

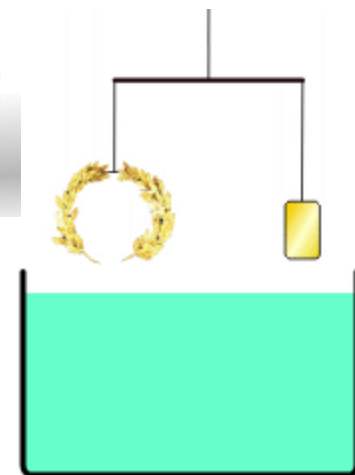
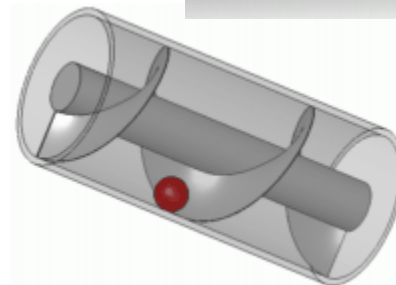
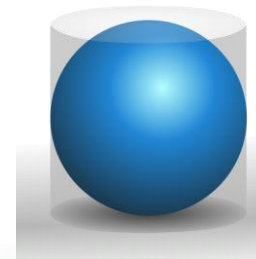
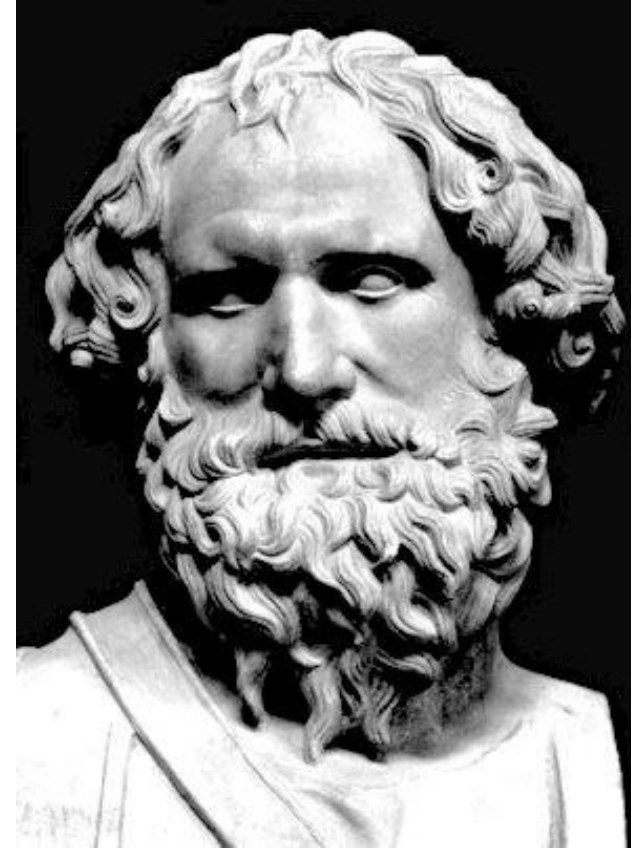
Historical Perspectives

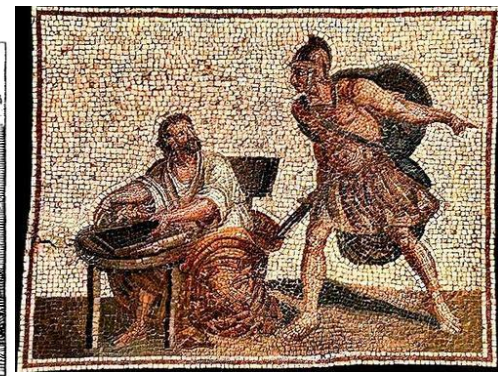
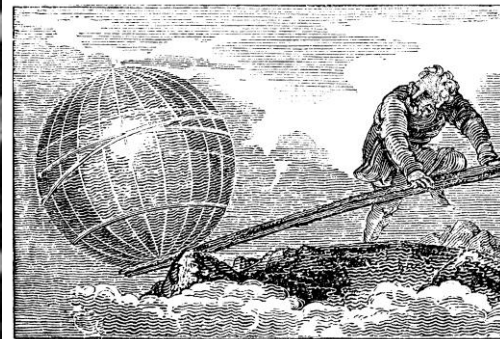
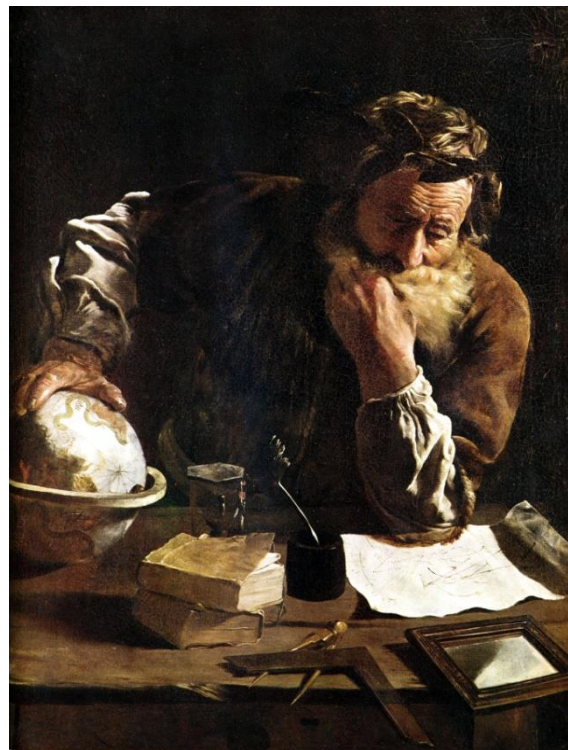
Archimedes of Syracuse (287-212 BC)

