

CSC242: Intro to AI

Lecture 17

Learning from Examples

Learning from Examples

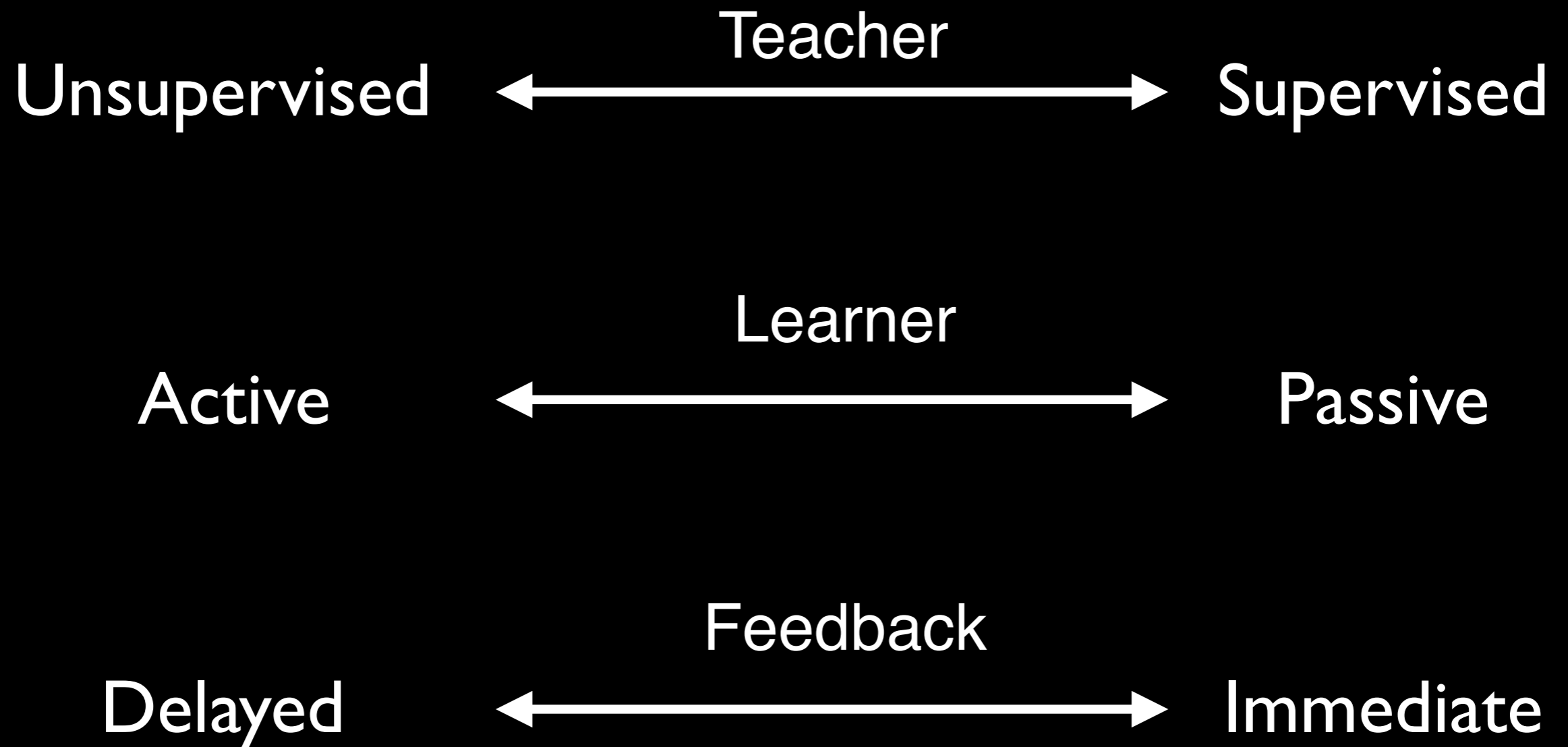
Learning



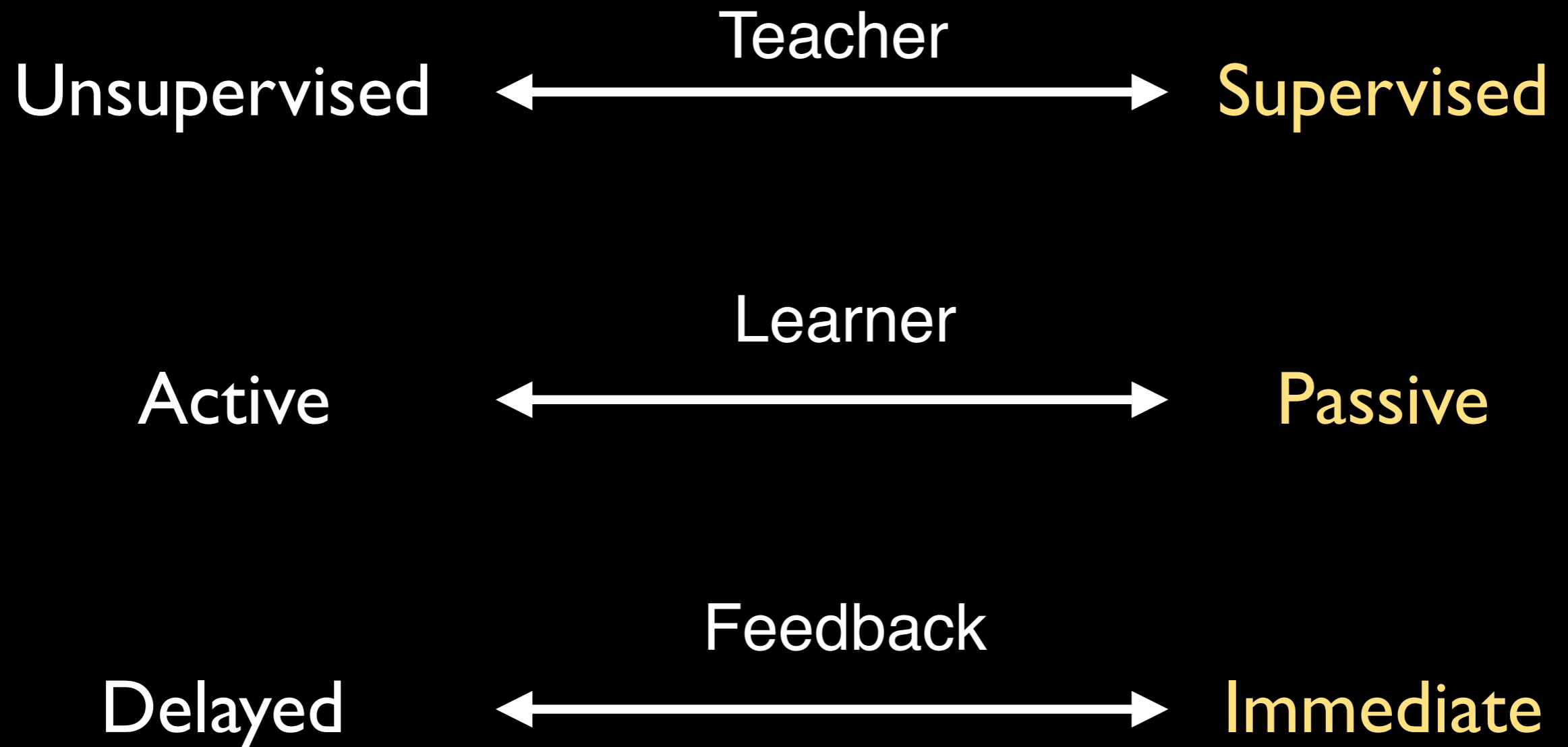
Why Learn?

- Can't anticipate all possible situations that the agent might find themselves in
- Cannot anticipate all changes that might occur over time
- Don't know how to program it other than by learning!

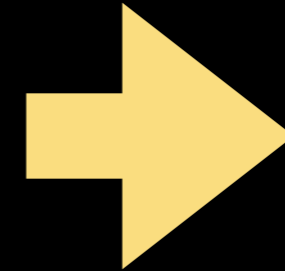
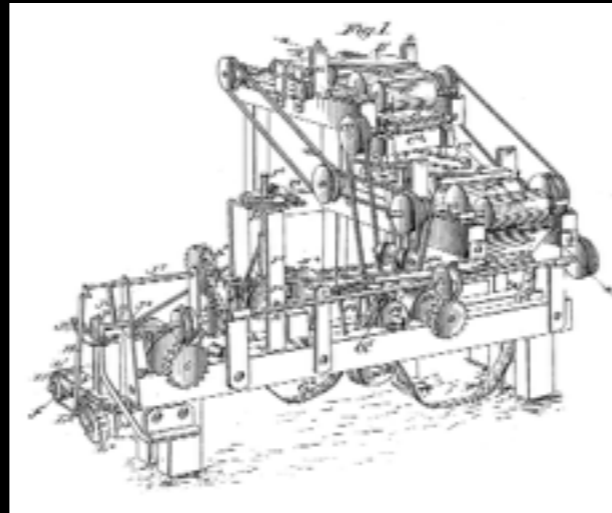
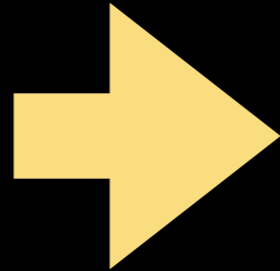
Dimensions of Learning



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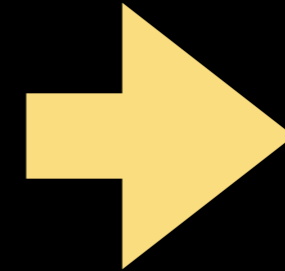
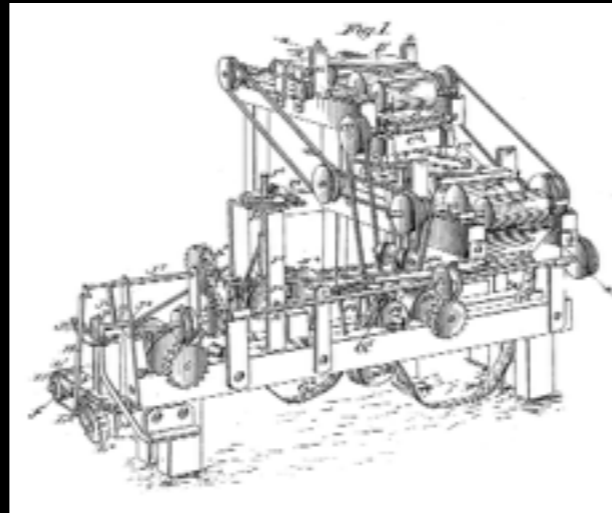
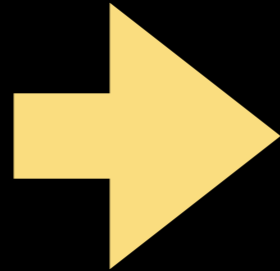


A Classifier



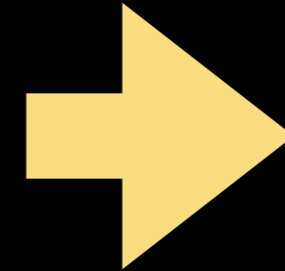
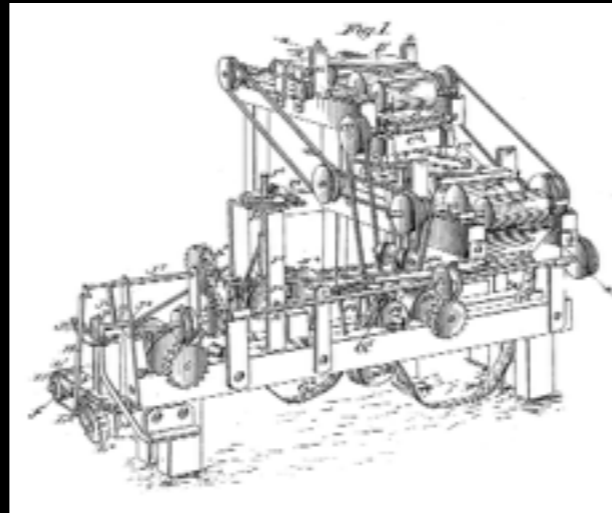
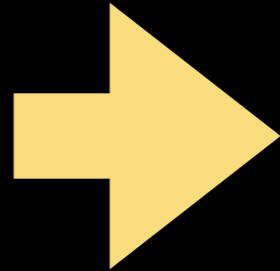
Edible

