Theory of Computation
CS3102 – Spring 2014
A tale of computers, math, problem solving, life, love and tragic death

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www.cs.virginia.edu/~njb2b/theory
Theory of Computation (CS3102) - Textbook

Textbook:


Good Articles / videos:

www.cs.virginia.edu/~robins/CS_readings.html
Supplemental reading:

*How to Solve It*, by George Polya (MIT), Princeton University Press, 1945

- A classic on problem solving

George Polya (1887-1985)
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Theory of Computation - Themes

- Nature of Computation
- Computing Machines
- Underlying Principles
- Applications
- “Standing on the shoulders of giants”
- Problem Solving and Creativity
- Self Discipline
- Fun!
Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?

• What approaches fail?
• What techniques work and why?
• Lessons and generalizations
Prerequisites

• Some **discrete math** knowledge
• Course will “**bootstrap**” from **first principles**
• **Tenacity, patience**
Course Organization

• **Exams**: probably take home
  • Decide by vote
  • Flexible exam schedule

• **Homeworks**:
  • Lots of problem solving
  • Work in groups!
  • Not formally graded
  • Many exam questions will come from homeworks!

• **Extra credit problems**
  • In class & take-home
  • Find mistakes in slides, handouts, materials

• Course materials posted on Web site
  www.cs.virginia.edu/~njb2b/theory
Grading Scheme

- Midterm 35%
- Final 35%
- Project 30%
- Extra credit 10%

Best strategy:
- Solve lots of problems!
Contact Information

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www.cs.virginia.edu/~njb2b/theory

Office hours: after class
• Any other time
• By email (preferred)
• By appointment
• Q&A blog posted on class Web site
TAs

**Mustafiz Rahman:**
Office: Rice 430
Office Hours: Tuesday/Thursday 11:15am-12:15pm
 Monday/Wednesday 11:30am-12:30pm

**Frank Feng:**
Office: Rice 430
Office Hours: Wednesday 1:30pm-2:30pm
 Friday 2:30pm-3:30pm

**Rachel Brown:**
Office: Thornton Stacks
Office Hours: Tuesday/Thursday 6:00pm-7:00pm
Problem Solving Session!

Mondays 7:30pm-8:30pm
Location TBA
Co-located with the graduate students
FREE PIZZA!
Good Advice

- Ask questions ASAP
- Do homeworks ASAP
- Work in study groups
- Do not fall behind
- “Cramming” won’t work
- Start on project early
- Attend every lecture
- Read Email often
- Solve lots of problems
Supplemental Readings
www.cs.virginia.edu/robins/CS_readings.html

• Great videos:
  – Randy Pausch's "Last Lecture", 2007
  – Randy Pausch's "Time Management“, 2007
  – "Powers of Ten", Charles and Ray Eames, 1977
Supplemental Readings
www.cs.virginia.edu/robins/CS_readings.html

• **Theory and Algorithms:**
  – *Who Can Name the Bigger Number*, Scott Aaronson, 1999
Supplemental Readings
www.cs.virginia.edu/robins/CS_readings.html

• Biological Computing:
  – Big Lab on a Tiny Chip, Charles Choi, Scientific American, October 2007, pp. 100-103.
Supplemental Readings
www.cs.virginia.edu/robins/CS_readings.html

• **Quantum Computing:**
  – Black Hole Computers, Seth Lloyd and Jack Ng, Scientific American, November 2004, pp. 52-61.
Supplemental Readings
www.cs.virginia.edu/robins/CS_readings.html

• **History of Computing:**

• **Security and Privacy:**
Supplemental Readings
www.cs.virginia.edu/robins/CS_readings.html

• Future of Computing:
  – Ballbots, Ralph Hollis, Scientific American, October 2006, pp. 72-77.
Supplemental Readings
www.cs.virginia.edu/robins/CS_readings.html

• **The Web:**

• **The Wikipedia Computer Science Portal:**
  – Theory of computation and Automata theory
  – Formal languages and grammars
  – Chomsky hierarchy and the Complexity Zoo
  – Regular, context-free & Turing-decidable languages
  – Finite & pushdown automata; Turing machines
  – Computational complexity
  – List of data structures and algorithms
Supplemental Readings
www.cs.virginia.edu/robins/CS_readings.html

• The Wikipedia Math Portal:
  – Problem solving
  – List of Mathematical lists
  – Sets and Infinity
  – Discrete mathematics
  – Proof techniques and list of proofs
  – Information theory & randomness
  – Game theory

• Mathematica's “Math World”
THE PROBLEM WITH WIKIPEDIA:

TACOMA NARROWS BRIDGE

SUSPENSION BRIDGE

STRUCTURAL COLLAPSE

THREE HOURS OF
FASCINATED CLICKING

24-HOUR ANALOG DIAL

WILLIAM HOWARD TAFT

LESBIANISM IN EROTICA

FATAL HILARITY

TAYLOR HANSON

COTTON T-SHIRT

WET-T-SHIRT CONTEST

WIKIFRIENDS:

I REALLY LIKED
THAT MOVIE.

I HATED
THAT MOVIE.

ME TOO.
What is a computer?
Wim Klein (1912-1986)
“CERN’s First Supercomputer”

• Circus Performer in the 40s and 50s
• Hired by CERN in 1958
• Replaced by a machine in 1976
• Additional theme: Whose jobs are safe?
• Murdered in 1986
Why Study Theory?

What if we used something other than steam?
Why Study Theory?

Nicolas Lèonard Sadi Carnot (1796-1832)

• Invented the Carnot Engine
• Model of any heat engine
• Independent of specifics of construction
• Provides fundamental limits on efficiency
• Motivated the Second Law of Thermodynamics
• Died of cholera at age 36
• Most of his notes buried with his body
**Problem:** Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?