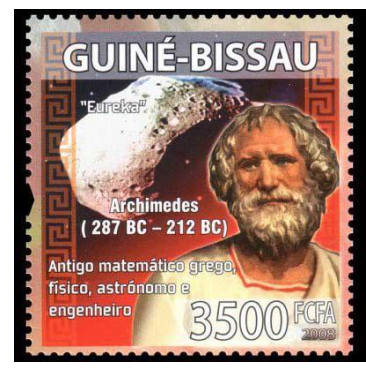
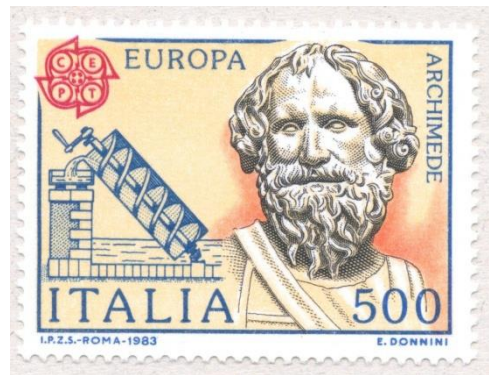
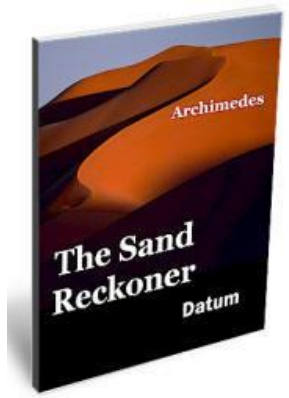
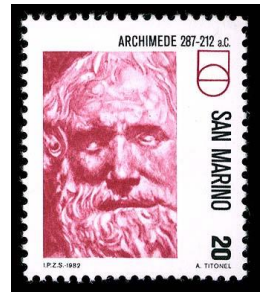
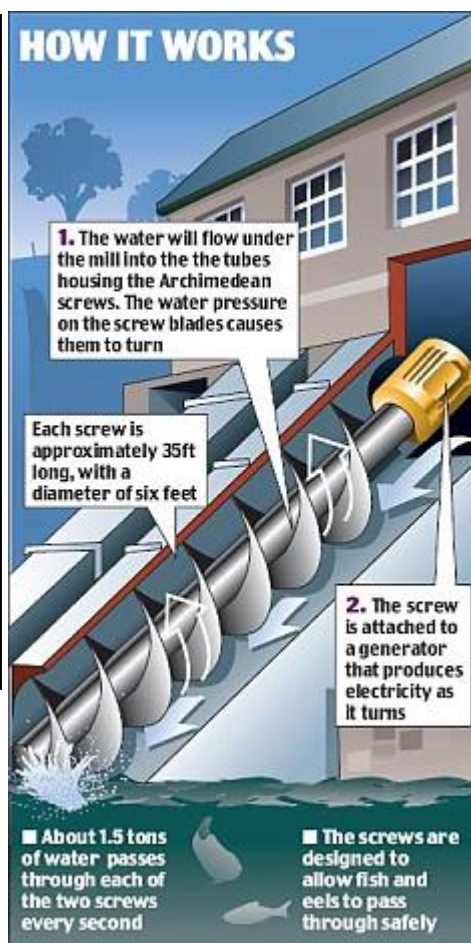
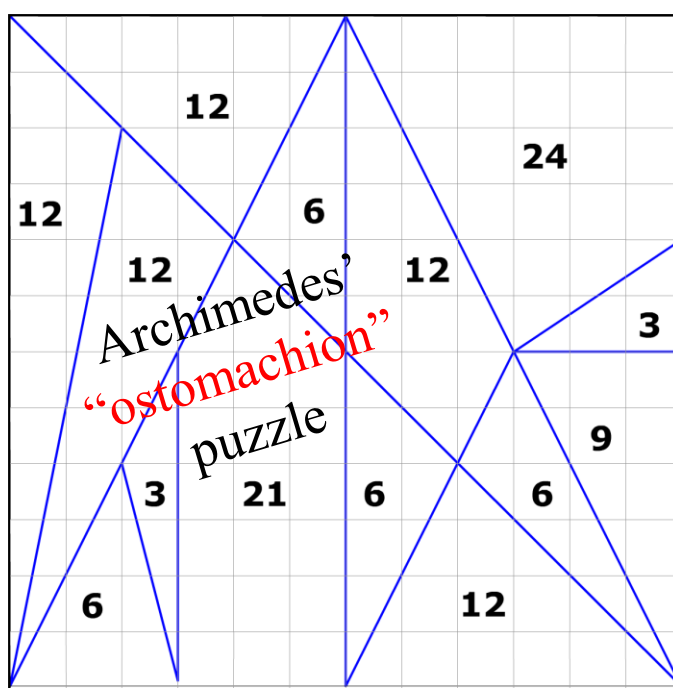
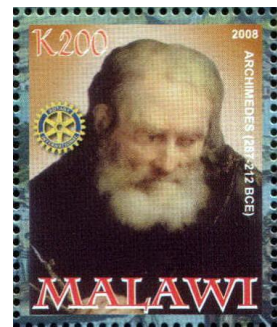
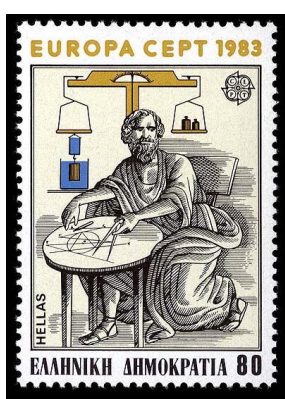
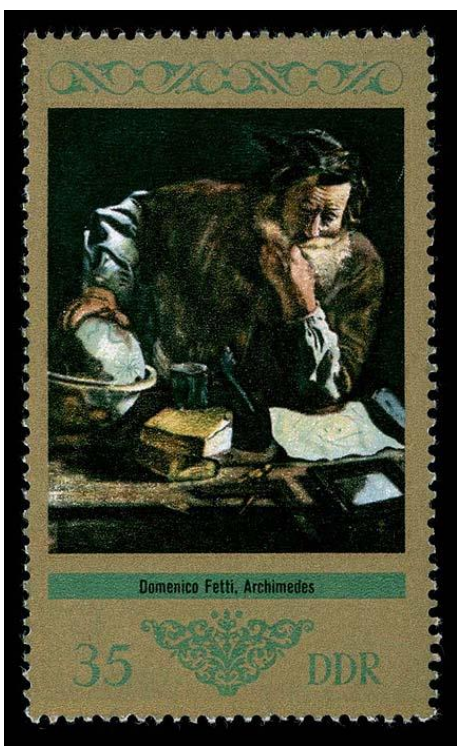
- Mathematician, physicist, engineer, inventor, astronomer
- Leading scientist of classical antiquity
- Originated **hydrostatics**, mechanics

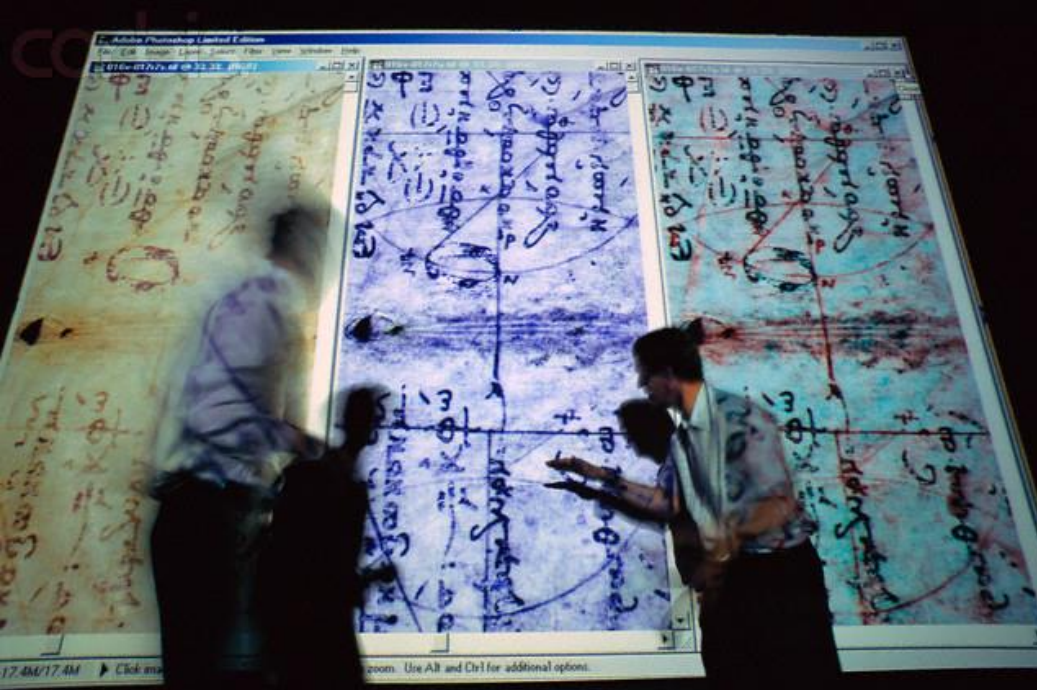
Archimedean screw, spiral, **lever**

- Discovered **Archimedes' principle**
- Used **infinitesimals**, approximated **Pi**
- Designed siege and naval **weapons**
- Invented large **number notation**









