Name: ____ CompID: ____________________

CS 2102 - DMT1 - Fall 2019 — Luther Tychonievich
Administered in class friday october 31, 2019

Quiz 08

Theorem 1 $\forall n \in \mathbb{N} \,.\, \sum_{x=0}^{n} \frac{1}{2^x} = \frac{2^{n+1}-1}{2^n}$

PROBLEM 1 *Proof by Induction*

Prove the above theorem using induction.

Proof.

We proceed by induction.

Base Case When $n = 0$ we have $\sum_{x=0}^{0} \frac{1}{2^x} = 1$ and $\frac{2^1 - 1 = 1}{2^0 = 1} = 1$, so the theorem holds for $n = 0$.

Inductive step Assume the theorem holds for some $n \in \mathbb{N}$: that is, $\sum_{x=0}^{n} \frac{1}{2^x} = \frac{2^{n+1}-1}{2^n}$. Adding $\frac{1}{2^{n+1}}$ to both sides, we have $\frac{1}{2^{n+1}} + \sum_{x=0}^{n} \frac{1}{2^x} = \frac{1}{2^{n+1}} + \frac{2^{n+1}-1}{2^n}$; the left-had side is equivalent to $\sum_{x=0}^{n+1} \frac{1}{2^x}$ by the definition of summation; the right-hand side can be rearranged to get $\frac{1+2(2^{n+1}-1)}{2^{n+1}} = \frac{2^{n+2}-1}{2^{n+1}}$; this means that $\sum_{x=0}^{n+1} \frac{1}{2^x} = \frac{2^{n+2}-1}{2^{n+1}}$, or in other words that the theorem holds for $n+1$.

By the principle of induction, the theorem holds for all $n \in \mathbb{N}$.

□