A Classifier



Edible

A Classifier



Poison

Learning a Classifier

Training Data



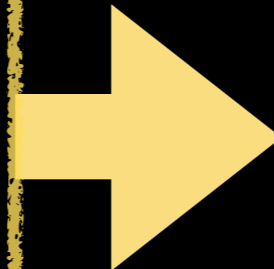
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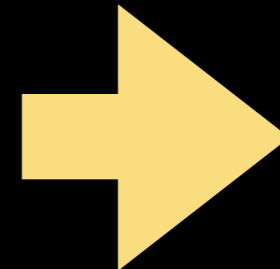
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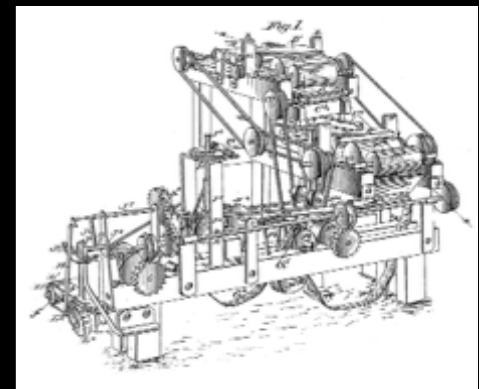
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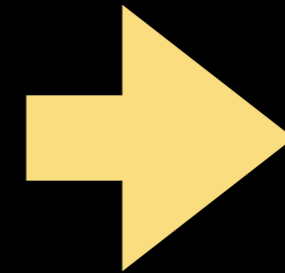
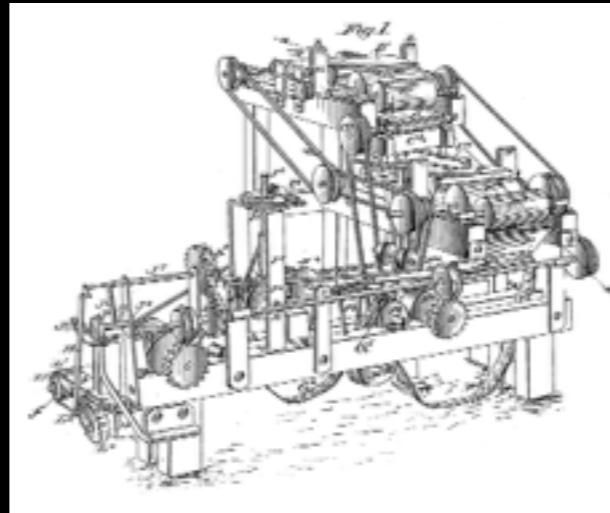
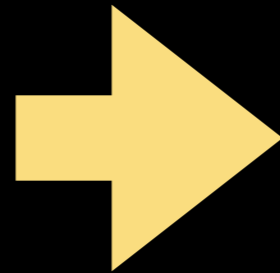
Learner



Classifier



Generalization



Poison

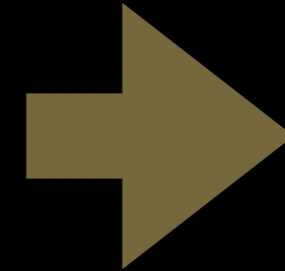
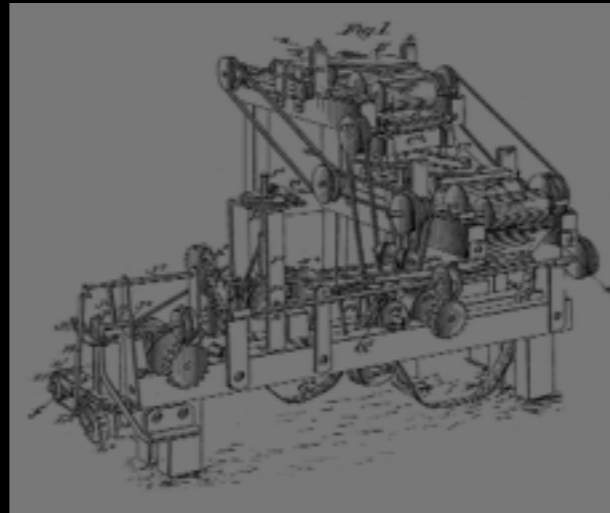
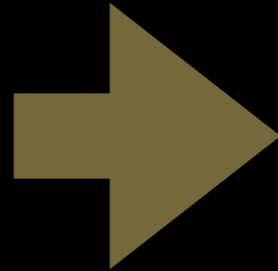
- Ability to classify items that were never seen before
- Going beyond simple memorization

Features



- img9201.jpg
 - (color=green,
leafs_per_stem=3,
leaf_edge=jagged)
-
- The observable properties of the things to classified
 - Also called “attributes”

Labels



Poison

- Classification: Symbols
- Regression: Numbers

Hypothesis Space

Training Data



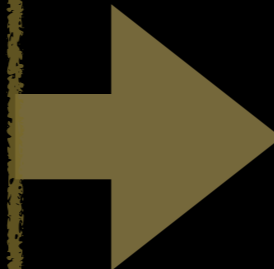
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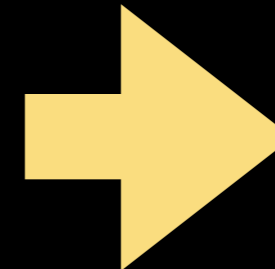
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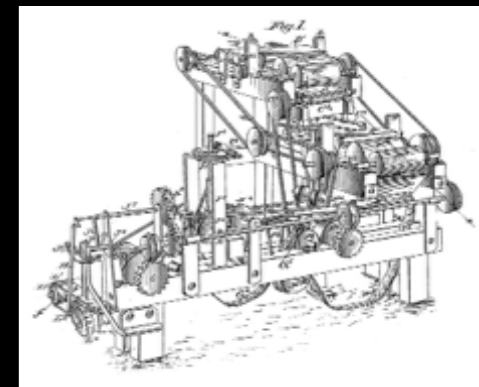
Poison



Learner



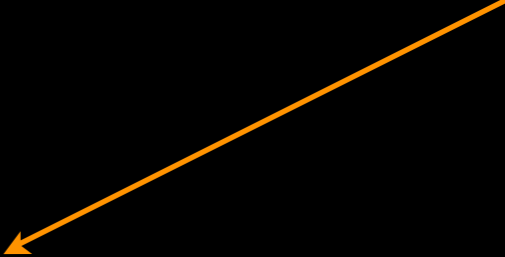
Classifier



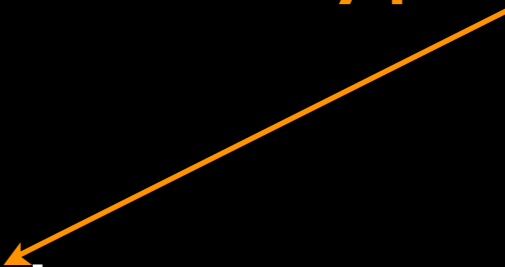
- Space of possible outputs of the learning system
- Polynomial functions, decision trees, neural networks, ...

Learning Functions from Examples

Function Learning

- There is some function $y = f(x)$
 - We don't know f
 - We want to learn a function h that approximates the true function f
- Hypothesis
- 

Function Learning

- There is some function $y = f(x)$
 - We don't know f
 - We want to learn a function h that approximates the true function f
 - Learning is a search through the space of possible hypotheses for one that will perform well
- Hypothesis
- 

Supervised Learning

- Given a training set of N example input-output pairs:

$$(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$$

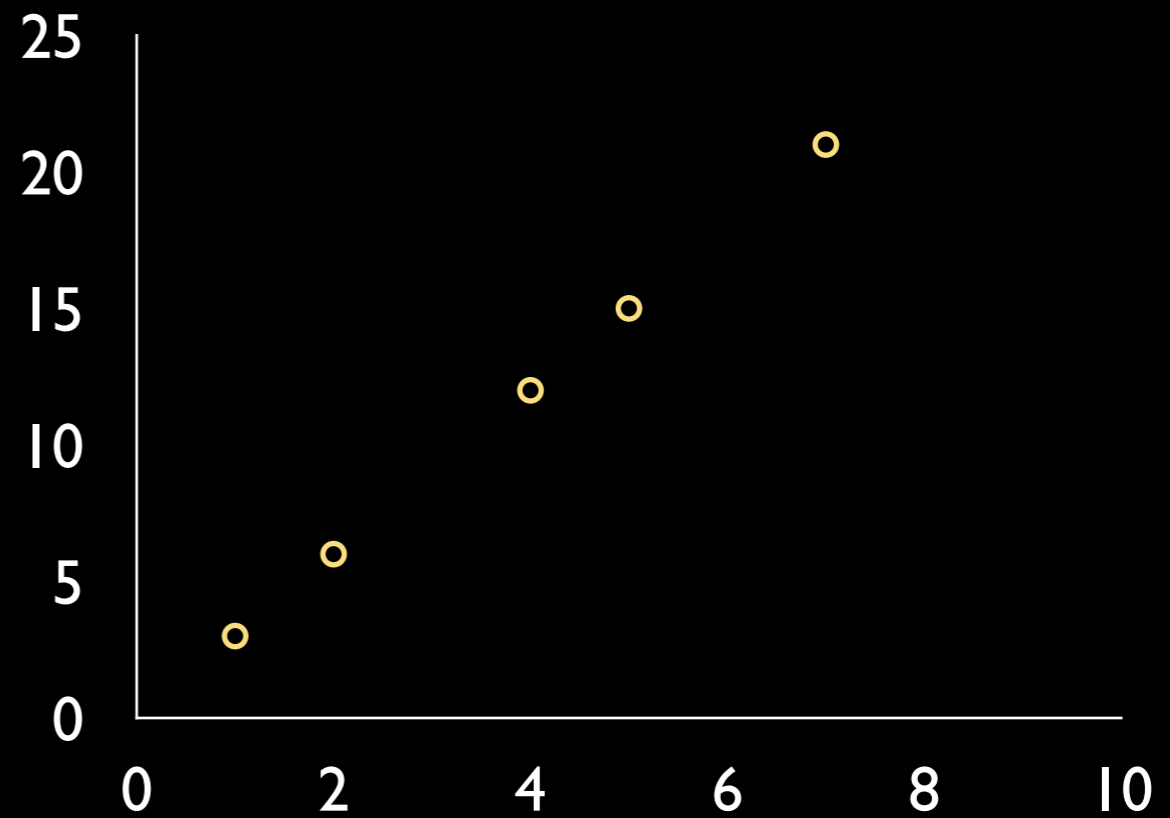
where each $y_j = f(x_j)$

- Discover function h that approximates f
- Search through the space of possible hypotheses for one that will perform well

Training Data

x	y
1	3
2	6
4	12
5	15
7	21

$$f(x) = y$$



$$h(x) = ?$$

Evaluating Accuracy

- Training set: $(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$
- **Test set:** Additional (x_j, y_j) pairs distinct from training set
- Test **accuracy** of h by comparing $h(x_j)$ to y_j for (x_j, y_j) from test set
- **Generalization:** Ability to handle examples in test set that were not in training test