ARCHIMEDES 83 km / 2150 m

97 / 10 / 09 D=254mm f/D=10

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a.cidadao@mail.telepac.pt

8



"THERE GOES ARCHIMEDES WITH HIS CONFOUNDED LEVER AGAIN"

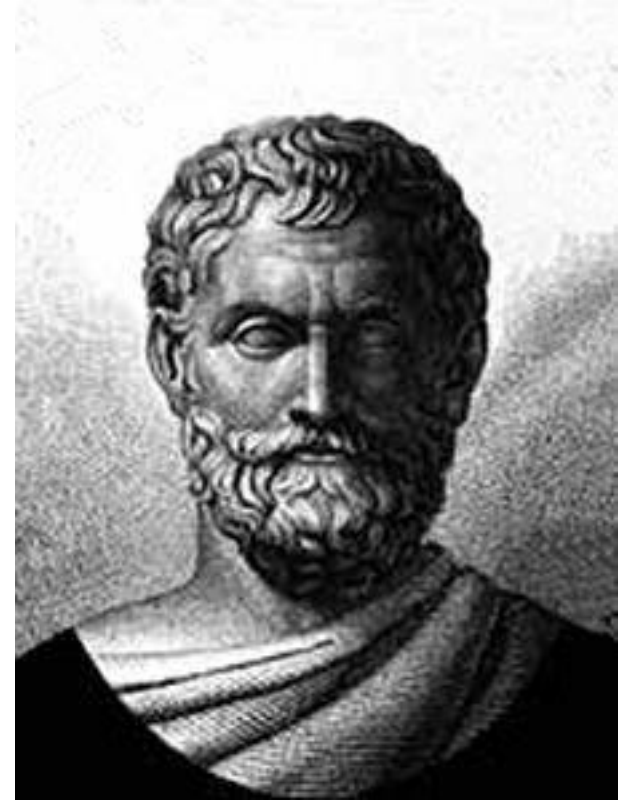


"The periodic table."

Historical Perspectives

Eratosthenes (276BC-194BC)

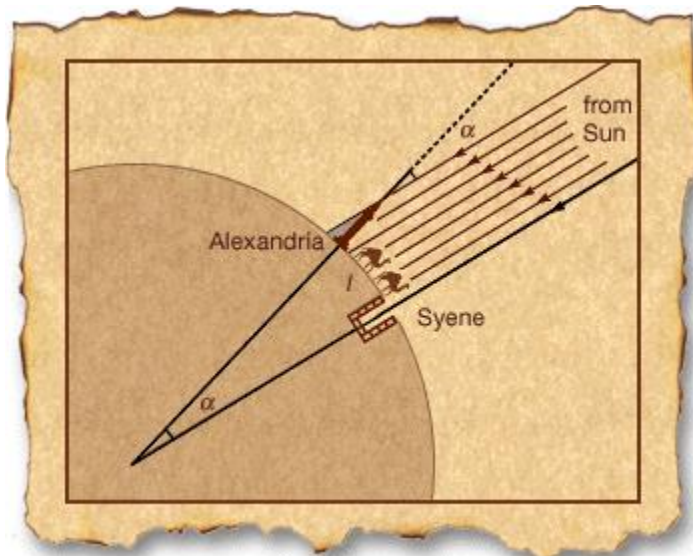
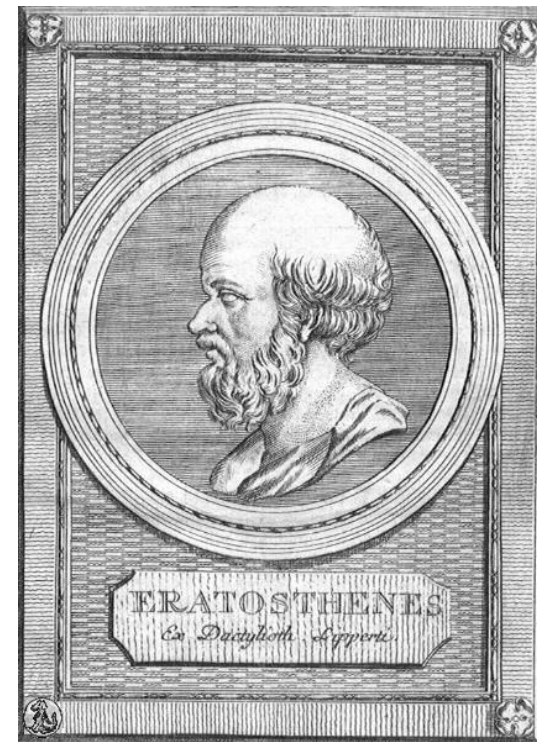
- Chief librarian at Library of Alexandria
- Measured the **Earth's size** (<1% error!)
- Calculated the Earth-Sun distance
- Invented **latitude** and **longitude**
- Primes - “**Sieve of Eratosthenes**”
- Chronology of ancient history
- Wrote on astronomy, geography, history, mathematics, philosophy, and literature



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
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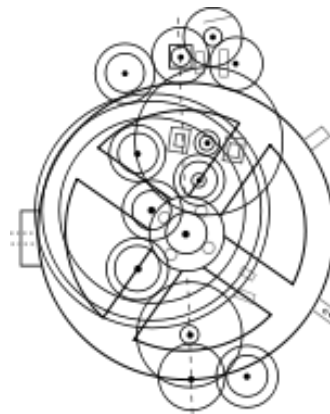
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11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

Prime numbers



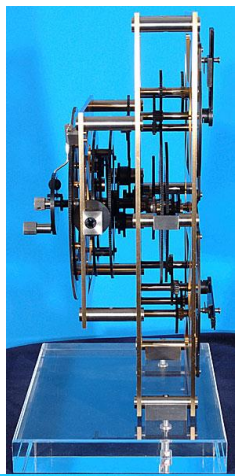
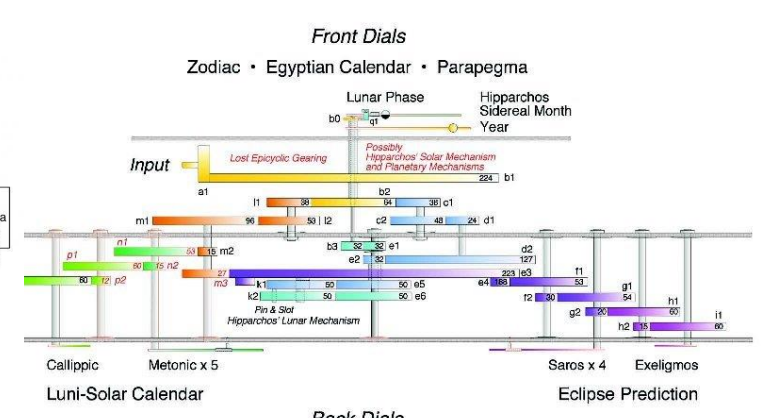
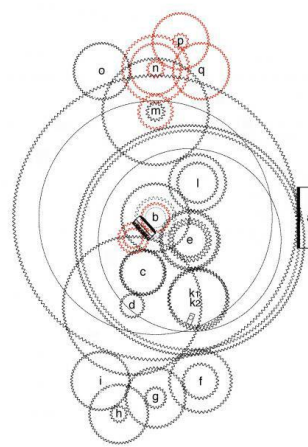
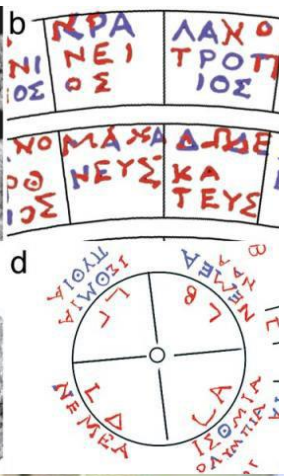
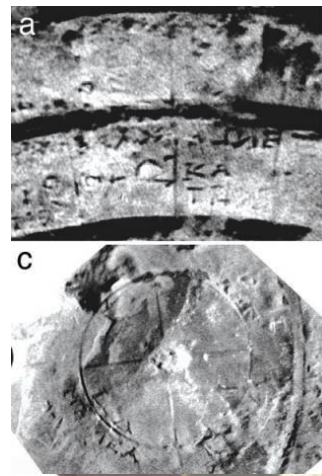
An Ancient Computer: The Antikythera

