assume　　contradiction

$\boxed{\neg A} \vdash \boxed{\bot}$

$\therefore A$

Well-ordering principle

$S \subseteq \mathbb{N} \models S$ has a smallest element

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

$1 + 2 + 3 + \dots + n-1 + n$

$$\sum_{i=-2}^{2n} i^2 + i$$

$((-2)^2 + -2) + ((-1)^2 + -1) + (0^2 + 0) + (1^2 + 1) + \dots + ((2n)^2 + 2n)$

$$\prod_{i=1}^{n} i^2$$

$1^2 \times 2^2 \times 3^2 \times \dots \times n^2$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Pf.

by contradiction w/ W.O.P.

$B = \{ x \mid \text{theorem false for } x \}$

1. made a set of counterexamples

Consider set of $B \subseteq \mathbb{N}$. theorem does not hold for $x \in B$

By W.O.P, there is a smallest element of B. call that element K.

$k \neq 0$ bcs. $\sum_{i=1}^{0} i = 0$

$\therefore k > 0$

$\therefore r \in \mathbb{N}$

2. Showed it has no smallest elem.

bcs $k \in B$, $\boxed{\sum_{i=1}^{k} i \neq \frac{k(k+1)}{2}}$ let $r = k-1$. Bcs $r \notin B$,

$\sum_{i=1}^{r} i = \frac{r(r+1)}{2}$

↳ B cant

$$\sum_{i=1}^{k-1} i = \frac{(k-1)(k)}{2}$$

$$k + \sum_{i=1}^{k-1} i = \frac{(k-1)(k)}{2} + k$$

$$\sum_{i=1}^{k} i = \frac{(k-1)(k) + 2k}{2}$$

$$= \frac{k^2 - k + 2k}{2}$$

$$= \frac{k^2 + k}{2}$$

$$\boxed{\sum_{i=1}^{k} i = \frac{k(k+1)}{2}}$$

contradict

3. no smallest falsify, none at all

$\therefore$ true

Bcs assum B had elements led to contr., B must $= \emptyset$

means $\sum_{i=1}^{n} = \frac{n(n-1)}{2}$ is true

for all $n \in \mathbb{N}$

$\frac{1}{2} \notin \mathbb{Z}$

assn $x \in \mathbb{Z}$ $x = \frac{1}{2}$

$2x = 1$

f.t.a. 2 is a factor of $2x$

2 is a facor of 1

$\perp$

$\therefore \nexists x \in \mathbb{Z} . x = \frac{1}{2}$

$\frac{1}{2} \notin \mathbb{Z}$

$\sqrt{2} \notin \mathbb{Q}$

assum $\sqrt{2} \in \mathbb{Q}$

$\exists x, y \in \mathbb{Z}.\ \frac{x}{y} = \sqrt{2} \quad \wedge \gcd(x, y) = 1 \wedge y \geq 1$

$60 = 2 \cdot 2 \cdot 3 \cdot 5$

$2^2 \cdot 3^1 \cdot 5^1 \cdot 7^0$

multiplicity

$$\frac{x}{y} = \sqrt{2}$$

$$x = y\sqrt{2}$$

$$x^2 = y^2 2$$

$x = 2^k \cdot w$

$x^2 = 2^{2k} \cdot w^2$

x^2 has an even multiplicity of 2s in its factors

y^2

$2y^2$ has odd

$\nexists$ smallest $x \in \mathbb{R}^+$

assume k is the smallest positive real number.

then $k > 0$

consider $\frac{k}{2}$

$\frac{k}{2} < k$ b/c $k > 0$

$\frac{k}{2} > 0$ b/c $k > 0$

$\frac{k}{2} \in \mathbb{R}$ b/c $\mathbb{R}^+$ closed under $\div$

$\therefore$ $\frac{k}{2}$ smaller element of $\mathbb{R}^+$ than it's smallest, $\perp$

Thm no largest addition function

Pf

Ass $\exists$ is a largest. let C be the code of that funct

define C' to be C, except the return $\underline{\qquad\qquad}$ is replaced

by return $\underline{\qquad\qquad}$ $+0$