Training Data

x	y
1	3
2	6
4	12
5	15
7	21

$$f(x) = y$$

$$h(x) = ?$$

Testing Data

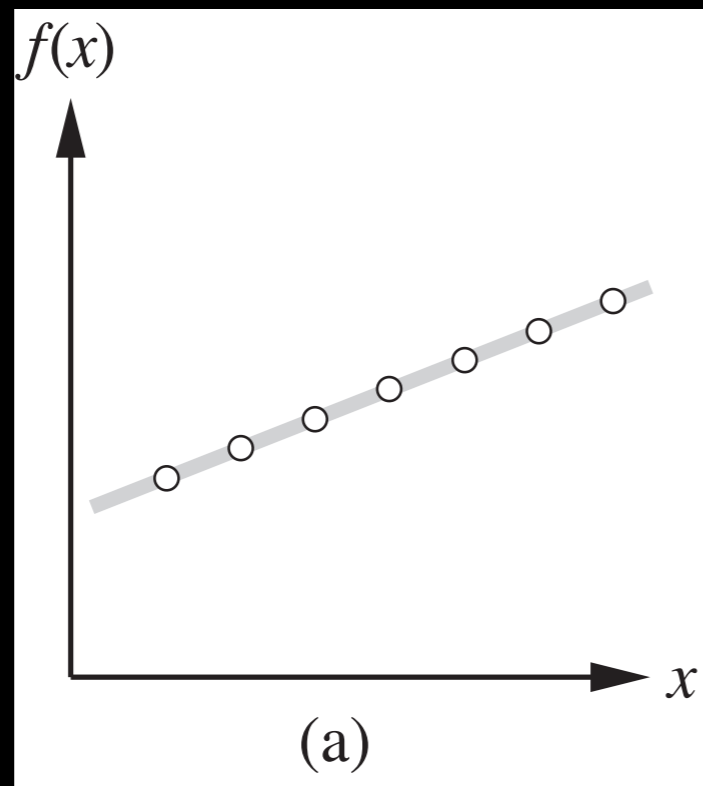
x	y
3	9
4	12
6	18

$$f(x) = y$$

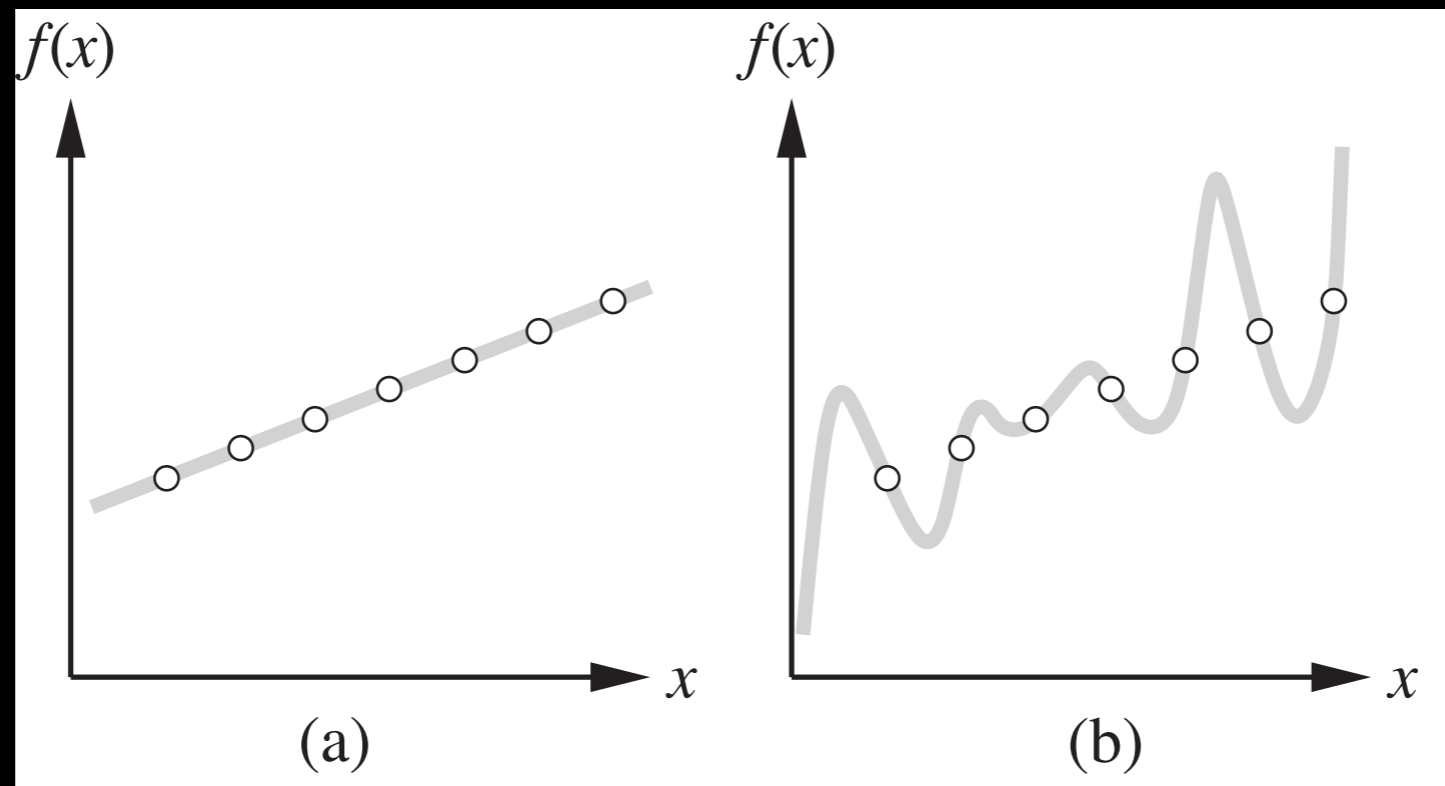
$$h(x) = y?$$

Hypothesis Space

- The class of functions that are acceptable as solutions, e.g.
 - Linear functions $y = mx + b$
 - Polynomials (of some degree)
 - Decision trees
 - Neural networks
 - Turing machines



$$y = -0.4x + 3$$



$$y = -0.4x + 3$$

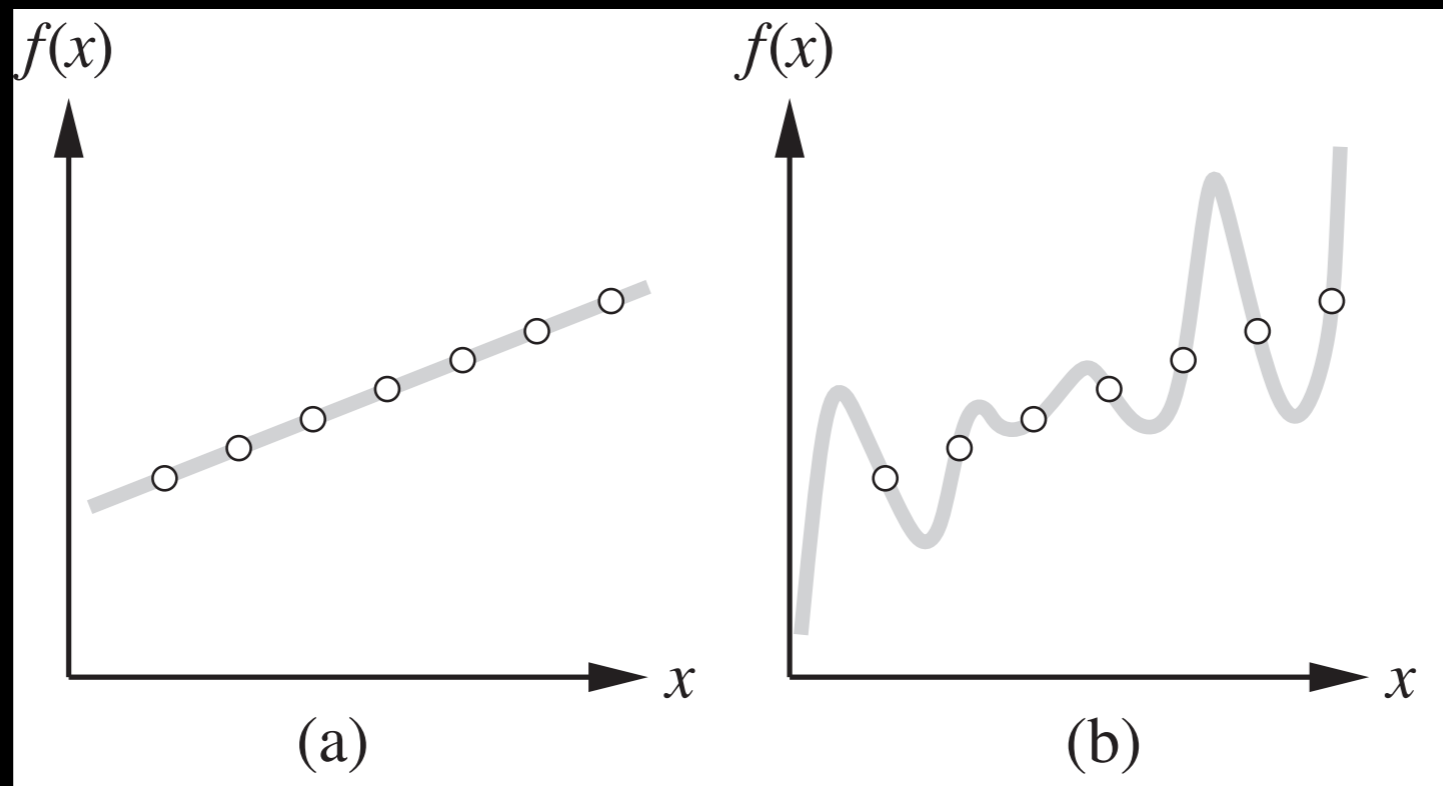
$$y = c_7x^7 + c_6x^6 + \dots + c_1x + c_0$$

$$= \sum_{i=0}^7 c_i x^i$$

Occam's Razor



William of Occam (or Ockham)
14th c.

