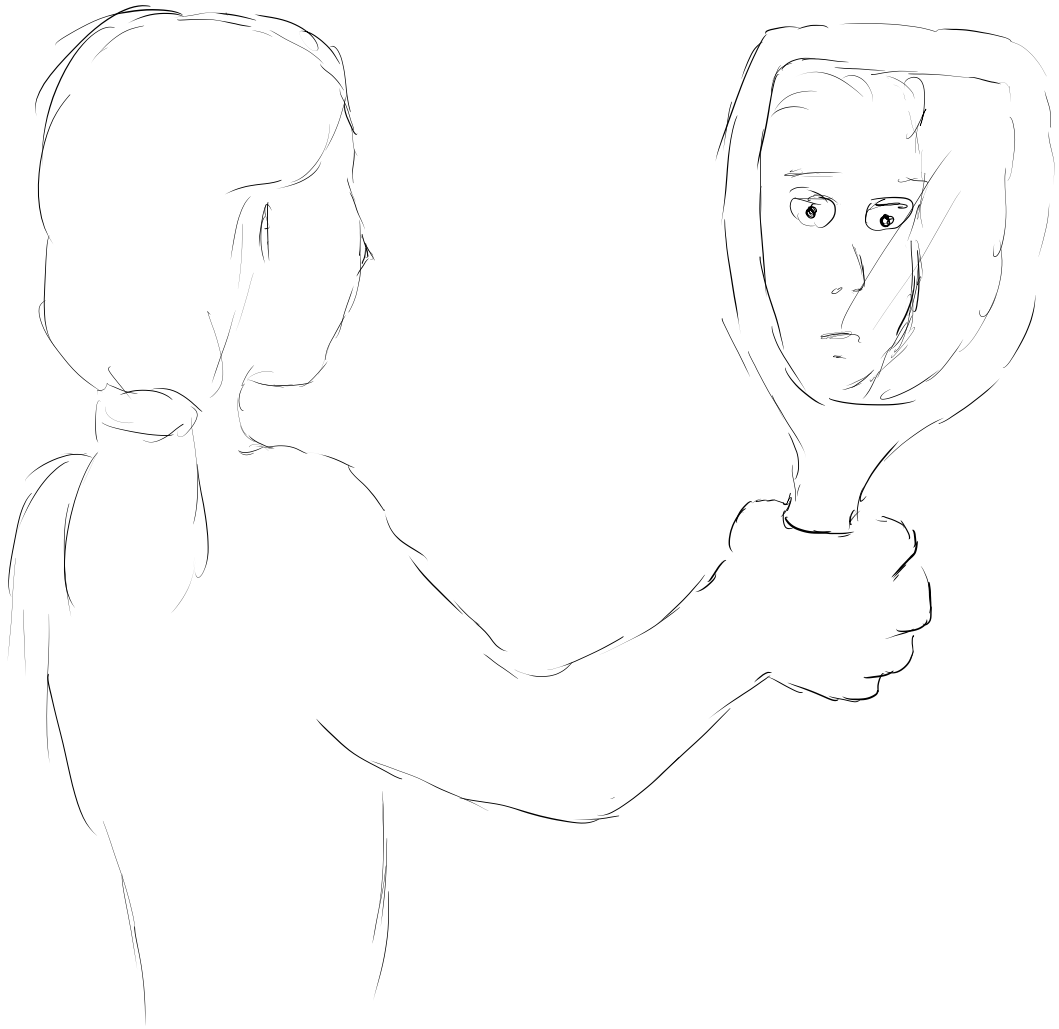


function - no state
no side-effects
1 result per args

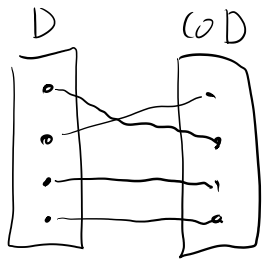
$\sqrt{4}$ → 2, -2

$f(x) = x^2$

f: ℝ → ℝ
f: fn, domain
ℝ → ℝ: type, codomain

total / Partial
codomain / range

surjective - onto → codomain = range - ok sampling
injective - 1-to-1 → diff arg → diff result - ok encoding
bijective - invertible correspondence → - ok representation



↳ invertible.