- Oldest known mechanical computer
- Built around **150-100 BCE !**
- Calculates eclipses and astronomical positions of sun, moon, and planets
- Very sophisticated for its era
- Contains dozens of intricate gears
- Comparable to 1700's Swiss clocks
- Has an attached "instructions manual"
- Still the subject of ongoing research

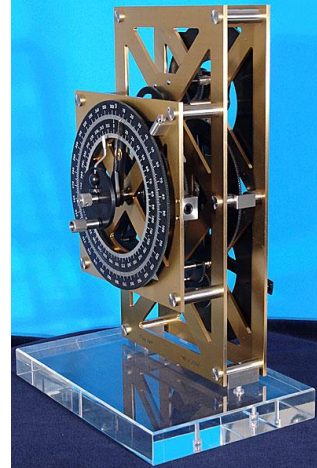




© Antikythera Mechanism Research Project



© Antikythera Mechanism Research Project



DECODING AN Ancient Computer

New explorations have revealed how the Antikythera mechanism modeled lunar motion and predicted eclipses, among other sophisticated tricks

By Tony Freeth



KEY CONCEPTS

- The Antikythera mechanism is a unique mechanical calculator from second-century B.C. Greece. Its sophistication surprised archaeologists when it was discovered in 1901. But no one had anticipated its true power.
- Advanced imaging tools have finally enabled researchers to reconstruct how the device predicted lunar and solar eclipses and the motion of the moon in the sky.
- Inscriptions on the mechanism suggest that it might have been built in the Greek city of Syracuse (now in modern Sicily), perhaps in a tradition that originated with Archimedes.

—The Editors

If it had not been for two storms 2,000 years apart in the same area of the Mediterranean, the most important technological artifact from the ancient world could have been lost forever.

The first storm, in the middle of the 1st century B.C., sank a Roman merchant vessel laden with Greek treasures. The second storm, in A.D. 1900, drove a party of sponge divers to shelter off the tiny island of Antikythera, between Crete and the mainland of Greece. When the storm subsided, the divers tried their luck for sponges in the local waters and chanced on the wreck. Months later the divers returned, with backing from the Greek government. Over nine months they recovered a hoard of beautiful ancient Greek objects—rare bronzes, stunning glassware, amphorae, pottery and jewelry—in one of the first major underwater archaeological excavations in history.

One item attracted little attention at first: an undistinguished, heavily calcified lump the size of a phone book. Some months later it fell apart, revealing the remains of corroded bronze gearwheels—all sandwiched together and with teeth just one and a half millimeters long—along with plates covered in scientific scales and Greek in-

scriptions. The discovery was a shock: until then, the ancients were thought to have made gears only for crude mechanical tasks.

Three of the main fragments of the Antikythera mechanism, as the device has come to be known, are now on display at the Greek National Archaeological Museum in Athens. They look small and fragile, surrounded by imposing bronze statues and other artistic glories of ancient Greece. But their subtle power is even more shocking than anyone had imagined at first.

I first heard about the mechanism in 2000. I was a filmmaker, and astronomer Mike Edmunds of Cardiff University in Wales contacted me because he thought the mechanism would make a great subject for a TV documentary. I learned that over many decades researchers studying the mechanism had made considerable progress, suggesting that it calculated astronomical data, but they still had not been able to fully grasp how it worked. As a former mathematician, I became intensely interested in understanding the mechanism myself.

Edmunds and I gathered an international collaboration that eventually included historians, astronomers and two teams of imaging experts. In the past few years our group has reconstruct-

ed how nearly all the surviving parts worked and what functions they performed. The mechanism calculated the dates of lunar and solar eclipses, modeled the moon's subtle apparent motions through the sky to the best of the available knowledge, and kept track of the dates of events of social significance, such as the Olympic Games. Nothing of comparable technological sophistication is known anywhere in the world for at least a millennium afterward. Had this unique specimen not survived, historians would have thought that it could not have existed at that time.

Early Pioneers

German philologist Albert Rehm was the first person to understand, around 1905, that the Antikythera mechanism was an astronomical calculator. Half a century later, when science historian Derek J. de Solla Price, then at the Institute for Advanced Study in Princeton, N.J., described the device in a *Scientific American* article, it still had revealed few of its secrets.

The device, Price suggested, was operated by turning a crank on its side, and it displayed its output by moving pointers on dials located on its front and back. By turning the crank, the user could set the machine on a certain date as indi-

cated on a 365-day calendar dial in the front. (The dial could be rotated to adjust for an extra day every four years, as in today's leap years.) At the same time, the crank powered all the other gears in the mechanism to yield the information corresponding to the set date.

A second front dial, concentric with the calendar, was marked out with 360 degrees and with the 12 signs representing the constellations of the zodiac [see box on pages 80 and 81]. These are the constellations crossed by the sun in its apparent motion with respect to the "fixed" stars—"motion" that in fact results from Earth's orbiting the sun—along the path called the ecliptic. Price surmised that the front of the mechanism probably had a pointer showing where along the ecliptic the sun would be at the desired date.

In the surviving fragments, Price identified the remains of a dozen gears that had been part of the mechanism's innards. He also estimated their tooth counts—which is all one can do given that nearly all the gears are damaged and incomplete. Later, in a landmark 1974 study, Price described 27 gears in the main fragment and provided improved tooth counts based on the first x-rays of the mechanism, by Greek radiologist Charalambos Karakalos.

ANCIENT GREEKS knew how to calculate the recurring patterns of lunar eclipses thanks to observations made for centuries by the Babylonians. The Antikythera mechanism would have done those calculations for them—or perhaps for the wealthy Romans who could afford to own it. The depiction here is based on a theoretical reconstruction by the author and his collaborators.

[THE PLACES]



The Greek and Roman worlds, circa 145 B.C.

Where Was It From?

The Antikythera mechanism was built around the middle of the 2nd century B.C., a time when Rome was expanding at the expense of the Greek-dominated Hellenistic kingdoms (green). Divers recovered its corroded remnants (including fragment at left) in A.D. 1901 from a shipwreck near the island of Antikythera. The ship sank around 65 B.C. while carrying Greek artistic treasures, perhaps from Pergamon to Rome. Rhodes had one of the major traditions of Greek astronomy, but the latest evidence points to a Corinthian origin. Syracuse, which had been a Corinthian colony in Sicily, is a possibility: the great Greek inventor Archimedes had lived there and may have left behind a technological tradition.