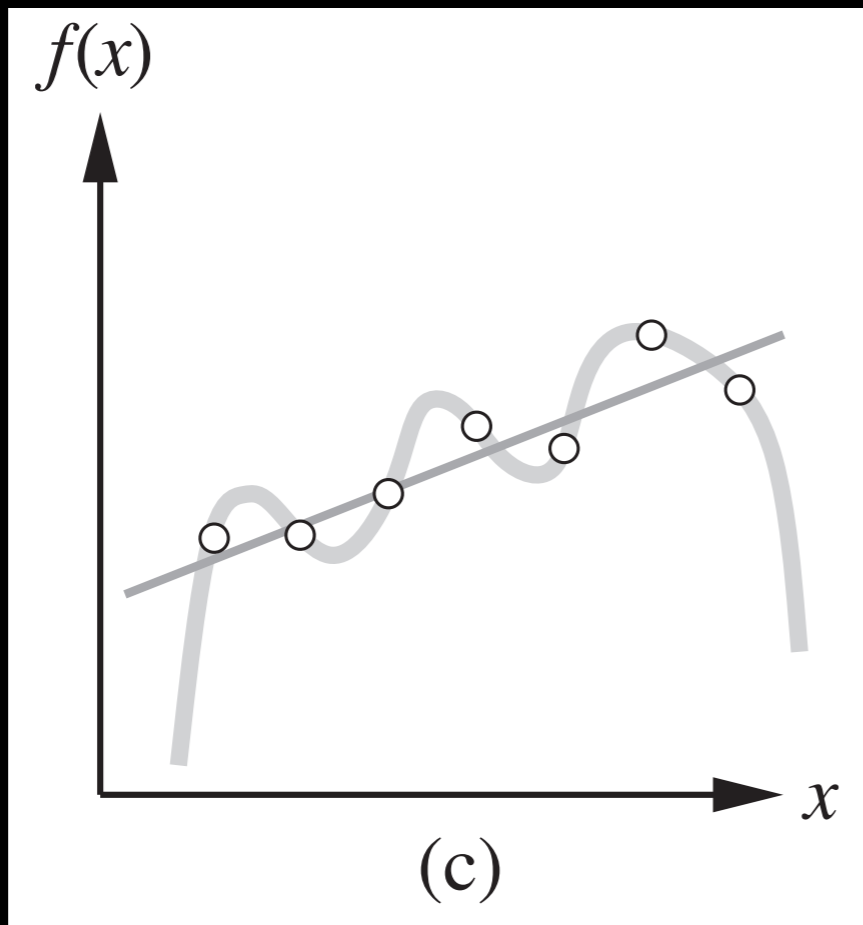


$$y = -0.4x + 3$$

$$y = c_7x^7 + c_6x^6 + \dots + c_1x + c_0$$

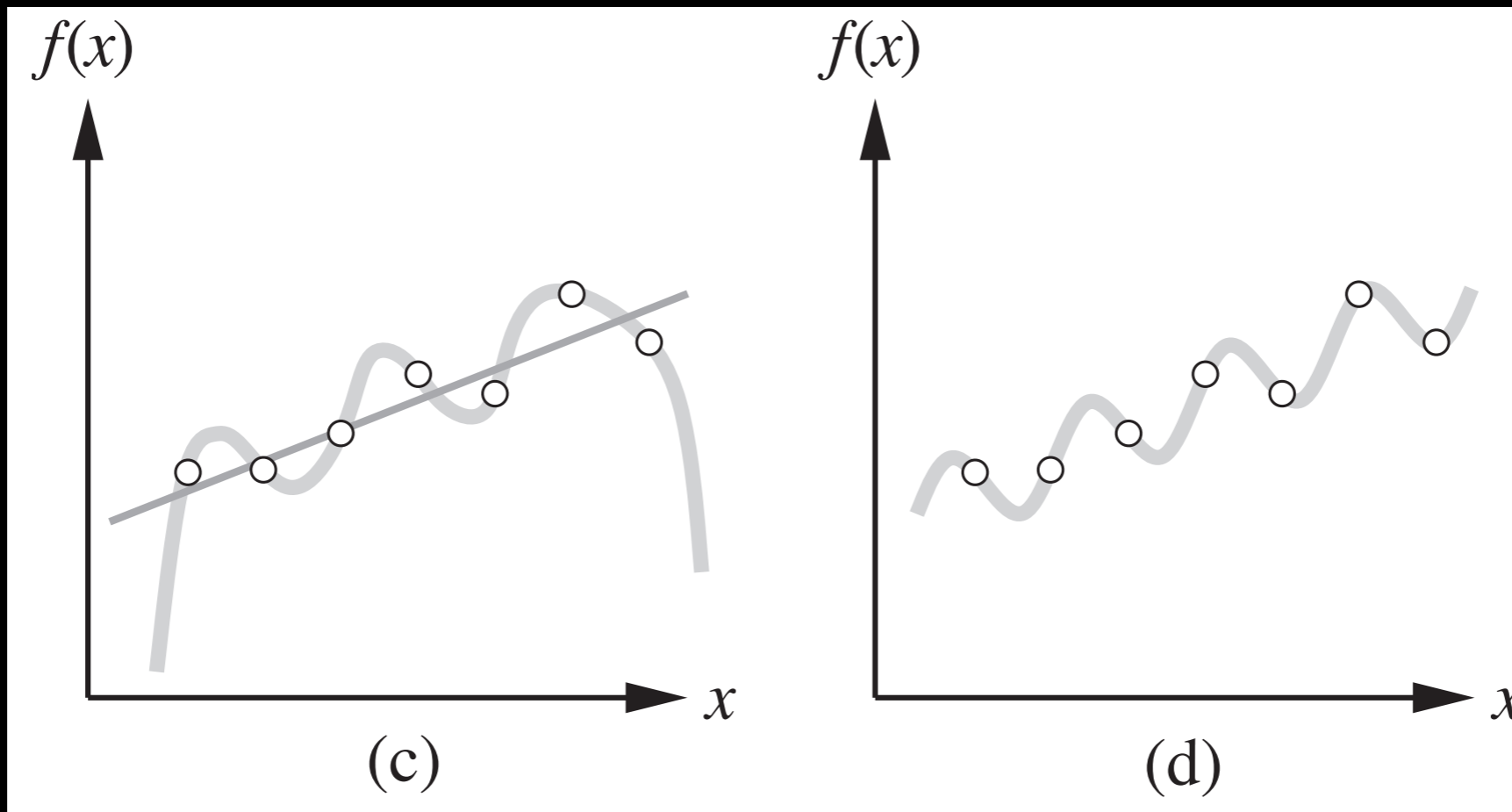
$$= \sum_{i=0}^7 c_i x^i$$





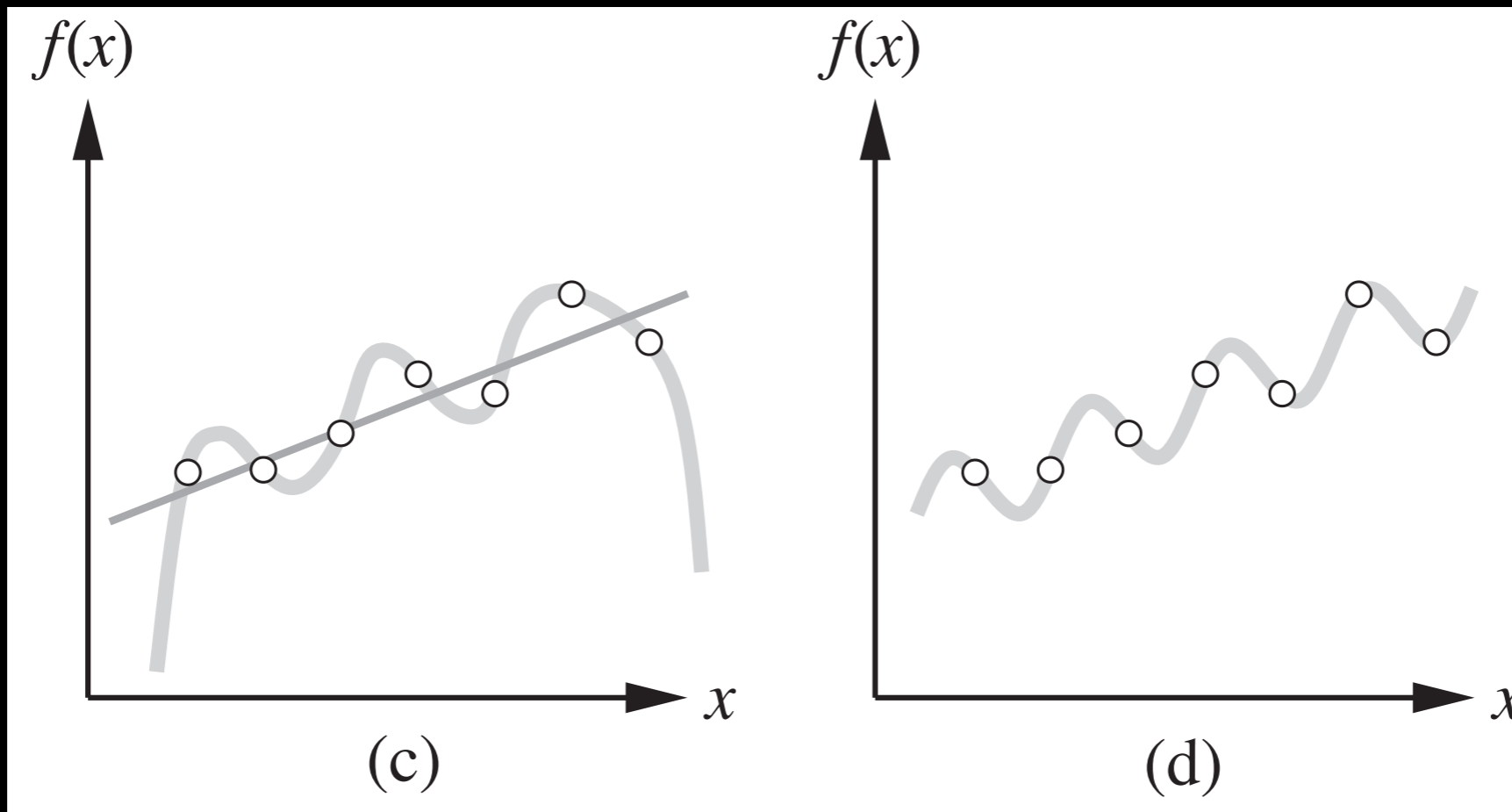
$$y = c_6x^6 + c_5x^5 \dots + c_1x + c_0$$

$$y = mx + b$$



$$y = c_6x^6 + c_5x^5 \dots + c_1x + c_0 \quad ax + b + c \sin(x)$$

$$y = mx + b$$

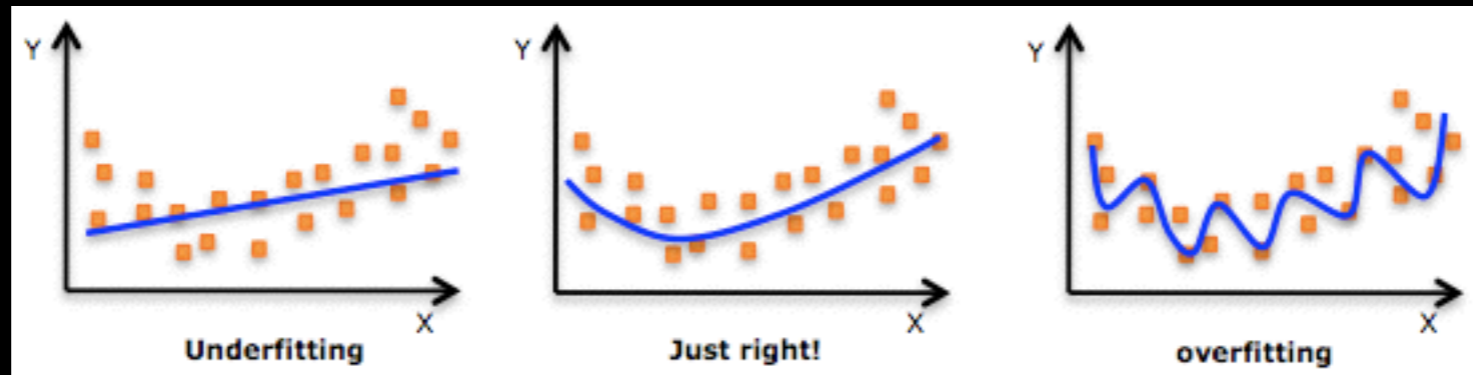


$$y = c_6x^6 + c_5x^5 \dots + c_1x + c_0 \quad ax + b + c \sin(x)$$

$$y = mx + b$$



Error and Overfitting



- It is often preferable to allow some error in the fit of the hypothesis to the training data in order to improve generalization
- Allowing too little error - resulting in a complex hypothesis with poor generalization - is **overfitting**
- Using too simple a hypothesis that has very high error - resulting again in poor generalization - is **underfitting**

Overfitting

- When a learned model adjusts to the **noise** in the input rather than the **signal**
- Becomes **more likely** as the hypothesis space and number of input attributes grows
- Becomes **less likely** as the number of training examples increases

Learning Decision Trees

Classification

- Output $y = f(x)$ is one of a finite set of values (classes, categories, ...)
 - Boolean classification: yes/no or true/false
- Input is vector \mathbf{x} of values for attributes
 - Factored representation

Example

- Going out to dinner with Stuart Russell
- Restaurants often busy in SF; sometimes have to wait for a table
- Decision: Do we wait or do something else?