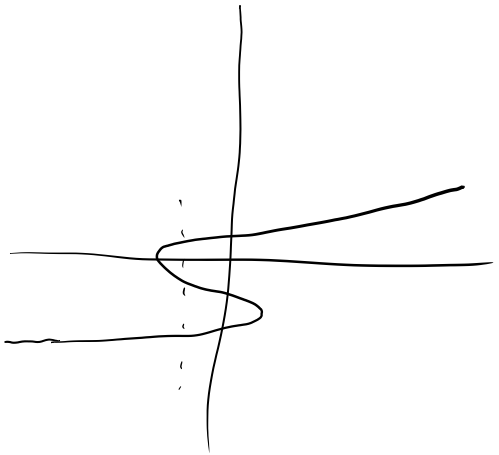
∀ bijective f . ∃ f^{-1} - undo
total, total

$f : \mathbb{R} \to \{\top, \bot\}$

$f(x) = x < 0$

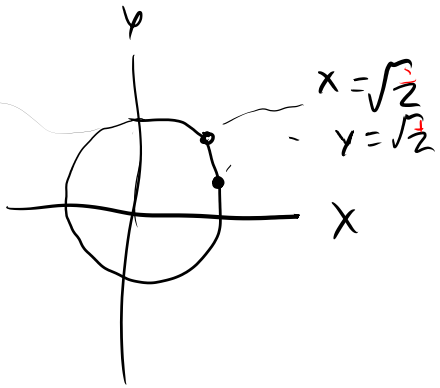
Relation

- function that can have multiple outputs
- function of 2+ arg that has Booleans as its codomain



$x^2 + y^2 = 1$

$y = \pm\sqrt{x^2 - 1}$



$R(x, y)$: T if $x^2 + y^2 = 1$, ⊥ otherwise

$\sqrt{\frac{1}{2}}$ is related to $\sqrt{\frac{1}{2}}$ by circle relation

- b is related to 8 by circle

$R(x, y)$ ~ Predicate

bool $R(\sim x, \sim y)$

$x = y \to \top, \bot$

$x < y \to \top, \bot$

mirror

- Reflexive : $\forall x. R(x,x)$ = $\quad \forall x\ \neg R(x,x)$ $<$
  irreflexive

- Symmetric : $\forall x,y.\ R(x,y) \iff R(y,x)$ =
  antisymmetric $\quad \forall x,y.\ x \neq y \to R(x,y) \to \neg R(y,x)$ $<$ $\leq$

- Transitive

$$\forall x,y,z.\ \big(R(x,y) \wedge R(y,z)\big) \to R(x,z) \quad \begin{matrix} = \\ < \\ \leq \end{matrix}$$

$R(x,y)$ x is an ancestor of y
- irreflexive
- antisymmetric
- transitive

$R(x,y)$ x is a friend of y
- ¬ reflexive
- ¬ irreflexive

def friend: two people who like eachother
- symmetric
- irreflexive

nor transitive

equivalence Relation

reflexive
symmetric
transitive

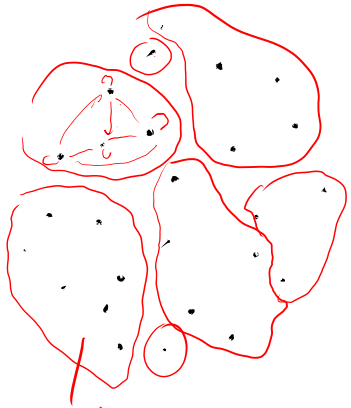
— Partition

==

.equals()

is

===



all equiv to each other

Partial
Total Order

antisymmetric
Transitive

< <= ... compareTo Comparator cmp

lessThan(x, y):

partial
not an order

given a function f, answer "is this ——"
relation