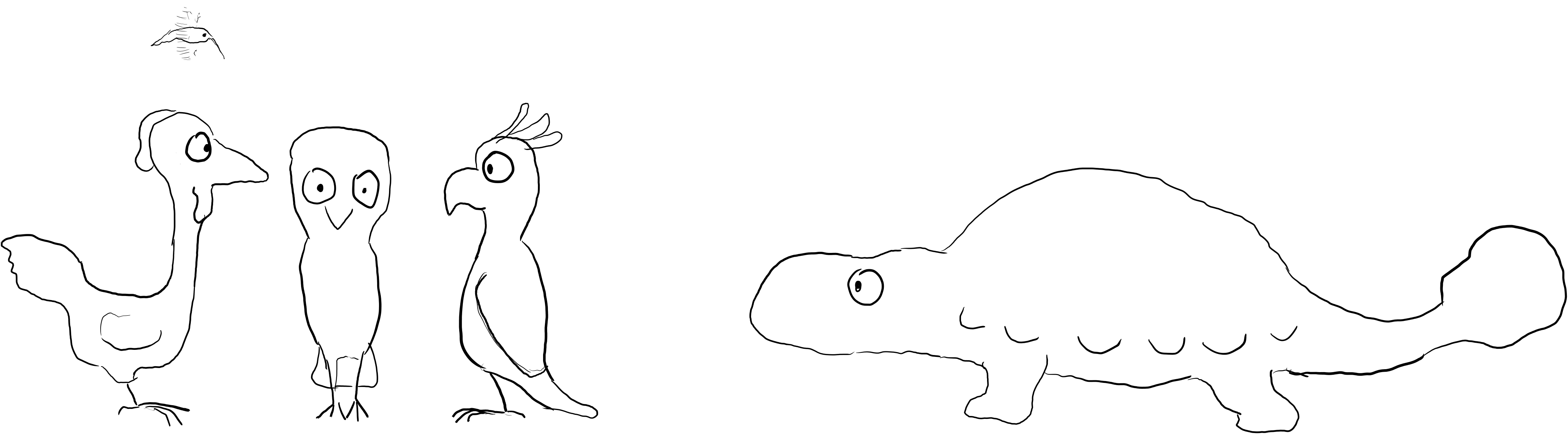

$$\sum_{k=1}^{4} k = 1+2+3+4 = 10$$

$$\sum_{k=a}^{b} f(k)$$

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sum = 0
k = a
while k ≤ b
    sum += f(k)
    k += 1
```

$$\sum_{k=-1}^{4} k = 9 \qquad = -1+0+1+2+3+4 = 9$$

$$f(n) = \sum_{k=1}^{n} k$$

n = -3

n = 3

0	-3
0	-2
0	-1
0	0
1	1
3	2
6	3

$$\sum_{k=4}^{4} k = \boxed{4}$$

0

$$\sum_{k=7}^{4} k = \cancel{22}$$

$\boxed{0}$

impossible

$4+5+6+7 = 22$

$$\sum_{k=1}^{4} 1 = 1+1+1+1 = 4$$

$$\sum_{k=-n}^{n} k = 0$$

$$-n + -(n-1) + \dots + (n-1) + n$$

$$\sum_{k=1}^{3} (2k-1) = 1+3+5 = 9$$

$$\sum_{k=3}^{6} k^2 = 9+16+25+36 = 86$$

$$P(0)$$

$$P(n) \vdash P(n+1)$$

$$\therefore \forall n \in \mathbb{N} .\ P(n)$$

$$Q(n) \stackrel{\text{def}}{=} P(n-3)$$

$$Q(3)$$

$$Q(n) \vdash Q(n+1)$$

$$\therefore \forall n \in \mathbb{Z} .\ n \geq 3 \rightarrow Q(n)$$

$P(0)$

$P(n) \to P(n+1)$

$\therefore \forall n \in \mathbb{N} . P(n)$

$$\boxed{\sum_{k=1}^{n} 1 = n \quad \forall n \in \mathbb{Z}^+}$$

$$\overset{k=1}{1} + \overset{2}{1} + \overset{3}{1} + \overset{4}{1} = 4$$

$$\sum_{k=a+1}^{b+1} f(k) = \cancel{f(a)} + f(a+1) + \ldots + f(b-1) + f(b) + f(b+1)$$

Base $n = 1$

$$\sum_{k=1}^{n} 1 = \sum_{k=1}^{1} 1 = 1 = n$$

Inductive

assume $\sum_{k=1}^{n} 1 = n$ for some $n \in \mathbb{Z}^+$

$$\underbrace{1 + \sum_{k=1}^{n}}= n + 1$$

$$\underbrace{1 + (\underbrace{1 + 1 + \ldots + 1}_{n \text{ 1's}})}_{n+1 \text{ 1's}}$$

$$\sum_{k=1}^{n+1} 1 = n + 1$$

By principle of induction, $\forall n \in \mathbb{Z}^+ . \sum_{k=1}^{n} 1 = n$

$P(0)$

$P(n) \rightarrow P(n+1)$

$\therefore \forall n \in \mathbb{N} \, . \, P(n)$

$\forall n \in \mathbb{Z}^+ , n \geq 3 \rightarrow$

$$\boxed{\sum_{k=3}^{n} 1 = n-2}$$