Attributes (Features)

Alternate : is there a suitable alternative nearby

Bar: does it have a comfy bar

FriSat: is it a Friday or Saturday

Hungry: are we hungry

Patrons: None, Some, Full

Price: \$, \$\$, \$\$\$

Raining: is it raining outside

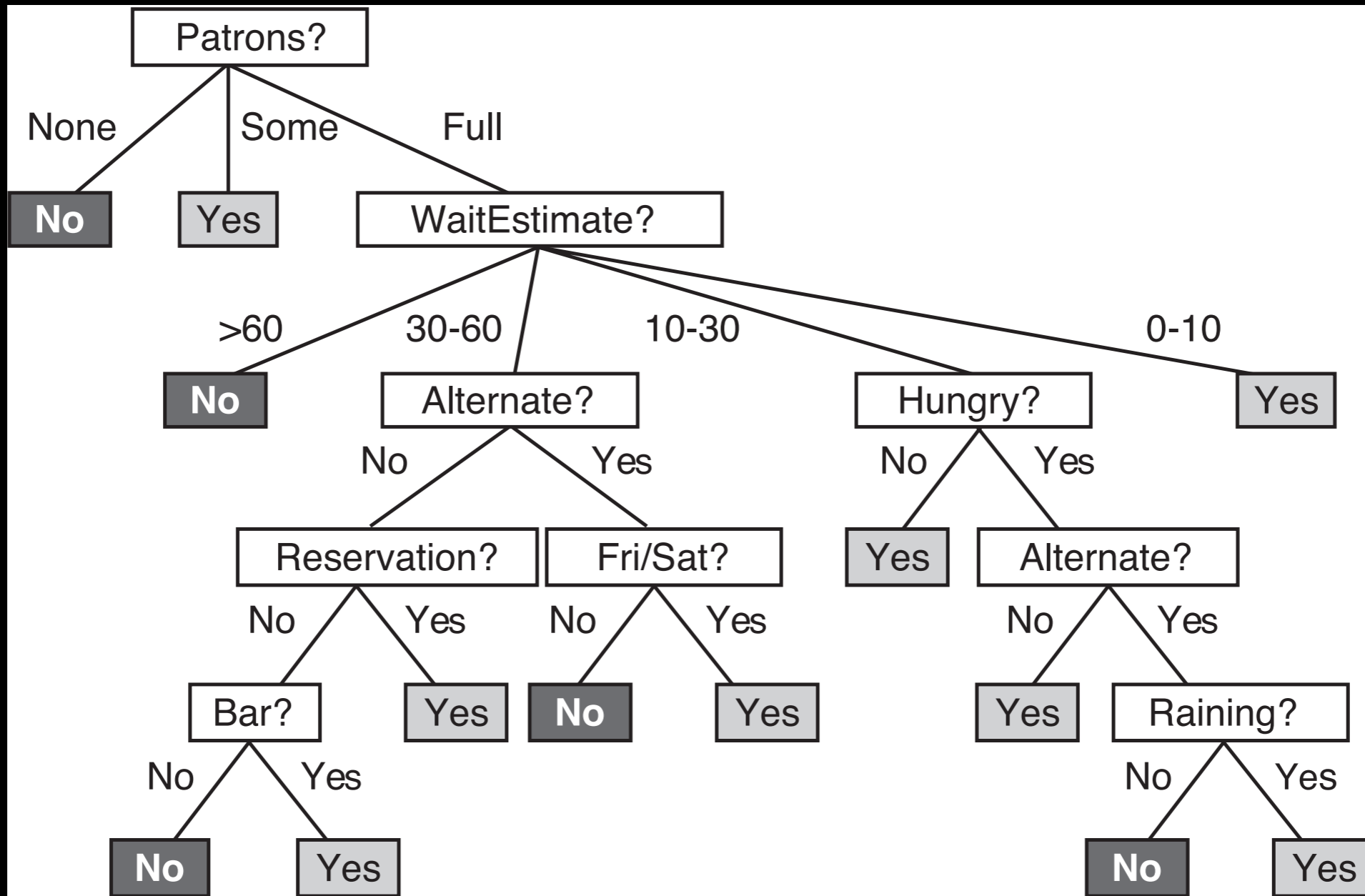
Reservation: do we have a reservation

Type: French, Italian, Thai, burger, ...

WaitEstimate: 0-10, 10-30, 30-60, >60

Decision Making

- If the host/hostess says you'll have to wait:
 - Then if there's no one in the restaurant you don't want to be there either;
 - But if there are a few people but it's not full, then you should wait
 - Otherwise you need to consider how long he/she told you the wait would be
 - ...



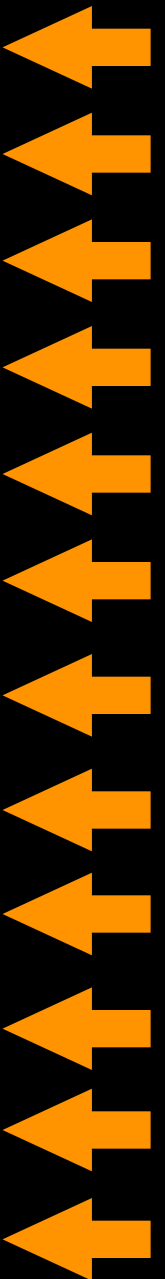
Decision Tree

- Each node in the tree represents a test on a single attribute
- Children of the node are labelled with the possible values of the feature
- Each path represents a series of tests, and the leaf node gives the value of the function when the input passes those tests

Inducing Decision Trees From Examples

- Examples: (\mathbf{x}, y) where \mathbf{x} is a vector of values for the input attributes and y is a single Boolean value (yes/no, true/false)

	Input Attributes										<i>Will Wait</i>
	<i>Alt</i>	<i>Bar</i>	<i>Fri</i>	<i>Hun</i>	<i>Pat</i>	<i>Price</i>	<i>Rain</i>	<i>Res</i>	<i>Type</i>	<i>Est</i>	
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>30-60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Some</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Thai</i>	<i>10-30</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>>60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Italian</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>>60</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>Italian</i>	<i>10-30</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>30-60</i>	<i>y</i>

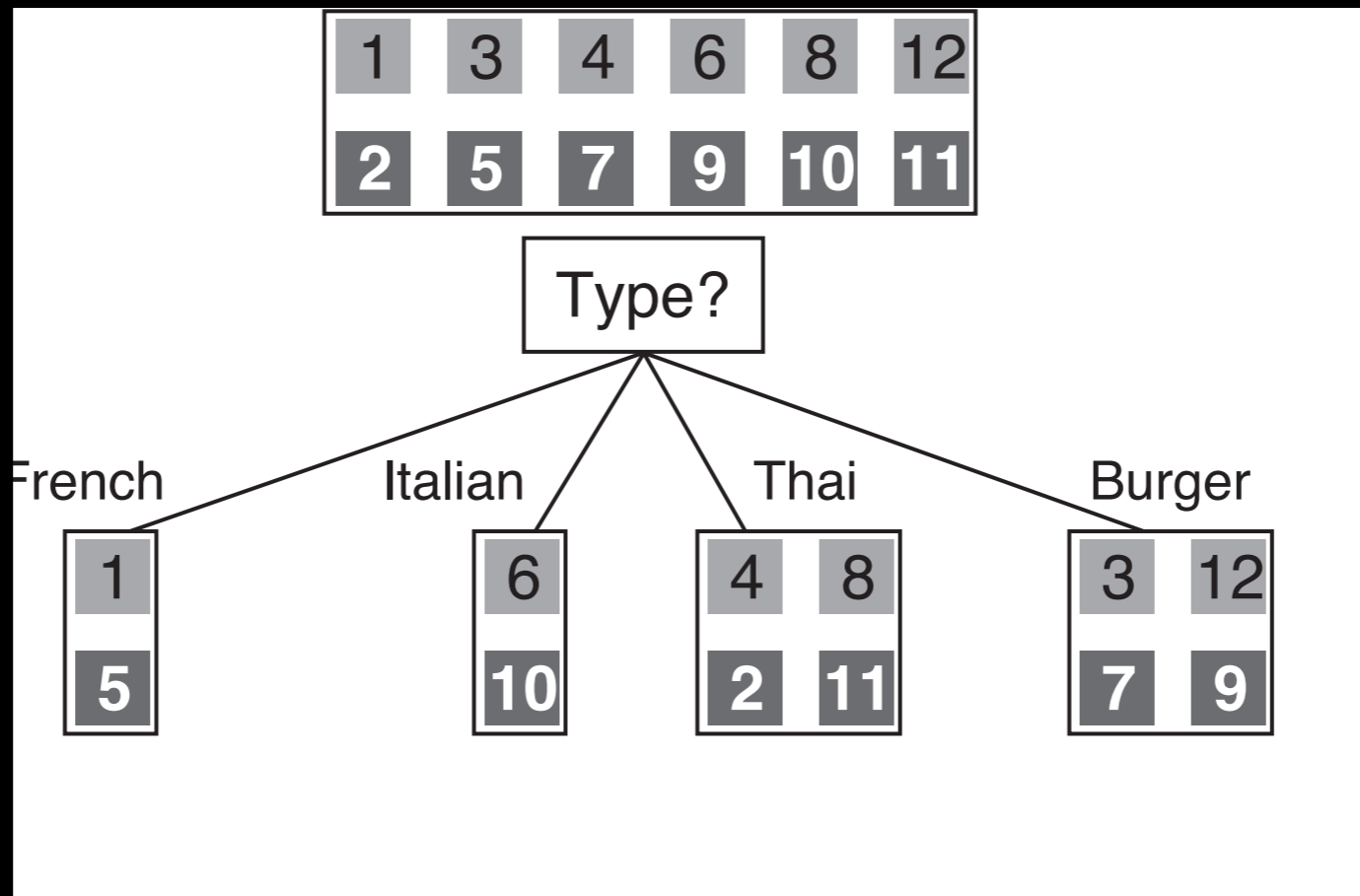


Inducing Decision Trees From Examples

- Examples: (\mathbf{x}, y)
- Want a shallow tree (short paths, fewer tests)
- Greedy algorithm (AIMA Fig 18.5)
 - Always test the most important attribute first
 - Makes the most difference to classification of an example

	Input Attributes										<i>Will Wait</i>
	<i>Alt</i>	<i>Bar</i>	<i>Fri</i>	<i>Hun</i>	<i>Pat</i>	<i>Price</i>	<i>Rain</i>	<i>Res</i>	<i>Type</i>	<i>Est</i>	
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>30-60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Some</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Thai</i>	<i>10-30</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>>60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Italian</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>>60</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>Italian</i>	<i>10-30</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>30-60</i>	<i>y</i>

	Input Attributes										<i>Will Wait</i>
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x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>30-60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Some</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Thai</i>	<i>10-30</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>>60</i>	<i>y</i>
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x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>>60</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>Italian</i>	<i>10-30</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>30-60</i>	<i>y</i>



Poor split: children very mixed!

