When you come to a fork in the road...

... take it.

(instantiation)

direct proof

$x = y + z$

$w = f(x)$

if

else

Case analysis

By cases

Contradiction

mainly for impossible

for

induction

thm: $\neg\neg P \equiv P$

Proof: Either P is true or it is false

case P is true

$\neg\neg P \equiv \neg\neg T \equiv \neg\bot \equiv T \equiv P$

case P is false

$\neg\neg P \equiv \neg\neg\bot \equiv \neg T \equiv \bot \equiv P$

Because $\neg\neg P \equiv P$ in both cases, $\neg\neg P \equiv P$ in general. $\square$

P	$\neg$	$\neg$	P
T	T	$\bot$	
$\bot$	$\bot$	T	

disjunction

$A \vee B \leftarrow A \vee \neg A$

$A \vdash C$

$B \vdash C$

$\therefore C$

if (A)

else $\leftarrow \neg A$

Bad: $\boxed{P \vee Q \equiv T}$

Either P or Q ← might be false

disjunctive Tautology

Case P: $T \vee Q \equiv T$

$\equiv T$

Case Q: $P \vee T \equiv T$

Because held in all, holds in general

f(n):

    x = 0
    for i from 1 to n
        x += 1
    return x

case: $f(1)$ runs loop once, return $0+1=1$

case $f(2)$ runs loop twice, " $0+1+1=2$

case $f(3)$ " " 3 times " $0+1+1+1=3$

⋮

$f(n)$ runs loop once more than $f(n-1)$, returning $f(n-1)+1$

$\boxed{\forall n \in \mathbb{Z}^+} .\ f(n) = n$

Thm: $A \to B$

Proof: assume A. .... therefore B.

either $n=1$ or $n>1$

Base case:

~~case $n=1$~~:

$f(1)$ runs loop once, return $0+1=1=n$ $\Big]$ $f(1)=1$

Induction step:

~~case $n>1$~~:

assume $f(n-1)=n-1$. then $f(n)$ runs loop once more than $f(n-1)$, return $f(n-1)+1=n$ $\Big]$ $f(n-1)=n-1 \to f(n)=n$

By the principle of induction, $\forall n \in \mathbb{Z}^+ .\ f(n)=n$

Previous slide

$P(n) := f(n+1) = n+1$

The Principle of Induction

$P(0)$

$\forall k \in \mathbb{N}.\ P(k) \rightarrow P(k+1)$

$\forall k \in \mathbb{Z}^+.$

$\equiv \quad P(k-1) \rightarrow P(k)$

$\therefore\ \forall x \in \mathbb{N}.\ P(x)$

$P(0)$

$P(0) \rightarrow P(1) \qquad \therefore\ P(1)$

$P(1) \rightarrow P(2) \qquad \therefore\ P(2)$

```
g(n):
    x := 0
    for i from 1 to n
        x += i
    return x
```

0: 0
1: 1
2: 3
3: 6
4: 10
5: 15
6: 21

$\frac{n(n+1)}{2}$

$\forall n \in \mathbb{N} .\ g(n) = \frac{n(n+1)}{2}$

Base case:
$g(0) = 0$ Becase it doesn't run the loop

Inducive step: $g(n-1) = \frac{(n-1)n}{2} \rightarrow g(n) = \frac{n(n+1)}{2}$

assume $g(n-1) = \frac{(n-1)n}{2}$. Then $g(n)$ runs the loop once more than $g(n-1)$, return $g(n-1) + n$. $g(n) = \frac{(n-1)n}{2} + n = \frac{n^2 - n}{2} + \frac{2n}{2}$

$= \frac{n^2+n}{2} = \frac{n(n+1)}{2}$

By the principle of inducton, $\forall n \in \mathbb{N} .\ g(n) = \frac{n(n+1)}{2}$