$$\begin{array}{cccc} k=1 & 2 & 3 & 4 \\ 1 + & 1 + & 1 + & 1 = 4 \end{array}$$

$$\sum_{k=a+1}^{b+1} f(k) = \cancel{f(a)} + f(a+1) + \ldots + f(b-1) + f(b) + f(b+1)$$

Base $n = 3$

$$\sum_{k=3}^{n} 1 = \sum_{k=3}^{3} 1 = 1 = n-2$$

Inductive

assume $\displaystyle\sum_{k=3}^{n} 1 = n-2$ ar some $n \in \mathbb{Z}^+ . n \geq 3$

$$\underbrace{1 + \sum_{k=3}^{n}}\ = \overset{n-2}{\cancel{n}} + 1$$

$$\underbrace{1 + \underbrace{(1 + 1 + \ldots + 1)}_{n \text{ 1's}}}_{n+1 \text{ 1's}}$$

$$\sum_{k=3}^{n+1} 1 = \overset{(n-2)}{\cancel{n}} + 1$$

$(n+1)-2$

By principle of induction, $\forall n \in \mathbb{Z}^+_{n \geq 3} \displaystyle\sum_{k=3}^{n} 1 = n-2$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$1 + 2 + 3 + \dots + (n-1) + n + (n+1)$

Base $n=1$

$$\sum_{k=1}^{1} k = 1 = \frac{1(2)}{2} = \frac{n(n+1)}{2}$$

Inducive step:

assume $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ for som $n \in \mathbb{Z}^+$

$$\underbrace{(n+1) + \sum_{k=1}^{n} k}_{\displaystyle\sum_{k=1}^{n+1} k} = \frac{n(n+1)}{2} + (n+1)$$

$$= \frac{n^2 + n + 2n + 2}{2} = \frac{n^2 + 3n + 2}{2}$$

$$= \frac{(n+1)(n+2)}{2}$$

$$\sum_{k=1}^{n+1} k = \frac{(n+1)((n+1)+1)}{2}$$

$$\sum_{k=2}^{n} 2k = n^2 + n - 2$$

Base case : $n = \underline{2}$

$$\sum_{k=2}^{n} 2k = \sum_{k=2}^{2} 2k = 4 = 4+2-2 = 2^2+2-2 = n^2+n-2$$

inducive step :

assume $\sum_{k=2}^{n} 2k = n^2 + n - 2$ for som $n \in \mathbb{Z}, n \geq 2$

$$\underbrace{2(n+1) + \sum_{k=2}^{n} 2k}_{\displaystyle \sum_{k=2}^{n+1} 2k} = n^2+n-2 + 2(n+1)$$

$$= \quad \vdots \text{ algebra}$$

$$\sum_{k=2}^{n+1} 2k = (n+1)^2 + (n+1) - 2$$

$$\sum_{k=n}^{2n} k = \frac{3n(n-1)}{2}$$

$$\cancel{n} + (n+1) + \dots + (2n-1) + 2n + (2n+1) + (2n+2)$$

$$-2 + 2n+1 + 2n+2$$