	Input Attributes										<i>Will Wait</i>
	<i>Alt</i>	<i>Bar</i>	<i>Fri</i>	<i>Hun</i>	<i>Pat</i>	<i>Price</i>	<i>Rain</i>	<i>Res</i>	<i>Type</i>	<i>Est</i>	
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>30-60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Some</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Thai</i>	<i>10-30</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>>60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Italian</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>>60</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>Italian</i>	<i>10-30</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>30-60</i>	<i>y</i>

	Input Attributes										<i>Will Wait</i>
	<i>Alt</i>	<i>Bar</i>	<i>Fri</i>	<i>Hun</i>	<i>Pat</i>	<i>Price</i>	<i>Rain</i>	<i>Res</i>	<i>Type</i>	<i>Est</i>	
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>30-60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Some</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Thai</i>	<i>10-30</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>>60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Italian</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>>60</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>Italian</i>	<i>10-30</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>30-60</i>	<i>y</i>

	Input Attributes										<i>Will Wait</i>
	<i>Alt</i>	<i>Bar</i>	<i>Fri</i>	<i>Hun</i>	<i>Pat</i>	<i>Price</i>	<i>Rain</i>	<i>Res</i>	<i>Type</i>	<i>Est</i>	
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>30-60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>Some</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Thai</i>	<i>10-30</i>	<i>y</i>
x	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>French</i>	<i>>60</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Italian</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>Yes</i>	<i>Some</i>	<i>\$\$</i>	<i>Yes</i>	<i>Yes</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>No</i>	<i>Yes</i>	<i>Yes</i>	<i>No</i>	<i>Full</i>	<i>\$</i>	<i>Yes</i>	<i>No</i>	<i>Burger</i>	<i>>60</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$\$\$</i>	<i>No</i>	<i>Yes</i>	<i>Italian</i>	<i>10-30</i>	<i>y</i>
x	<i>No</i>	<i>No</i>	<i>No</i>	<i>No</i>	<i>None</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Thai</i>	<i>0-10</i>	<i>y</i>
x	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Yes</i>	<i>Full</i>	<i>\$</i>	<i>No</i>	<i>No</i>	<i>Burger</i>	<i>30-60</i>	<i>y</i>

1	3	4	6	8	12
2	5	7	9	10	11

Patrons?

None

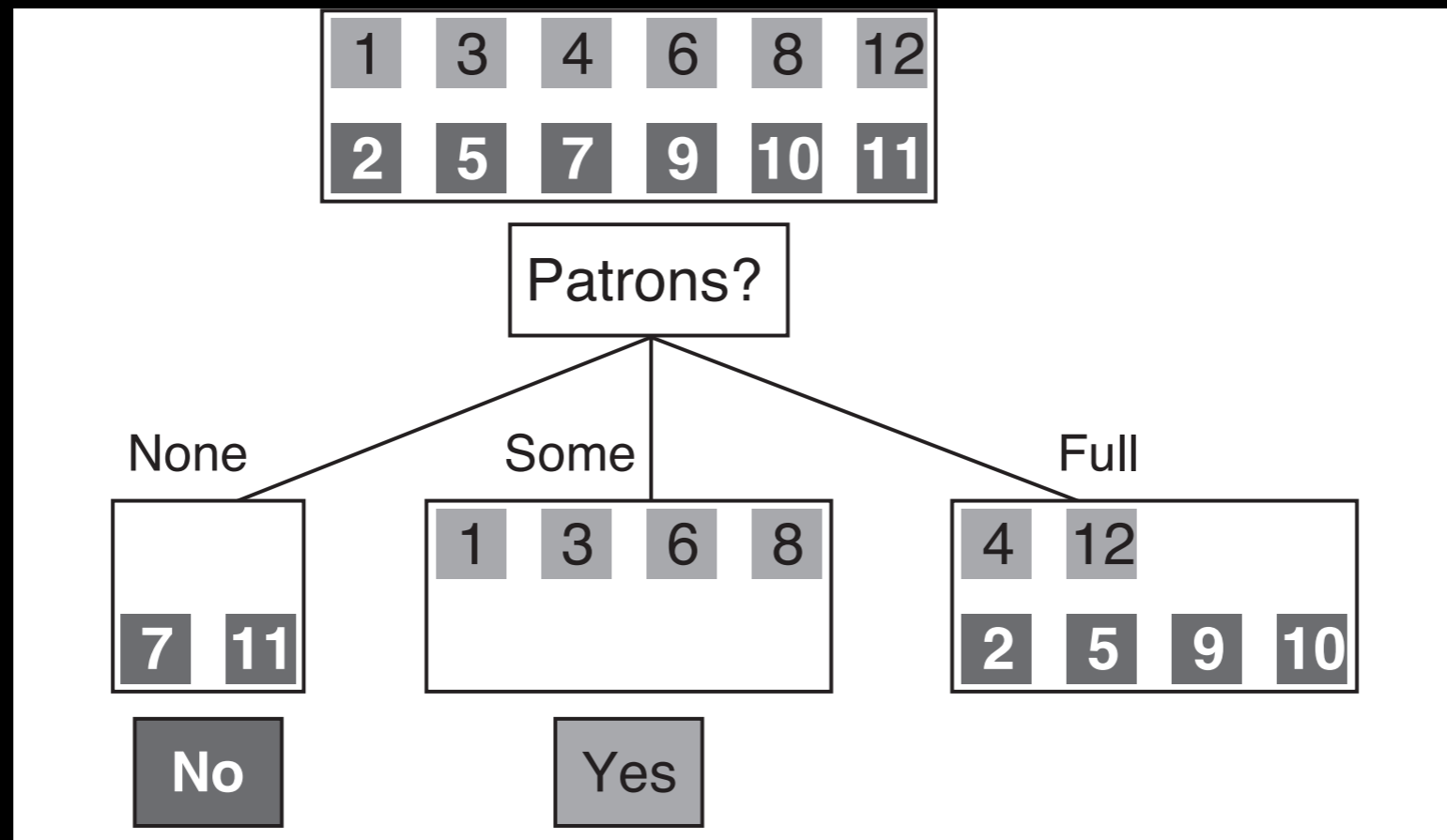
7	11

Some

1	3	6	8

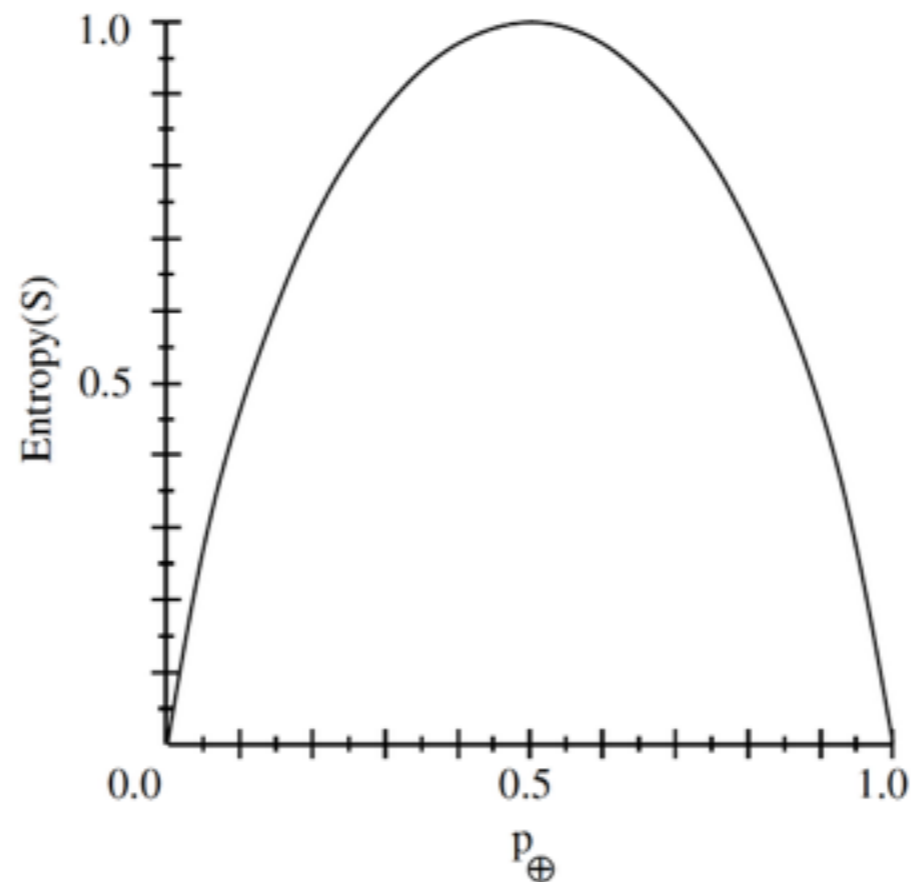
Full

4	12		
2	5	9	10



Good split: children very unbalanced!

Entropy



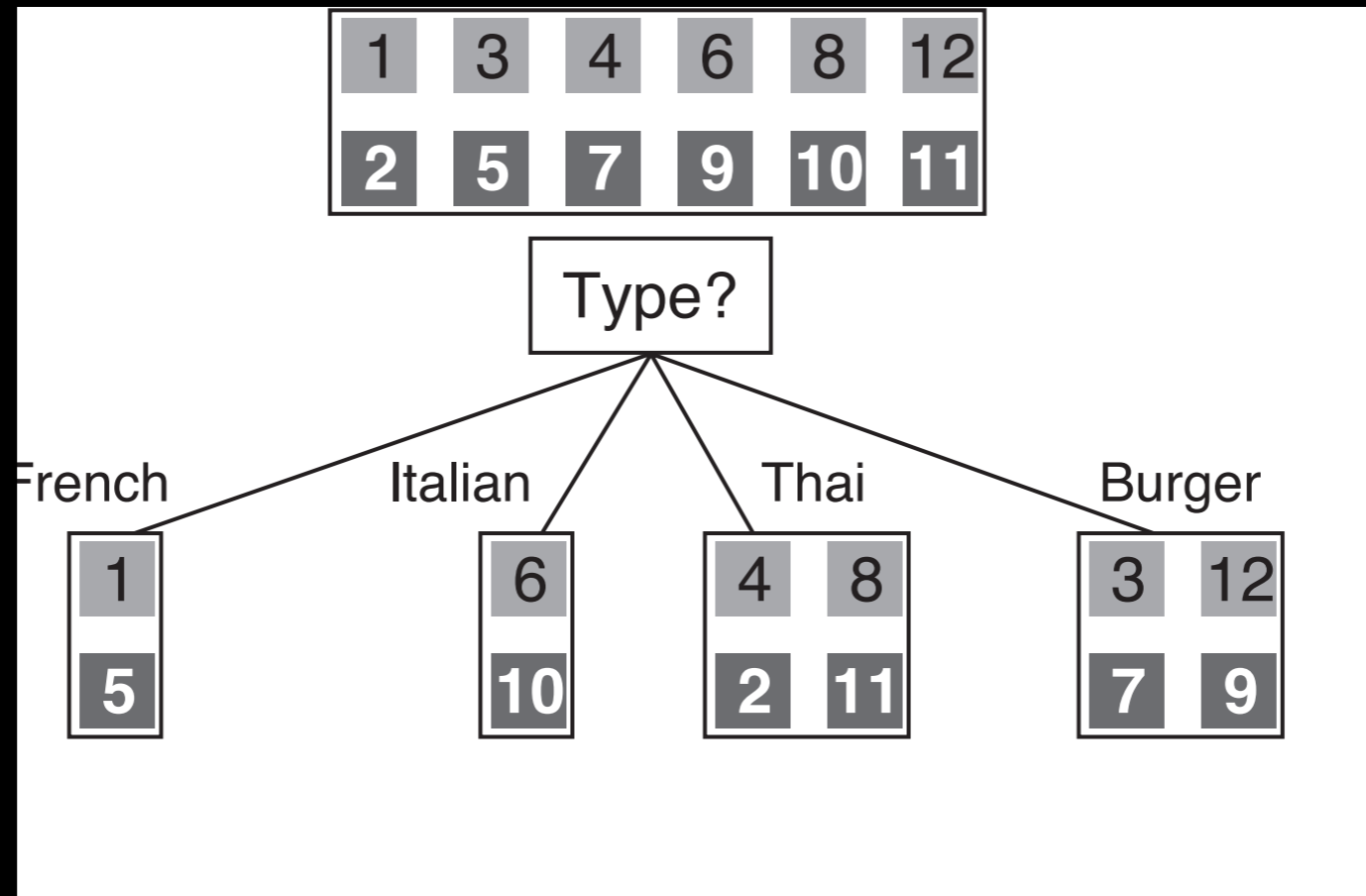
- S is a sample of training examples
- p_{\oplus} is the proportion of positive examples in S
- p_{\ominus} is the proportion of negative examples in S
- Entropy measures the impurity of S

$$\text{Entropy}(S) \equiv -p_{\oplus} \log_2 p_{\oplus} - p_{\ominus} \log_2 p_{\ominus}$$