Tooth counts indicate what the mechanism calculated. For example, turning the crank to give a full turn to a primary 64-tooth gear represented the passage of a year, as shown by a pointer on the calendar dial. That primary gear was also paired to two 38-tooth secondary gears, each of which consequently turned by 64/38 times for every year. Similarly, the motion relayed from gear to gear throughout the mechanism; at each step, the ratio of the numbers of gear teeth represents a different fraction. The motion eventually transmitted to the pointers, which thus turned at rates corresponding to different astronomical cycles. Price discovered that the ratios of one of these gear trains embodied an ancient Babylonian cycle of the moon.

Price, like Rehm before him, suggested that the mechanism also contained epicyclic gearing—gears spinning on bearings that are themselves attached to other gears, like the cups on a Mad Hatter teacup ride. Epicyclic gears extend the range of formulas gears can calculate beyond multiplications of fractions to additions and subtractions. No other example of epicyclic gearing is known to have existed in Western technology for another 1,500 years.

Several other researchers studied the mechanism, most notably Michael Wright, a curator at the Science Museum in London, in collaboration

with computer scientist Allan Bromley of the University of Sydney. They took the first three-dimensional x-rays of the mechanism and showed that Price's model of the mechanism had to be wrong. Bromley died in 2002, but Wright persisted and made significant advances. For example, he found evidence that the back dials, which at first look like concentric rings, are in fact spirals and discovered an epicyclic mechanism at the front that calculated the phase of the moon.

Wright also adopted one of Price's insights, namely that the dial on the upper back might be a lunar calendar, based on the 19-year, 235-lunar-month cycle called the Metonic cycle. This calendar is named after fifth-century B.C. astronomer Meton of Athens—although it had been discovered earlier by the Babylonians—and is still used today to determine the Jewish festival of Rosh Hashanah and the Christian festival of Easter. Later, we would discover that the pointer was extensible, so that a pin on its end could follow a groove around each successive turn of the spiral.

BladeRunner in Athens

As our group began its efforts, we were hampered by a frustrating lack of data. We had no access to the previous x-ray studies, and we did not even have a good set of still photographs.

Two images in a science magazine—x-rays of a goldfish and an enhanced photograph of a Babylonian clay tablet—suggested to me new ways to get better data.

We asked Hewlett-Packard in California to perform state-of-the-art photographic imaging and X-Tek Systems in the U.K. to do three-dimensional x-ray imaging. After four years of careful diplomacy, John Seiradakis of the Aristotle University of Thessaloniki and Xenophon Moussas of the University of Athens obtained the required permissions, and we arranged for the imaging teams to bring their tools to Athens, a necessary step because the Antikythera mechanism is too fragile to travel.

Meanwhile we had a totally unexpected call from Mary Zafeiropoulou at the museum. She had been to the basement storage and found boxes of bits labeled "Antikythera." Might we be interested? Of course we were interested. We now had a total of 82 fragments, up from about 20.

The HP team, led by Tom Malzbender, assembled a mysterious-looking dome about five feet across and covered in electronic flashbulbs that provided lighting from a range of different angles. The team exploited a technique from the computer gaming industry, called polynomial texture mapping, to enhance surface details. In-

scriptions Price had found difficult to read were now clearly legible, and fine details could be enhanced on the computer screen by controlling the reflectance of the surface and the angle of the lighting. The inscriptions are essentially an instruction manual written on the outer plates.

A month later local police had to clear the streets in central Athens so that a truck carrying the BladeRunner, X-Tek's eight-ton x-ray machine, could gain access to the museum. The BladeRunner performs computed tomography similar to a hospital's CT scan, but with finer detail. X-Tek's Roger Hadland and his group had specially modified it with enough x-ray power to penetrate the fragments of the Antikythera mechanism. The resulting 3-D reconstruction was wonderful: whereas Price could see only a puzzle of overlapping gears, we could now isolate layers inside the fragment and see all the fine details of the gear teeth.

Unexpectedly, the x-rays revealed more than 2,000 new text characters that had been hidden deep inside the fragments. (We have now identified and interpreted a total of 3,000 characters out of perhaps 15,000 that existed originally.) In Athens, Moussas, and Yanis Bitsakis, also at the University of Athens, and Agamemnon Telikas of the Center for History and Palaeography be-

Historians would have thought that SOMETHING SO COMPLEX could not have existed at the time.

[THE AUTHOR]

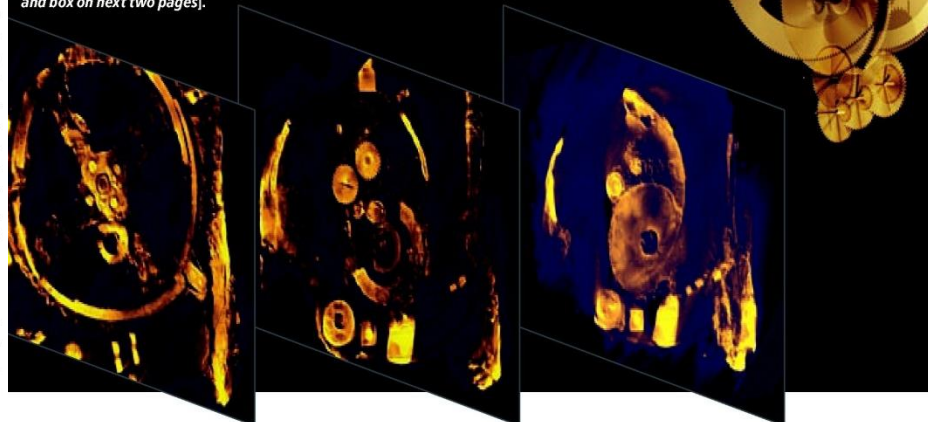
Tony Freeth's academic background is in mathematics and mathematical logic (in which he holds a Ph.D.). His award-winning career as a filmmaker culminated in a series of documentaries about increasing crop yields in sub-Saharan Africa, featuring the late Nobel Peace Prize Laureate Norman Borlaug. Since 2000 Freeth has returned to an academic focus with research on the Antikythera mechanism. He is managing director of the film and television production company Images First, and he is now developing a film on the mechanism.



[THE RECONSTRUCTION]

Anatomy of a Relic

Computed tomography—a 3-D mapping obtained from multiple x-ray shots—enabled the author and his colleagues to get inside views of the Antikythera mechanism's remnants. For example, a CT scan can be used to virtually slice up an object (below, slices of main fragment). The information helped the team see how the surviving gears connected and estimate their tooth counts, which determined what calculations they performed. The team could then reconstruct most of the device [see model at right and box on next two pages].



[INSIDE THE ANTIKYTHERA MECHANISM]

Astronomical Clockwork

gan to discover inscriptions that had been invisible to human eyes for more than 2,000 years. One translated as "... spiral subdivisions 235..." confirming that the upper back dial was a spiral describing the Metonic calendar.

Babylon System

Back at home in London, I began to examine the CT scans as well. Certain fragments were clearly all part of a spiral dial in the lower back. An estimate of the total number of divisions in the dial's four-turn spiral suggested 220 to 225.

