$$\forall x \in (\mathbb{N} \cup \{-1\}) \sum_{i=n}^{2n} i = \frac{3n(n+1)}{2}$$

$$n + (n+1) + \dots + (2n-1) + (2n)$$

$$(n+1) + \dots + (2n-1) + (2n) + (2n+1) + (2n+2)$$

0

1

$$\sum_{i=1}^{2} i = 1 + 2 = 3$$

$$\boxed{\sum_{i=0}^{0} i = 0}$$

$$\sum_{i=-1}^{-2} i = 0$$

Base case: $n = -1$

$$\sum_{i=-1}^{-2} i = 0 = \frac{3(-1)(0)}{2}$$

Base case: $n = 0$

$$\sum_{i=0}^{0} i = 0 = \frac{3(0)(+1)}{2}$$

Inductive Step:

Assume $\sum_{i=n}^{2n} i = \frac{3n(n+1)}{2}$ for some $n \geq 0$

$$(-n + (2n+1) + (2n+2)) + \sum_{i=n}^{2n} i = \frac{3n(n+1)}{2} + \overbrace{(-n + (2n+1) + (2n+2))}^{3n+3}$$

$$\frac{3n^2 + 3n + 6n + 6}{2}$$

$$\sum_{i=n+1}^{2(n+1)} i = \frac{3(n^2 + 3n + 2)}{2} = \frac{3(n+1)\overset{((n+1)+1)}{(n+2)}}{2}$$

Choose A $\overset{\text{one from}}{\text{or}}$ B $\boxed{+}$

Choose A $\boxed{\text{and}}$ B $\boxed{\times}$

$W = \{2, 3\} = T \setminus \{x\} \quad x \in T$

$T = \{2, 3, 4\}$

$$P(T) = \{\underbrace{\boxed{\{\}, \{2\}, \{3\}, \{2,3\}}}_{P(w)}, \underbrace{\{4\}, \{4,2\}, \{4,3\}, \{4,2,3\}}_{S \in P(w),\ S \cup \{4\}}\}$$

$|\mathcal{P}(S)| = 2^{|S|}$

Base: $S = \{\}$ $\quad |S| = 0$

$\mathcal{P}(\{\}) = \{\{\}\}$ $\quad |\mathcal{P}(S)| = 1$ $\quad 2^0 = 1$

Inductive:

Assume $|\mathcal{P}(S)| = 2^n$ when $|S| = n \overset{\in \mathbb{N}}{\geq 0}$

Consider T where $|T| = n+1$.

let $x \in T$ and $W = T \setminus \{x\}$

then $|W| = n$ so $|\mathcal{P}(W)| = 2^n$

$$\mathcal{P}(T) = \mathcal{P}(W) \cup \{Q \cup \{x\} \mid Q \in \mathcal{P}(W)\}$$

$$|\mathcal{P}(T)| = |\mathcal{P}(W)| + \underbrace{|\{Q \cup \{x\} \mid Q \in \mathcal{P}(W)\}|}_{|P(W)|} = 2|P(W)| = 2 \cdot 2^n$$

$$\boxed{|\mathcal{P}(T)| = 2^{n+1}}$$

Proof.

We proceed by induction on the cardinality of S

Base case: $|S| = 0$. Then $S = \{\}$, $P(S) = \{\{\}\}$ and $|P(S)| = 1 = 2^0 = 2^{|S|}$

Inductive step:

Assume $|P(S)| = 2^n$ for all S where $|S| = n$

Consider T where $|T| = n+1$. Because $|T| \geq 1$, $\exists x \in T$. Let $W = T \setminus \{x\}$, meaning $|W| = n$ so $|\mathcal{P}(W)| = 2^n$. Every subset of T either doesn't contain x and is thus part of $P(W)$, or does contain x and is thus part of $M = \{Q \cup \{x\} \mid Q \in P(W)\}$.

$|M| = |P(W)|$ because each elem of $P(W)$ corresponds to one element of M, and $M \cap P(W) = \{\}$.

$P(T) = P(W) \cup M$, and $|P(T)| = |P(W)| + |M| = 2|P(W)| = 2 \cdot 2^n = 2^{n+1}$

By principle of induction, it follows that $|P(S)| = 2^{|S|}$ $\forall S$ with $|S| \in \mathbb{N}$. $\square$