Information Gain

$Gain(S, A)$ = expected reduction in entropy due to sorting on A

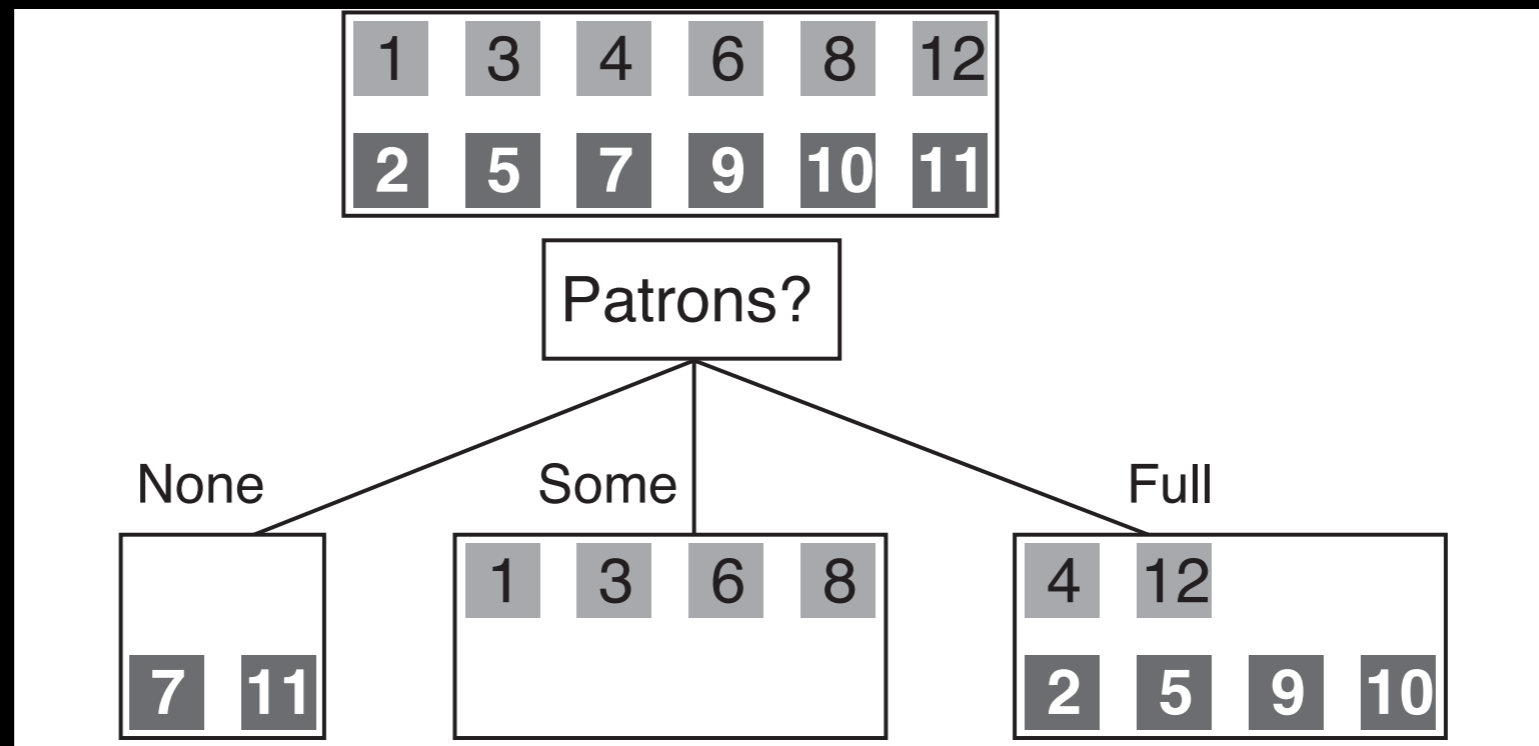
$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$



$$Entropy(S) = -0.5 \log_2 0.5 - 0.5 \log_2 0.5 = 1$$

$$Entropy(S_F) = Entropy(S_I) = Entropy(S_T) = Entropy(S_B) = 1$$

$$Gain(Type) = Entropy(S) - \sum_{v \in Type} \frac{|S_v|}{|S|} Entropy(S_v) = 1 - 1 = 0$$



$$Entropy(S) = -0.5 \log_2 0.5 - 0.5 \log_2 0.5 = 1$$

$$Entropy(S_N) = -0 \log_2 0 - (1) \log_2 1 = 0$$

$$Entropy(S_S) = -(1) \log_2 1 - 0 \log_2 0 = 0$$

$$Entropy(S_F) = -(\frac{1}{3}) \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = 0.92$$

$$Gain(Patron) = 1 - \sum_{v \in Patron} \frac{|S_v|}{|S|} Entropy(S_v) = 1 - (\frac{1}{2})(0.92) = 0.54$$

1	3	4	6	8	12
2	5	7	9	10	11

Patrons?

None

7	11

No

Some

1	3	6	8

Yes

Full

4	12		
2	5	9	10

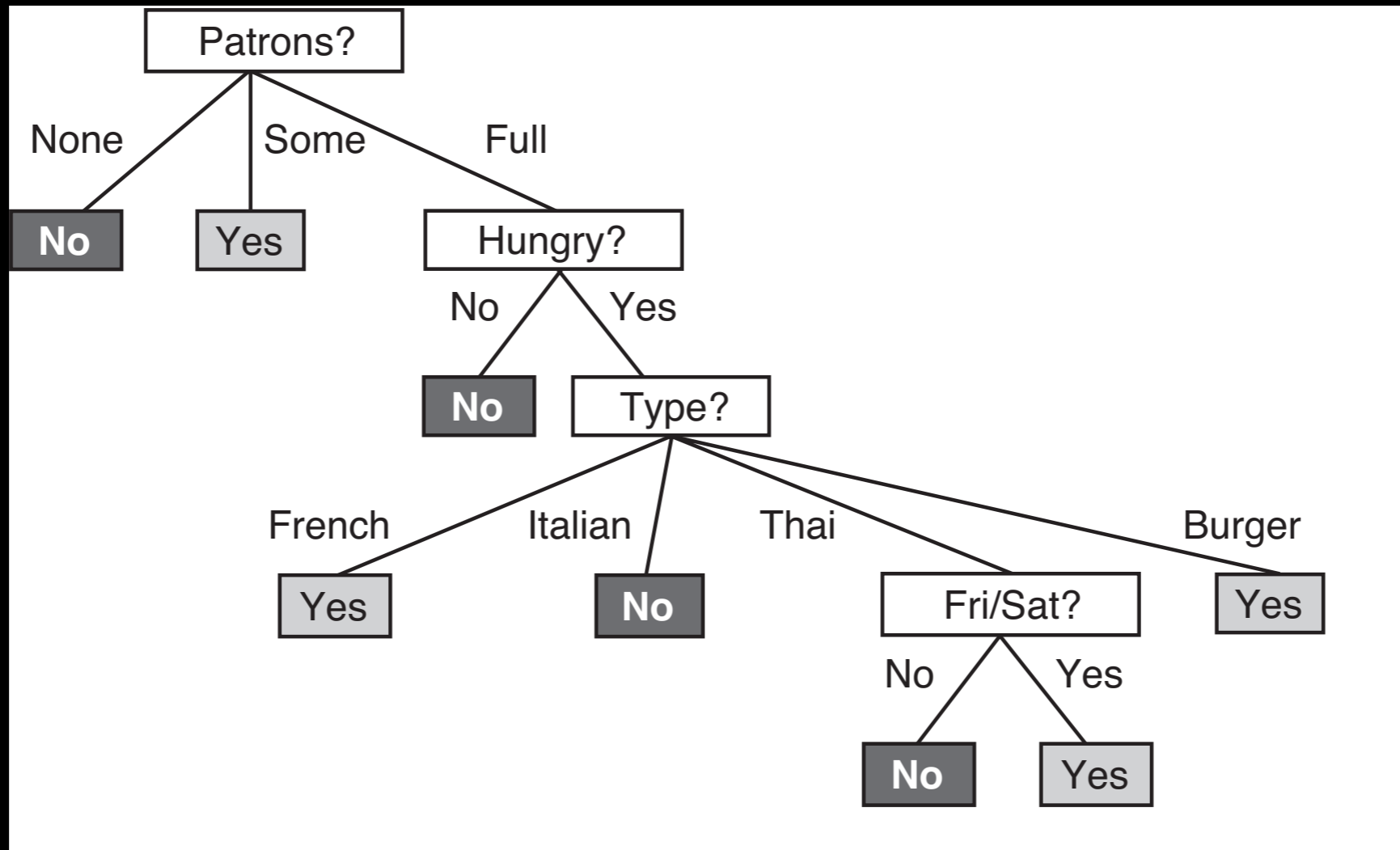
Hungry?

