

Throughout this quiz, use quotes to delimit the ends of strings.

PROBLEM 1 *Products and Powers*

Write out the following in full.

$$\{1, 2\} \times \{3\} \times \{1, 4\} = \{(1, 3, 1), (1, 3, 4), (2, 3, 1), (2, 3, 4)\}$$

$$\{56\}^3 = \{(56, 56, 56)\}$$

$$\{1, 2\} \times \mathcal{P}(\{1\}) = \{(1, \{\}), (1, \{1\}), (2, \{\}), (2, \{1\})\}$$

$$\{(1, 2)\} \times \mathcal{P}(\{(1, 2, 3, 4)\}) = \{(1, 2, \{\}), (1, 2, \{(1, 2, 3, 4)\})\}; \text{ some uses will treat singleton sets as their one element to get } \{(1, 2, \{\}), (1, 2, 1, 2, 3, 4)\} \text{ or chose not to flatten to get } \{((1, 2), \{\}), ((1, 2), \{(1, 2, 3, 4)\})\} \text{ or } \{((1, 2), \{\}), ((1, 2), (1, 2, 3, 4))\} \text{ instead.}$$

$$\{a, b\}^2 = \{“aa”, “ab”, “ba”, “bb”\}$$

PROBLEM 2 *Members of Products and Powers*

Give two different example members of each of the following sets. Make them different from one another: different lengths, different internal patterns, etc., is the set allows that. If there are not enough elements of the set to give two different elements, leave some blanks blank.

$$\{a, b, c\}^4 \text{ contains “} \underline{aaaa} \text{” and “} \underline{cbac} \text{”}$$

$$\{a, b, c\}^1 \text{ contains “} \underline{a} \text{” and “} \underline{c} \text{”}$$

$$\{a, b, c\}^0 \text{ contains “} \underline{”} \text{ and } \underline{\hspace{2cm}}$$

$$\{a, b, c\}^* \text{ contains “} \underline{ba} \text{” and “} \underline{cababacc} \text{”}$$

$$\{“good”, “fun”\}^2 \text{ contains “} \underline{goodgood} \text{” and “} \underline{fungood} \text{”}$$

PROBLEM 3 *Subsequences*

Definition 1 A *subsequence* is a sequence that can be derived from another sequence by deleting zero or more elements without changing the order of the remaining elements.

What are the subsequences of the string "OK"? "", "O", "K", "OK"

What is the longest subsequence shared by "MATHEMATICS" and "COMPUTERS"? "MTES"

PROBLEM 4 *Images of Functions*

What is the image of $\{1, 2, 3\}$ under $R(x) = |x|$? $\{1, 2, 3\}$

What is the image of $\{-1, 0, 1\}$ under $R(x) = |x|$? $\{0, 1\}$

If $R(x) = |x|$, what is the image of $\{1, 2, 3\}$ under R^{-1} ? $\{-3, -2, -1, 1, 2, 3\}$

If $R(x) = |x|$, what is the image of $\{-2, 1, 3\}$ under $R^{-1} \circ R$? $\{-3, -2, -1, 1, 2, 3\}$

If $R(x) = |x|$, what is the image of $\{-2, 1, 3\}$ under $R \circ R^{-1}$? $\{1, 3\}$

PROBLEM 5 *Properties of Relations*

For the following, assume the domain and codomain are \mathbb{R} and you are limited to pre-calc math unless otherwise specified.

Give an example function that is not total: $f(x) = \sqrt{x}$

Give an example function that is total but not invertible: $f(x) = x^2$

Give an example function that is invertible: $f(x) = x$

See also §4 Problems 4.12–4.33