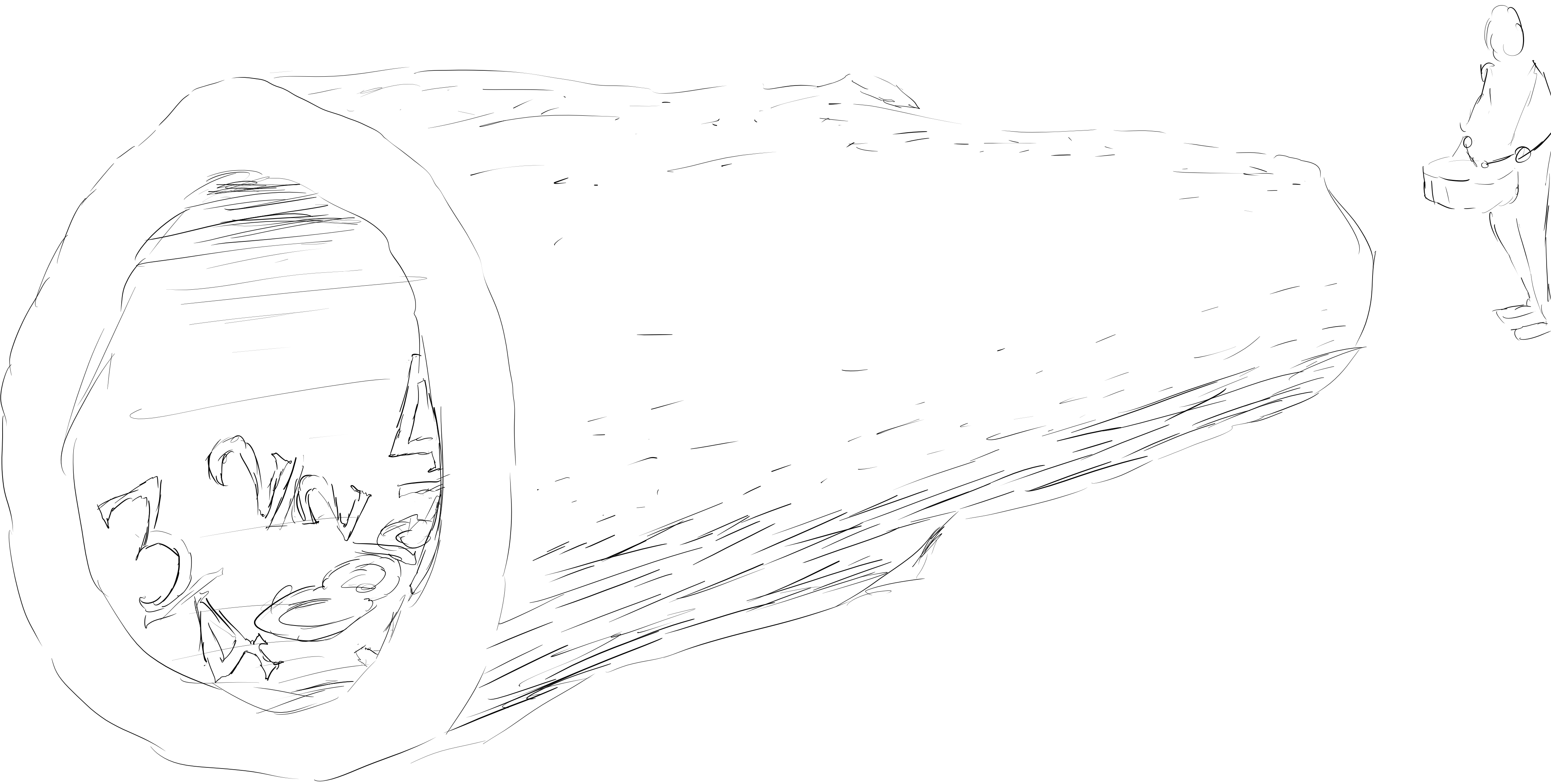

Theorem: $\log_2(3) \notin \mathbb{Q}$

Proof.

By contradiction, assum $\log_2(3) \in \mathbb{Q}$.

then $\overset{\text{Positive}}{\text{integer}}$ x and y such that $\log_2(3) = \frac{x}{y}$

$\hookrightarrow \neq 0$

$$3 = 2^{x/y} = \sqrt[y]{2^x}$$

$$y \log_2(3) = x$$

$$\log_2(3^y) = x$$

$$3^y = 2^x$$

$$3^y = 2^x$$

F.T. of Arithmatic 3^y and 2^x must have same prime factors

Theorem: $\log_8(9) \notin \mathbb{Q}$

Proof.