- What does “balanced” mean?
- Why are 3 test tubes balanced?
- **Symmetry!**
- Can you merge solutions?
- **Superposition!**
- **Linearity!** \( f(x + y) = f(x) + f(y) \)
- Can you spin 7 test tubes?
- **Complementarity!**
- Empirical testing…

No vector calculus / trig
Truth is guaranteed!
Fundamental principles exposed!
Easy to generalize!
High elegance / beauty!
Problem: $1 + 2 + 3 + 4 + \ldots + 100 = ?$

Proof: Induction…

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$1 + 2 + 3 + \ldots + 99 + 100 = \frac{100\times101}{2} = 5050$$
Drawbacks of Induction

- You must a priori know the formula / result
- Easy to make mistakes in inductive proof
- Mostly “mechanical” – ignores intuitions
- Tedious to construct
- Difficult to check
- Hard to understand
- Not very convincing
- Generalizations not obvious
- Does not “shed light on truth”
- Obfuscates connections

Conclusion: only use induction as a last resort!
I.e., almost never!
Problem: $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = ?$

$$\sum_{i=1}^{n} i^3 = ?$$

Extra Credit: find a short, geometric, induction-free proof.
Problem: \((1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?\)

\[\sum_{i=1}^{\infty} \frac{1}{4^i} = ?\]

Extra Credit:
Find a short, **geometric**, induction-free proof.
Problem: \((1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ?\)

\[
\sum_{i=1}^{\infty} \frac{1}{8^i} = ?
\]

Extra Credit:
Find a short, geometric, induction-free proof.
Problem: Are the complex numbers closed under exponentiation? E.g., what is the value of $i^i$?
Problem: Prove that there are an infinity of primes.

Extra Credit: Find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations
Problem: True or false: there arbitrary long blocks of consecutive composite integers.

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations
Problem: Prove that $\sqrt{2}$ is irrational.

Extra Credit: find a short, induction-free proof.

• What approaches fail?
• What techniques work and why?
• Lessons and generalizations
Problem: Does exponentiation preserve irrationality? i.e., are there two irrational numbers \(x\) and \(y\) such that \(x^y\) is rational?

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations
Historical Perspectives

- Science and mathematics builds heavily on past
- Often the simplest ideas are the most subtle
- Most fundamental progress was done by a few
- We learn much by observing the best minds
- Research benefits from seeing connections
- The field of computer science has many “parents”
- We get inspired and motivated by excellence
- The giants can show us what is possible to achieve
- It is fun to know these things!
Euclid (325BC-265BC)

- Founder of geometry & the axiomatic method
- “Elements” – oldest and most impactful textbook
- Unified logic & math
- Introduced rigor and “Euclidean” geometry
- Influenced all other fields of science: Copernicus, Kepler, Galileo, Newton, Russell, Lincoln, Einstein & many others
Euclid’s Straight-Edge and Compass Geometric Constructions
Euclid’s Axioms

1: Any two points can be connected by exactly one straight line.

2: Any segment can be extended indefinitely into a straight line.

3: A circle exists for any given center and radius.

4: All right angles are equal to each other.

5: The parallel postulate: Given a line and a point off that line, there is exactly one line passing through the point, which does not intersect the first line.

The first 28 propositions of Euclid’s Elements were proven without using the parallel postulate!

Theorem [Beltrami, 1868]: The parallel postulate is independent of the other axioms of Euclidean geometry.

The parallel postulate can be modified to yield non-Euclidean geometries!
Non-Euclidean Geometries

**Hyperbolic geometry**: Given a line and a point off that line, there are an infinity of lines passing through that point that do not intersect the first line.

- Sum of triangle angles is less than 180°
- Not all triangles have the same angle sum
- Triangles with same angles have same area
- There are no similar triangles
- Used in relativity theory
Non-Euclidean Geometries

Spherical / Elliptic geometry: Given a line and a point off that line, there are no lines passing through that point that do not intersect the first line.

• Lines are geodesics - “great circles”
• Sum of triangle angles is $> 180^\circ$
• Not all triangles have same angle sum
• Figures can not scale up indefinitely
• Area does not scale as the square
• Volume does not scale as the cube
• The Pythagorean theorem fails
• Self-consistent, and complete
Founders of Non-Euclidean Geometry

János Bolyai (1802-1860)

Nikolai Ivanovich Lobachevsky (1792-1856)
Non-Euclidean Non-Orientable Surfaces

- Möbius strip: one side, one boundary!
- Klein bottle: one side, no boundary!
- Projective plane: one side, no boundary!
THE GEOMETRY OF EVERYDAY LIFE

TUNA SANDWICH  SNEAKER  GRANDMA
Problem: A man leaves his house and walks one mile south. He then walks one mile west and sees a Bear. Then he walks one mile north back to his house. What color was the bear?

Problem: Is the house location unique?
Problem: \((1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?\)

Find a short, geometric, induction-free proof.

\[
\sum_{i=1}^{\infty} \frac{1}{4^i} = \frac{1}{3}
\]
Problem: \( (1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ? \)

Find a short, geometric, induction-free proof.

\[
\sum_{i=1}^{\infty} \frac{1}{8^i} = \frac{1}{7}
\]
Problem: Are the complex numbers closed under exponentiation? E.g., what is the value of $i^i$?

$$i^i = \frac{1}{\sqrt{e^{\pi}}} = 0.207879...$$

$$e^{ix} = \cos(x) + i \sin(x)$$

$$i^i = \frac{1}{\sqrt{e^{\pi + 2k\pi}}}$$

$i^i$ is multi-valued!