Contradiction

3 sym

- $\forall x \forall y R(x,y) \rightarrow R(y,x)$
 - $R(1, -1)$
 - $\neg R(-1, 1)$

$$x = 1, y = -1$$

$$\underbrace{R(1, -1)}_{\text{ }} \rightarrow R(-1, 1)$$

A hand-drawn diagram in red ink on a white background. It consists of a horizontal line segment with three vertical tick marks. Two tick marks are located on the left side of the segment, and one tick mark is located on the right side. The tick marks are drawn as short vertical lines extending slightly beyond the horizontal line.

$R(1, -1)$ but not $R(-1, 1)$

$$\cancel{x = -1}, \quad y = 1$$

~~$$R(-1,1) \rightarrow R(1,-1)$$~~

A hand-drawn diagram in red ink. It features a horizontal line with two vertical tick marks labeled 'T'. Above this line is a curved line that starts from the left, goes up to the right, and then curves back down towards the right.

$R(x,y) : x = |y|$ is not symmetric

$$\neg \forall_{x,y \in \mathbb{Z}}. R(x,y) \rightarrow R(y,x)$$

$$\exists_{x,y \in \mathbb{Z}}. \neg (R(x,y) \vee R(y,x))$$

$$\exists_{x,y \in \mathbb{Z}}. R(x,y) \wedge \neg R(y,x)$$

$$\frac{\begin{array}{c} x=1 \\ y=-1 \end{array} \in \mathbb{Z}}{R(1,-1) \wedge \neg R(-1,1)} \quad \begin{array}{c} + \\ T \\ + \end{array}$$

Proof. consider 1 and -1. $\underline{R(1,-1)}$ is true because $|1| = |-1|$; but $\underline{R(-1,1)}$ is false thus $\underline{R(1,-1) \wedge \neg R(-1,1)}$ because $-1 \neq |1|$. Because both 1 & -1 are integers, by existential instantiation we have $\exists_{x,y \in \mathbb{Z}}. \underline{R(x,y) \wedge \neg R(y,x)}$. Using double negation, De Morgan's law, and the def of Imp, thus is equivalent
 $\neg \forall_{x,y \in \mathbb{Z}}. R(x,y) \rightarrow R(y,x)$, which is the negation of the definition of R being symmetric. thus, R is non symmetric.

$$P(1) \models \exists x \in \mathbb{Z}. P(x)$$

$$\begin{array}{c} | \quad | \\ | \quad / \end{array} \quad \begin{array}{c} | \quad | \\ - \quad / \end{array} \quad P(3)$$

$$R(x,y) \wedge R(y,x)$$

$$\models \exists x \in \mathbb{Z}. P(x)$$

$$\models \exists x \in \mathbb{N}. P(x)$$

(x,y)

$P(x)$

$x \in S$

$\therefore \exists x \in S. P(x)$

} Existential
Instantiation