The prime number 223 was the obvious contender. The ancient Babylonians had discovered that if a lunar eclipse is observed—something that can happen only during a full moon—usually a similar lunar eclipse will take place 223 full moons later. Similarly, if the Babylonians saw a solar eclipse—which can take place only during a new moon—they could predict that 223 new moons later there would be a similar one (although they could not always see it: solar eclipses are visible only from specific locations, and ancient astronomers could not predict them reliably). Eclipses repeat this way because every 223 lunar months the sun, Earth and the moon return to approximately the same alignment with respect to one another, a periodicity known as the Saros cycle.

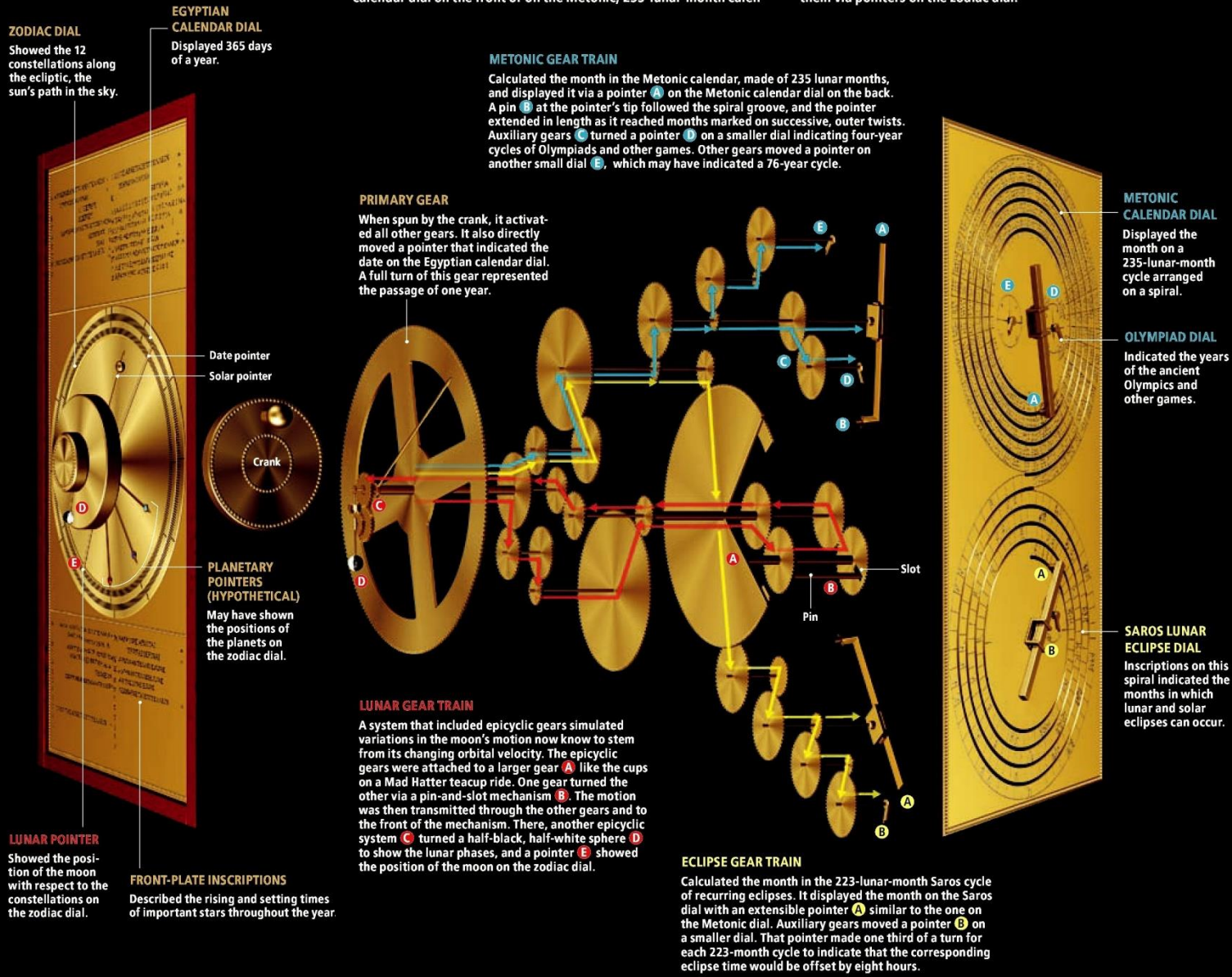
Between the scale divisions were blocks of symbols, nearly all containing Σ (*sigma*) or *H* (*eta*), or both. I soon realized that Σ stands for $\Sigma\epsilon\lambda\eta\nu\eta$ (*selene*), Greek for "moon," indicating a lunar eclipse; *H* stands for $\text{H}\lambda\omega\sigma$ (*helios*), Greek for "sun," indicating a solar eclipse. The Babylonians also knew that within the 223-month period, eclipses can take place only in particular months, arranged in a predictable pattern and separated by gaps of five or six months; the distribution of symbols around the dial exactly matched that pattern.

I now needed to follow the trail of clues into the heart of the mechanism to discover where this new insight would lead. The first step was to find a gear with 223 teeth to drive this new Saros dial. Karakalos had estimated that a large gear visible at the back of the main fragment had 222 teeth. But Wright had revised this estimate to 223, and Edmunds confirmed this. With plausible tooth counts for other gears and with the addition of a small, hypothetical gear, this 223-tooth gear could perform the required calculation.

But a huge problem still remained unsolved and proved to be the hardest part of the gearing to crack. In addition to calculating the Saros cy-

This exploded view of the mechanism shows all but one of the 30 known gears, plus a few that have been hypothesized. Turning a crank on the side activated all the gears in the mechanism and moved pointers on the front and back dials: the arrows colored blue, red and yellow explain how the motion transmitted from one gear to the next. The user would choose a date on the Egyptian, 365-day calendar dial on the front or on the Metonic, 235-lunar-month calen-

dar on the back and then read the astronomical predictions for that time—such as the position and phases of the moon—from the other dials. Alternatively, one could turn the crank to set a particular event on an astronomical dial and then see on what date it would occur. Other gears, now lost, may have calculated the positions of the sun and of some or all of the five planets known in antiquity and displayed them via pointers on the zodiac dial.

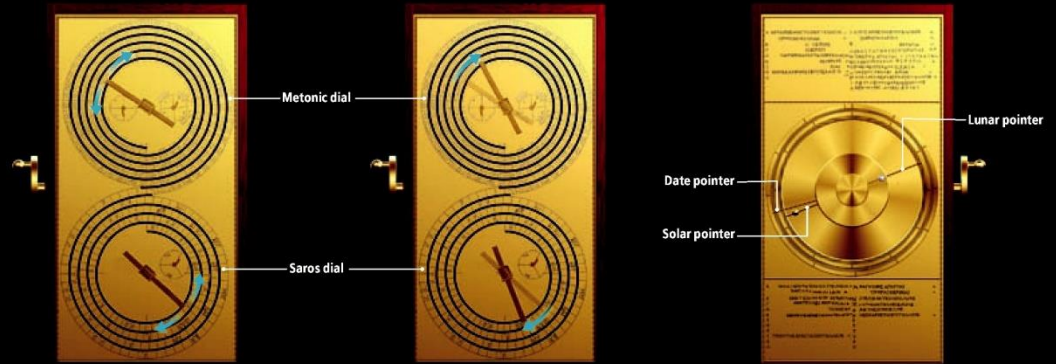


GRAPHIC BY WASHINGTON AND TONY FREETH

[A USER'S MANUAL]

How to Predict an Eclipse

Operating the Antikythera mechanism may have required only a small amount of practice and astronomical knowledge. After an initial calibration by an expert, the mechanism could provide fairly accurate predictions of events several decades in the past or future. The inscriptions on the Saros dial, coming at intervals of five or six months, corresponded to months when Earth, the sun and the moon come to a near alignment (and so represented potential solar and lunar eclipses) in a 223-lunar-month cycle. Once the month of an eclipse was known, the actual day could be calculated on the front dials using the fact that solar eclipses always happen during new moons and lunar eclipses during full moons.