By contradiction, assume $\log_8(9) \in \mathbb{Q}$.

then positive integer x and y such that $\log_8(9) = \frac{x}{y}$

$\hookrightarrow \neq 0$

$y \log(9) = x$

$\log(9^y) = x$

$9^y = 8^x$

$(3 \cdot 3)^y = (2 \cdot 2 \cdot 2)^x \longrightarrow 3^{2y} = 2^{3x}$

$3 = 2^{x/y} = \sqrt[y]{2^x}$

$3^y = 2^x$

F.T. of Arithmetic 3^{2y} and 2^{3x} must have same prime factors

Theorem: $\log_8(4) \cancel{\notin \mathbb{Q}} = \frac{2}{3}$

$\log_8(4) = \frac{2}{3}$

$\log_8(4^3) = 2$

$\log_8(8^2) = 2$

Proof.

By contradiction, assume $\log_8(4) \in \mathbb{Q}$.

then positive integer x and y such that $\log_8(4) = \frac{x}{y} = \frac{2}{3}$

↳ $\neq 0$

$y \log(4) = x$

$\log(4^y) = x$

$4^y = 8^x$

$2^{2y} = 2^{3x}$

$x = 2$

$y = 3$

~~F.T. of Arithmetic 3^{2y} and 2^{3x} must have same prime factors~~

$$\log_{\sqrt{3}}(5) = \log_3(\overset{25}{\cancel{x}})$$

$$\log(a \cdot b) \qquad \log(a) + \log(b)$$

$$\log(a^b) \qquad b\log(a)$$

$$\log\left(\frac{a}{b}\right) \qquad \log(a) - \log(b)$$

$$\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$$

$$\frac{\log_3(5)}{\frac{1}{2}} = \log_3(x)$$

$$2\log_3(5) = \log_3(x)$$

$$\log_3(5^2)$$

$$\log_3(25) = \log_3(x)$$

$$25 = x$$

$$\log_{\sqrt{3}}(5) = \log_3\left(3^{\log_{\sqrt{3}}(5)}\right)$$

$$\boxed{3^{\log_{\sqrt{3}}(5)}} = 3^{\log_3(x)} = x$$

$$\log_{b^a}(x^a) \stackrel{?}{=} \log_b(x)$$

$$\frac{\log_b(x^a)}{\log_b(b^a)} = \frac{\cancel{a}\log_b(x)}{\cancel{a}\,\cancel{\log_b(b)}} = \log_b(x)$$

$\log_b(x) = \left(\log_b(a)\right)^2$

$\frac{\log_b(x)}{\log_b(a)} = \log_b(a)$

$x = \boxed{a^{\log_b(a)}}$

$\log_a(x) = \frac{\log_a(a)}{\log_a(b)} = \log_b(a)$

$\log(a \cdot b) = \log(a) + \log(b)$ ✗

$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$ ✗

$\log(a^b) = b \log(a)$ ✗

$\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$ ✗

$\log_a(b) = x \equiv a^x = b$

$\log_a(b) \cdot \log_a(x) = 1 = \log_a(a)$

$\log_a\left(x^{\log_a(b)}\right) = 1$

$x^{\log_a(b)} = a$

$x = \boxed{a^{\frac{1}{\log_a(b)}}} = \boxed{a^{(\log_a(b))^{-1}}}$

$\log_a(b) \cdot \log_a(a-b) = 1$

$\log_a\left((a-b)^{\log_a(b)}\right) = 1$

$$3^{\log_5(7)} = 7^{\log_{\boxed{5}}(\boxed{3})}$$

$$7^{\log_7\left(3^{\log_5(7)}\right)}$$

$$\log_7\left(3^{\log_5(7)}\right) = \log_{\square}(\square)$$

$$\log_5(7) \cdot \log_7(3) = \log_{\square}(\square)$$

$$\cancel{\log_5(7)} \cdot \frac{\log_5(3)}{\cancel{\log_5(7)}}$$

$$\log_3\left(3^{\log_5(7)}\right) = \log_3\left(7^{\log_{\square}(\square)}\right)$$

$$\log_5(7) = \log_{\square}(\square) \cdot \log_3(7)$$

$$\log_5(3) = \frac{\log_5(7)}{\frac{\log_5(7)}{\log_5(3)}} = \frac{\log_5(7)}{\log_3(7)} = \log_{\square}(\square)$$