No

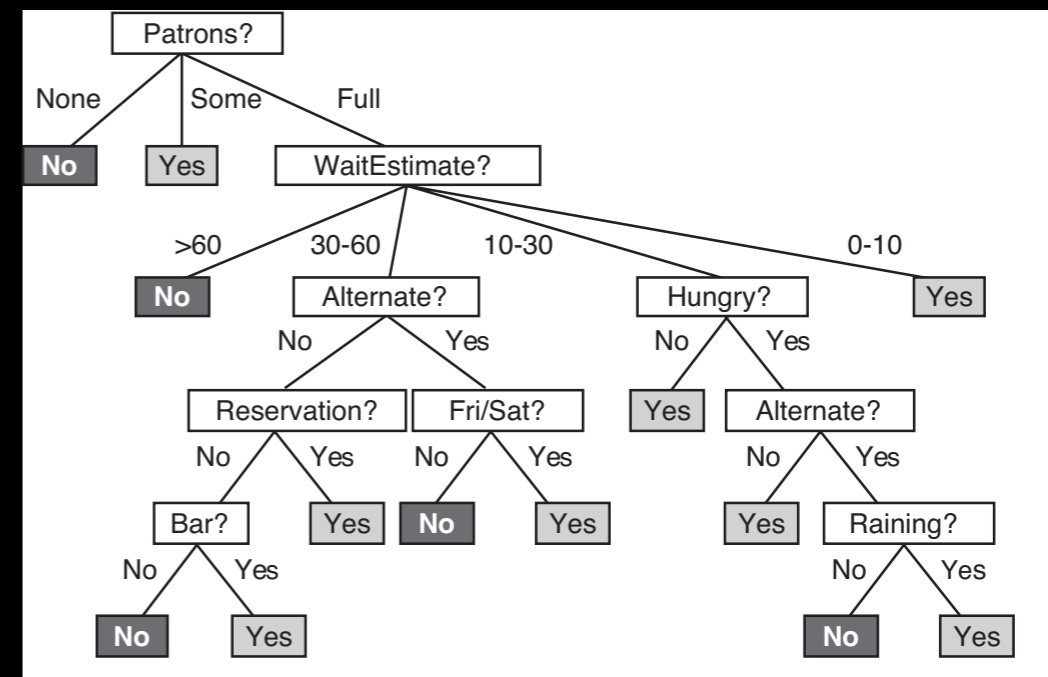
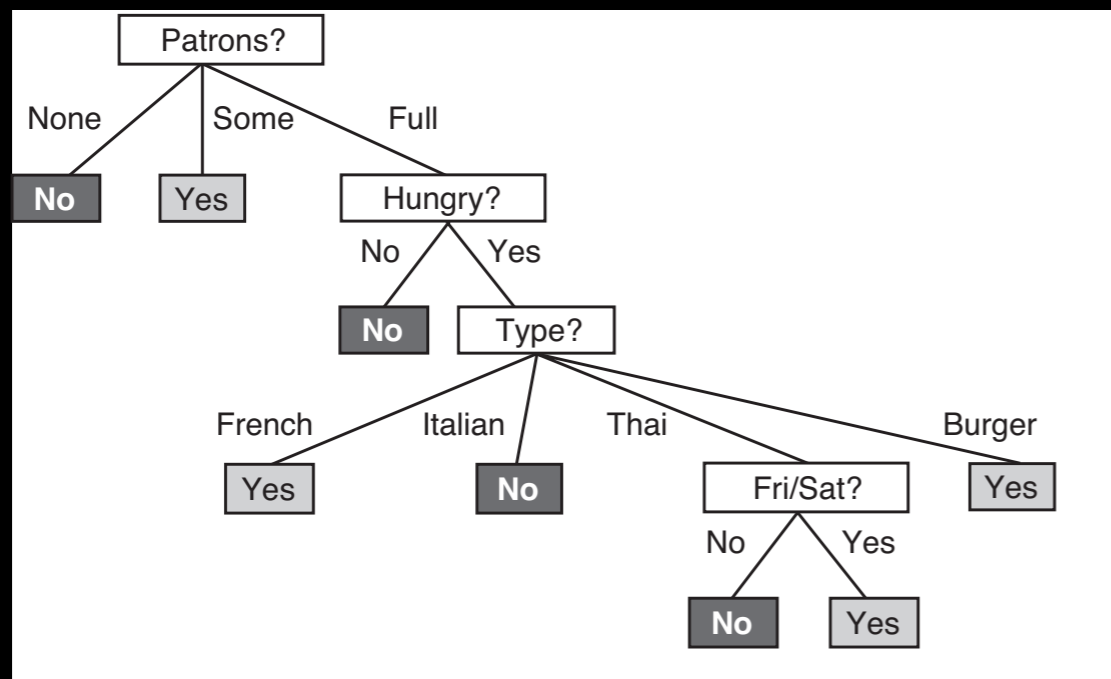
5	9

Yes

4	12
2	10



Avoiding Overfitting



- Problem: How to determine when to stop growing the decision tree?

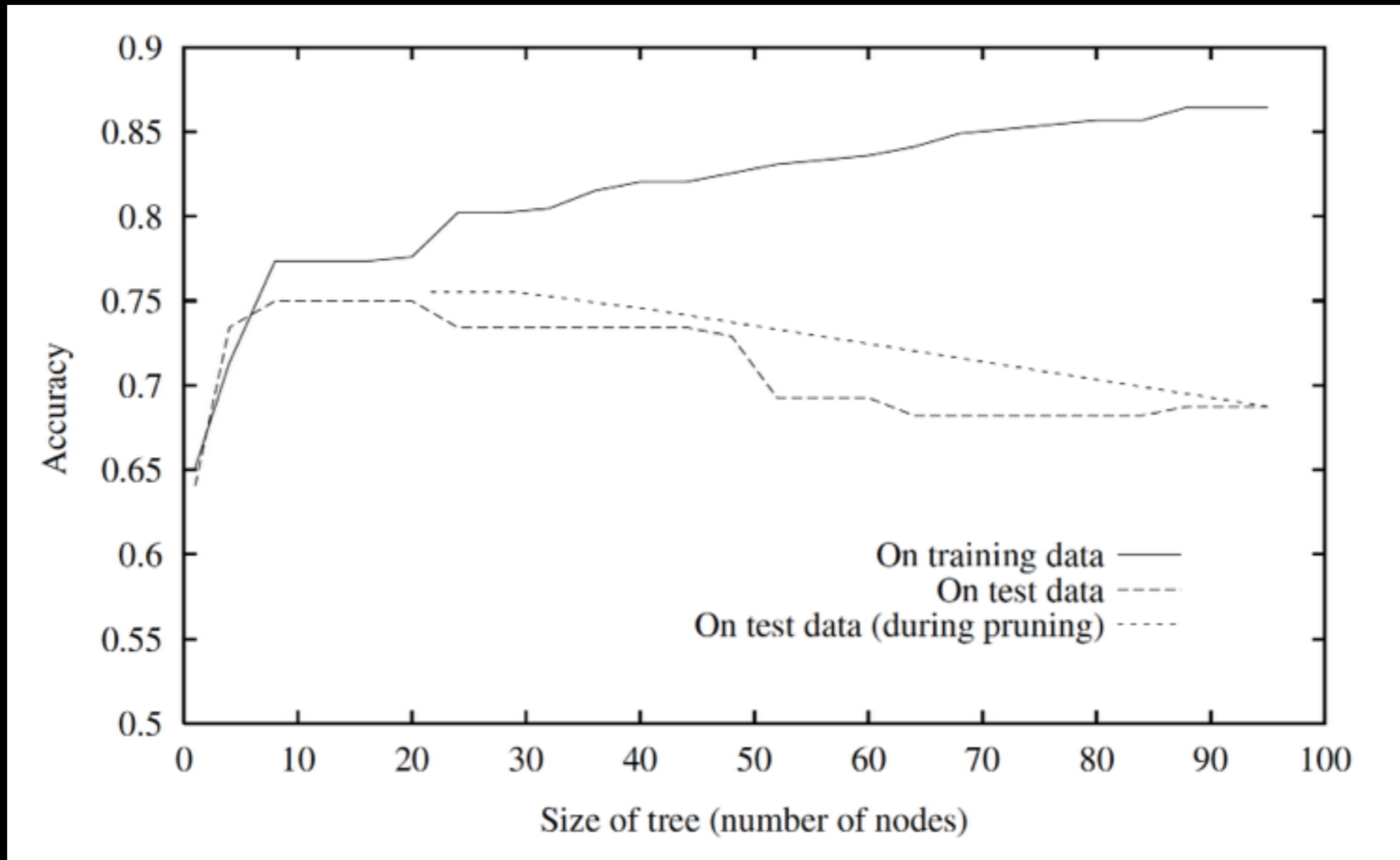
Reduced-Error Pruning

Split data into *training* and *validation* set

Do until further pruning is harmful:

1. Evaluate impact on *validation* set of pruning each possible node (plus those below it)
 2. Greedily remove the one that most improves *validation* set accuracy
- produces smallest version of most accurate subtree

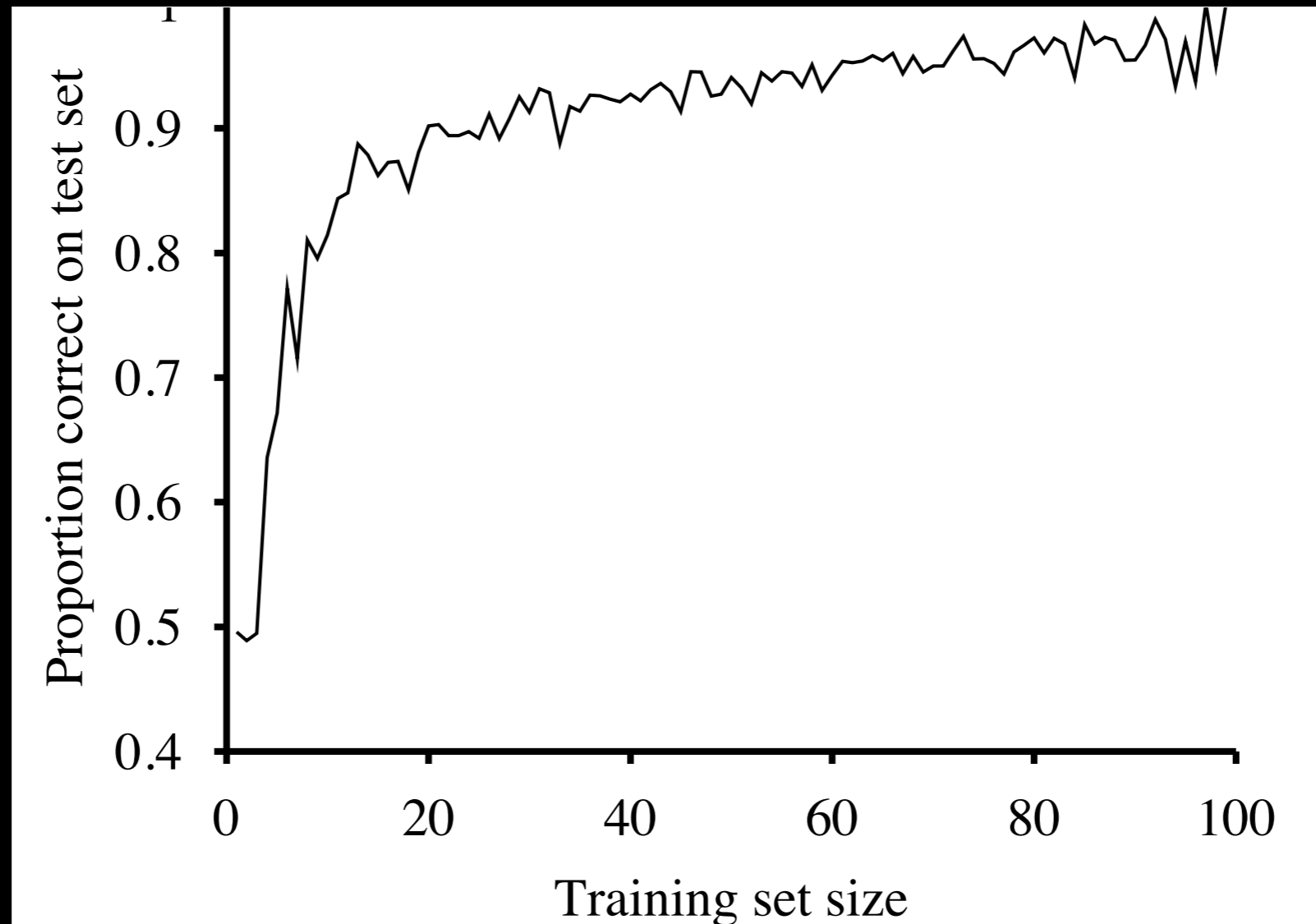
Effect of Reduced-Error Pruning



Evaluating Learning Mechanisms

Evaluating Learning

- Split data into training set and testing set
- Learn a hypothesis h using the training set and evaluate it on the testing set
- Start with training set of size 1 up to size $N-1$



Learning Curve

Error Rate

- Error rate: proportion of times $h(x) \neq y$ for an (x, y) example
- Inverse of proportion correct (accuracy)
- Need to evaluate error rate on examples not used in training

Cross-Validation

- Randomly split data into training and testing (in some proportion)
 - Hold out test data during training
- Doesn't use all data for training

k-Fold Cross-Validation

- Divide data into k equal subsets
- Perform k rounds of learning
 - Leave out 1 subset ($1/k$ of the data) each round; use for testing that round
- Average test scores over k rounds

Learning
(from Examples)
Summary

Learning

- Kinds and dimensions of learning
- General framework for supervised, passive, immediate feedback learning
- Classification and Regression
- Data: training, testing, (pruning)
- Generalization, error, overfitting
- Hypothesis space: lines, curves, decision trees, ...

Coming Up

- April 8: Exam 3, Probability & Introduction to Learning
- Project 3: Learning to Recognize Faces using Neural Networks
 - Assigned: April 10th
 - Due on April 29
- April 11: Neural Networks Part I