RESET DATE

Begin by turning the crank to set the current month and year on the Metonic calendar. The lower pointer will turn to the corresponding month on the Saros (eclipse) dial.

FIND ECLIPSE MONTH

Turn the crank to move time forward until the pointer on the Saros dial points to an eclipse inscription. The inscription will indicate month and time of the day (but not the day) of an eclipse and whether it will be solar or lunar.

CALCULATE DAY

Adjust the crank until the lunar and solar pointers are aligned (for a solar eclipse) or at 180 degrees (for a lunar eclipse). The Egyptian calendar pointer will move correspondingly and indicate the day of the eclipse.

cle, the large 223-tooth gear also carried the epicyclic system noticed by Price: a sandwich of two small gears attached to the larger gear in teacup-ride fashion. Each epicyclic gear also connected to another small gear. Confusingly, all four small gears appeared to have the same tooth count—50—which seemed nonsensical because the output would then be the same as the input.

After months of frustration, I remembered that Wright had observed that one of the two epicyclic gears has a pin on its face that engages with a slot on the other. His key idea was that the two gears turned on slightly different axes, separated by about a millimeter. As a consequence, the angle turned by one gear alternated between being slightly wider and being slightly narrower than the angle turned by the other gear. Thus, if one gear turned at a constant rate, the other gear's rate kept varying between slightly faster and slightly slower.

Ask for the Moon

Although Wright rejected his own observation, I realized that the varying rotation rate is precisely what is needed to calculate the moon's motion according to the most advanced astronomical theory of the second century B.C., the one often attributed to Hipparchos of Rhodes. Before Archimedes (A.D. 1605), no one understood that orbits are elliptical and that the moon accelerates toward the perigee—its closest point to Earth—and slows down toward the apogee, the opposite point. But the ancients did know that the moon's motion against the zodiac appears to periodically slow down and speed up. In Hipparchos's model, the moon moved at a constant rate around a circle whose center itself moved around a circle at a constant rate—a fairly good approximation of the moon's apparent motion. These circles on circles, themselves called epicycles, dominated astronomical thinking for the next 1,800 years.

There was one further complication: the apogee and perigee are not fixed, because the ellipse of the moon's orbit rotates by a full turn about every nine years. The time it takes for the body to get back to the perigee is thus a bit longer than the time it takes it to come back to the same point in the zodiac. The difference was just 0.112579655 turns a year. With the input gear having 27 teeth, the rotation of the large gear was slightly too big; with 26 teeth, it was slightly too small. The right result seemed to be about halfway in between. So I tried the impossible idea that the input gear had 26 1/2 teeth. I pressed the key on my calculator, and it gave 0.112579655—

exactly the right answer. It could not be a coincidence to nine places of decimals! But gears cannot have fractional numbers of teeth.

Then I realized that $26 \frac{1}{2} \times 2 = 53$. In fact, Wright had estimated a crucial gear to have 53 teeth, and I now saw that that count made everything work out. The designer had mounted the pin and slot epicyclically to subtly slow down the period of its variation while keeping the basic rotation the same, a conception of pure genius. Thanks to Edmunds, we also realized that the epicyclic gearing system, which is in the back of the mechanism, moved a shaft that turned inside another, hollow shaft through the rest of the mechanism and to the front, so that the lunar motion could be represented on the zodiac dial and on the lunar phase display. All gear counts were now explained, with the exception of one small gear that remains a mystery to this day.

Further research has caused us to make some modifications to our model. One was about a small subsidiary dial that is positioned in the back, inside the Metonic dial, and is divided into four quadrants. The first clue came when I read the word "NEMEA" under one of the quadrants. Alexander Jones, a New York University historian, explained that it refers to the Nemean Games, one of the major athletic events in ancient Greece. Eventually we found, engraved round the four sectors of the dial, most of "ISTHMIJA," for games at Corinth, "PYTHIA," for games at Delphi, "NAA," for minor games at Dodona, and "OLYMPIA," for the most important games of the Greek world, the Olympics. All games took place every two or four years. Previously we had considered the mechanism to be

purely an instrument of mathematical astronomy, but the Olympiad dial—as we named it—gave it an entirely unexpected social function.

Twenty-nine of the 30 surviving gears calculate cycles of the sun and the moon. But our studies of the inscriptions at the front of the mechanism have also yielded a trove of information on the risings and settings of significant stars and of the planets. Moreover, on the "primary" gear-wheel at the front of the mechanism remnants of bearings stand witness to a lost epicyclic system that could well have modeled the back-and-forth motions of the planets along the ecliptic (as well as the anomalies in the sun's own motion). All these clues strongly support the inclusion of the sun and of at least some of the five planets known in ancient times—Mercury, Venus, Mars, Jupiter and Saturn.

Wright built a model of the mechanism with epicyclic systems for all five planets. But his ingenious layout does not agree with all the evidence. With its 40 extra gears, it may also be too complex to match the brilliant simplicity of the rest of the mechanism. The ultimate answer may still lie 50 meters down on the ocean floor.

Eureka?

The question of where the mechanism came from and who created it is still open. Most of the cargo in the wrecked ship came from the eastern Greek world, from places such as Pergamon, Kos and Rhodes. It was a natural guess that Hipparchos or another Rhodian astronomer built the mechanism. But text hidden between the 235 monthly scale divisions of the Metonic calendar contradicts this view. Some of the month names

were used only in specific locations in the ancient Greek world and suggest a Corinthian origin. If the mechanism was from Corinth itself, it was almost certainly made before Corinth was completely devastated by the Romans in 146 B.C. Perhaps more likely is that it was made to use in one of the Corinthian colonies in northwestern Greece or Sicily.

Sicily suggests a remarkable possibility. The island's city of Syracuse was home to Archimedes, the greatest scientist of antiquity. In the first century B.C. Roman statesman Cicero tells how in 212 Archimedes was killed at the siege of Syracuse and how the victorious Roman general, Marcellus, took away with him only one piece of plunder—an astronomical instrument made by Archimedes. Was that the Antikythera mechanism? We believe not, because it appears to have been made many decades after Archimedes died. But it could have been constructed in a tradition of instrument making that originated with the eureka man himself.

Many questions about the Antikythera mechanism remain unanswered—perhaps the greatest being why this powerful technology seems to have been so little exploited in its own era and in succeeding centuries.

In *Scientific American*, Price wrote:

It is a bit frightening to know that just before the fall of their great civilization the ancient Greeks had come so close to our age, not only in their thought, but also in their scientific technology.

Our discoveries have shown that the Antikythera mechanism was even closer to our world than Price had conceived.

MORE TO EXPLORE

An Ancient Greek Computer. Derek J. de Solla Price in *Scientific American*, Vol. 200, No. 6, pages 60–67; June 1959.

