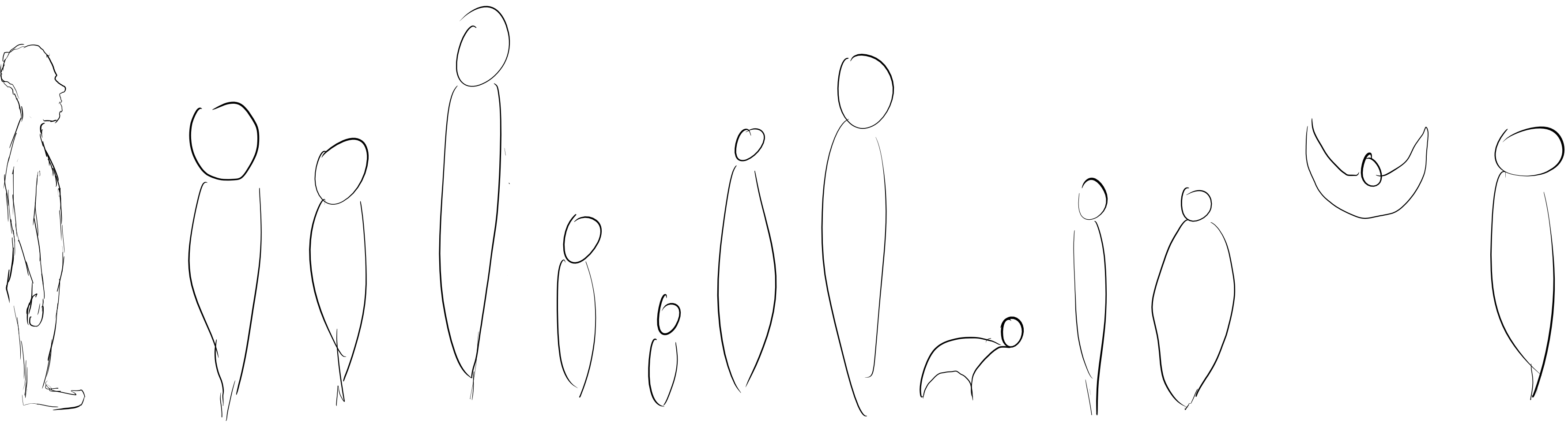

Fund Thm of Arithmetic

every pos int
- has a prime factorization
- unique prime factorization

P is set of primes

$\forall x \in \mathbb{Z}^+ . \exists S \subseteq P . \prod_{x \in S} x$

no duplicates

$\neg \left(\forall x \in \mathbb{Z}^+ . \; x \text{ has a prm factrzn} \right)$

$\equiv \; \exists x \in \mathbb{Z}^+ . \; x$ does not hav prm fa

let C be set of vales that lack prm factrizatn

$C \neq \{\}$

Cont

assume $C \neq \{\}$

by W.O.P C has a smallest membr

let n be that smallest member of C

n is not prime (it would be an prm fact)

n is not 1 (has empty prme factrzn)

$\exists p, q \in \mathbb{Z}^+ . \quad p \cdot q = n$

$p > 1$

$q > 1$

$\boxed{p \text{ or } q \text{ is non-prime}}$

$\boxed{p < n \quad q < n}$ means n is not smallest in C

why? why?

is n non-prime? why

Prime factorization

$$x = \underbrace{x_1 \cdot x_2 \cdot x_3 \cdots x_n}_{\text{all prime}}$$

let C be the set of positive integers that lack a prime factorization.

1. $1 \notin C$ $\qquad 1 = 2^0 \cdot 3^0 \cdot 5^0 \cdot 7^0 \cdot 11^0 \cdots$

$\qquad\qquad\qquad \boxed{2 = 2^1} \cdot 3^0 \cdot 5^0 \cdot 7^0 \cdot 11^0 \cdots$

2. x is prime $\rightarrow x \notin C$

3. $C \subset \mathbb{N}$

Assume $C \neq \{\}$ $\rightarrow$ By well ordering principle, C has a smallest element n.

by 1., $n > 1$

by 2., n is not prime

$\Big\}$ n is composite

$p \neq n$ composite
$p \leq n$ factor

$$\exists p, q \in (\mathbb{Z}^+ \setminus \{1\}) \,.\, p < n \wedge q < n \wedge p \cdot q = n$$

by def of C, n is not a product of primes

$p \cdot q = n \rightarrow$ p is not prime or q is not prime

if p and q both have prime factorization, so does n, $\bot$

otherwise, p or q (or both) lack a prime factorization, meaning they are in C, which contradicts n being the smallest $\in C$ $\bot$

$1+2+3+\dots+n = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

let C be set of integer $n \geq 1$ for which $\sum_{i=1}^{n} i \neq \frac{n(n+1)}{2}$

assume $C \neq \{\}$

by w.o.p C has a smallest element, x.

then $\sum_{i=1}^{x} i \neq \frac{x(x+1)}{2}$

$x \neq 1$ bks $\boxed{\sum_{i=1}^{1} i = 1 = \frac{1(2)}{2} = 1}$

Consd $y = x-1$

$$\boxed{\sum_{i=1}^{y} i} = \sum_{i=1}^{x-1} i = \left(\sum_{i=1}^{x} i\right) - x \neq \frac{x(x+1)}{2} - x =$$

$$\frac{x(x+1)-2x}{2} = \frac{x^2+x-2x}{2} = \frac{x^2-x}{2} = \frac{(x-1)x}{2} = \boxed{\frac{y(y+1)}{2}}$$

$y \in C \quad y < x \quad \perp$

$$\sum_{x \in S} f(x) = f(s_1) + f(s_2) + f(s_3) + \dots + f(s_n)$$

where $S = \{s_1, s_2, \dots s_n\}$

$$\sum_{x \in \{1,3\}} (x^2+1) = (1^2+1) + (3^2+1) = 12$$

$$\sum_{x=a}^{b} f(x) = \sum_{x \in S} f(x)$$

where $S = \{x \mid x \in \mathbb{Z} \wedge x \geq a \wedge x \leq b\}$

$$\sum_{x=-1}^{2} 2x = -2 + 0 + 2 + 4 = 4$$

$$\sum_{x=2}^{-1} 2x = \sum_{x \in \{\}} 2x = 0$$

$\prod$ is $\sum$ except multiply, not add

$$\prod_{x \in \{1,3,5\}} x = 15$$

$$\prod_{x=1}^{n} x = n!$$