
MY OTHER

Proof by Contradiction

1. Proof.
2. We proceed by contradiction
3. → Assume $\underbrace{\qquad\qquad\qquad\qquad}_{\text{negation of theorem}}$.
4. ⋮ math + logic ⋮ A

   .... which is a contradiction. ¬A ... which contradicts $\underline{A\quad}$.
5. → Because assuming ____ led to contradiction, ________ .

1. $\square$

Thm. $\frac{1}{2} \notin \mathbb{Z}$

assum $\neg\left(\frac{1}{2} \notin \mathbb{Z}\right)$

$\downarrow \neg\neg$

$\frac{1}{2} \in \mathbb{Z} \xrightarrow{\text{def}} \exists x \in \mathbb{Z}.\ x = \frac{1}{2}$

$\downarrow$ instantiation

$\boxed{x = \frac{1}{2}}$

$\downarrow$ algebra

$2x = 1$

by Fundamental Theorem of Arithmetic, both sides have same unique prime factorization

but $2x$ has 2 in its prim factorization, but 1 does not.

$120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 1 \cdot 1 \cdot 1$

multiplicity 3

$1 =$

Every pos int has one and only one prime factorization

Proof.

We proceed by contradiction.

Assume $\frac{1}{2}$ is an integer. Let $x \in \mathbb{Z}$ be $\frac{1}{2}$, i.e. $x = \frac{1}{2}$ and $x \in \mathbb{Z}$. Then $2x = 1$. By F.T.o.A, $2x$ and 1 must have sam prime factors. But 2 is a factor of $2x$, and not a factr of 1, which is a contradiction.

Because assum $\frac{1}{2} \in \mathbb{Z}$ led to a contr, $\frac{1}{2} \notin \mathbb{Z}$. $\square$

$\frac{6}{2} \in \mathbb{Z}$

$\frac{\cancel{2} \cdot 3}{\cancel{2}} = 3$

$\frac{6}{2} = 3$

$6 = 6$

$2 \cdot 3 = 2 \cdot 3$

$\sqrt{2} \in \mathbb{Z}$

$(a \cdot b \cdot c \cdot c)^2 = a^2 \cdot b^2 \cdot c^4$

$x \in \mathbb{Z}$

$\sqrt{2} = x$

$\downarrow$

$2 = x^2$

F.T.o.A

2 has prim factor 2 w/ multiplicity 1, which is odd

x^2 has prime factor 2 w/ even multiplicity

$$\sqrt{2} \notin \mathbb{Q}$$

assum $\exists x \in \mathbb{Q} . \sqrt{2} = x$

$\exists n, d \in \mathbb{Z} . \frac{n}{d} = \sqrt{2}$

$n = 9 \quad 3^2$

$$\frac{n}{d} = \sqrt{2}$$

$n^2 = 3^4 \cdot 2^0$

$$n = d\sqrt{2}$$

2 w/ odd mult

$$n^2 = d^2 \cdot 2$$

2 w/ een mult

$\sqrt[3]{4} \notin \mathbb{Q}$

$\frac{n}{d} = \sqrt[3]{4}$

$\frac{n^3}{d^3} = 4$

$n^3 = 4d^3$

$n^3 = 2^2 d^3$

FTA

$\underbrace{2^{3k} \cdot m^3}_{\text{mult of 2}} = \underbrace{2^2 \cdot (2^{3c}}_{\text{mult of 2}} \cdot p^3)$

$3k = 3c+2$

Proof.

We proceed by contradiction.

assume $\sqrt[3]{4} \in \mathbb{Q}$. Then there is some $n, d \in \mathbb{Z}$. $\sqrt[3]{4} = \frac{n}{d}$.

rearranging, that means $4d^3 = n^3$. By FTA, $4d^3$ and n^3 must have same prime factorization. Consider the multiplicity of 2 in that factorization. Because n^3 is the cube of an integer, 2 must have a multiplicity that is a multiple of 3. Because $4d^3 = 2^2 d^3$, 2 must have a mult. that is 2 + a multiple of 3, which is a contradiction.

Because assume $\sqrt[3]{4} \in \mathbb{Q}$ leads to a contr, $\sqrt[3]{4} \notin \mathbb{Q}$. ∎