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Decoding the Ancient Greek Astronomical Calculator Known as the Antikythera Mechanism. Tony Freeth et al. in *Nature*, Vol. 444, pages 587–591; November 30, 2006.

Calendars with Olympiad Display and Eclipse Prediction on the Antikythera Mechanism. Tony Freeth, Alexander Jones, John M. Steele and Yanis Bitsakis in *Nature*, Vol. 454, pages 614–617; July 31, 2008.

The Antikythera Mechanism Research Project: www.antikythera-mechanism.gr

DATA BY XTEC SYSTEMS, SOFTWARE BY VOLUME GRAPHICS. © 2005 ANTIKYTHERA MECHANISM RESEARCH PROJECT

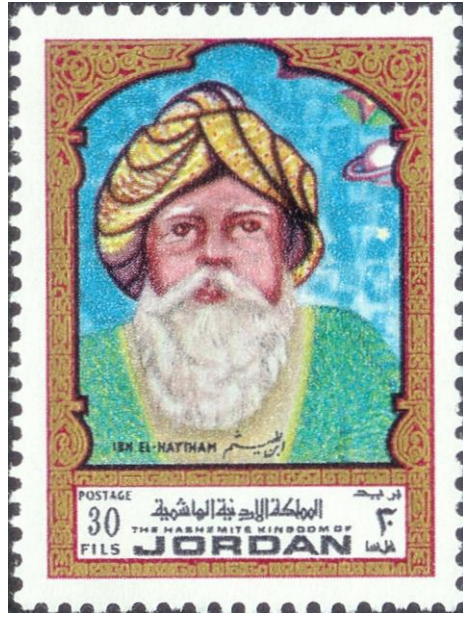
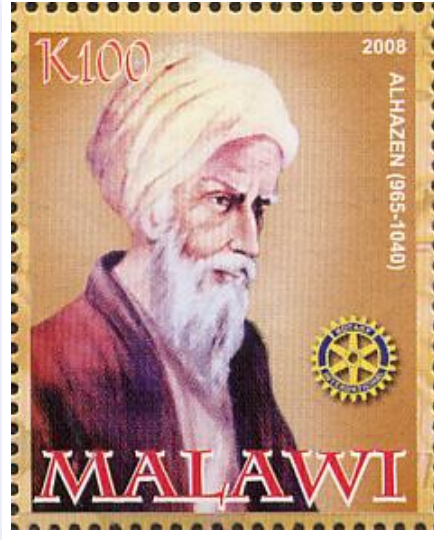
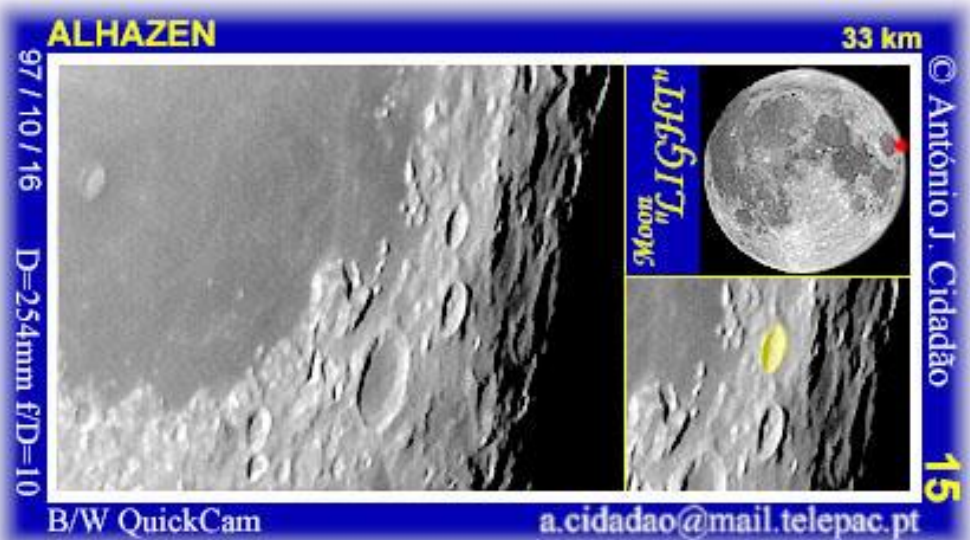
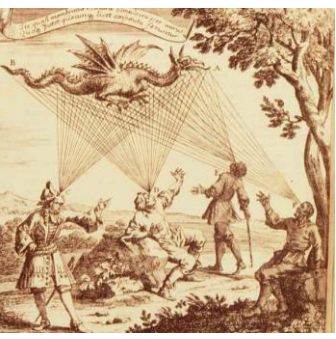
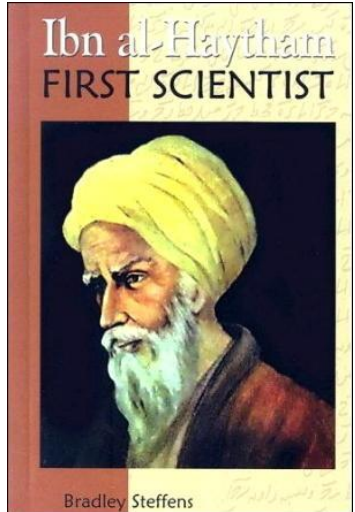
GRIFF WATSON AND TONY FREETH

Historical Perspectives

Abu Ali al-Hasan ibn al-Haytham (965-1039)

- AKA **Alhazen** or “The Physicist”
- **Greatest scientist of the middle ages**
- Contributed to mathematics, physics, optics, astronomy, anatomy, medicine, engineering, philosophy, psychology
- Pioneered the **scientific method**, modern **optics** and **experimental physics**
- Polymath: authored over 200 treatises, including influential “**Book of Optics**”
- Influenced Leonardo da Vinci, Bacon, Descartes, Kepler, Galileo and Newton





THE OLD SCIENTIFIC METHOD

Formulate a hypothesis.
Accumulate data.
Do extensive
experimentation.



THE NEW SCIENTIFIC METHOD

Formulate a hypothesis.
Patent it.
Raise \$17 million.



J. HARRIS

Historical Perspectives

Leonardo of Pisa (1170–1250)

- Better known as “**Fibonacci**”
- Considered the most talented mathematician of the middle ages
- Published (1202) “**Liber Abaci**” – “The Book of Calculation”
- Introduced Hindu-Arabic **positional number system** in Europe
- Popularized **Fibonacci sequence**



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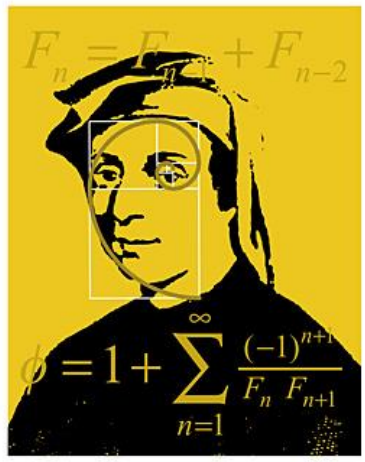
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Fibonacci
Un ponte sul Mediterraneo
Die arabische Wissenschaft
und die Wiedergeburt der
Mathematik im Abendland

AUSSTELLUNGEN HAUPTHALLE ETH ZENTRUM
27. OKTOBER 2003 BIS 14. NOVEMBER 2003
AUSSTELLUNGSBOYER DER ETH-BIBLIOTHEK
27. OKTOBER 2003 BIS 7. FEBRUAR 2004
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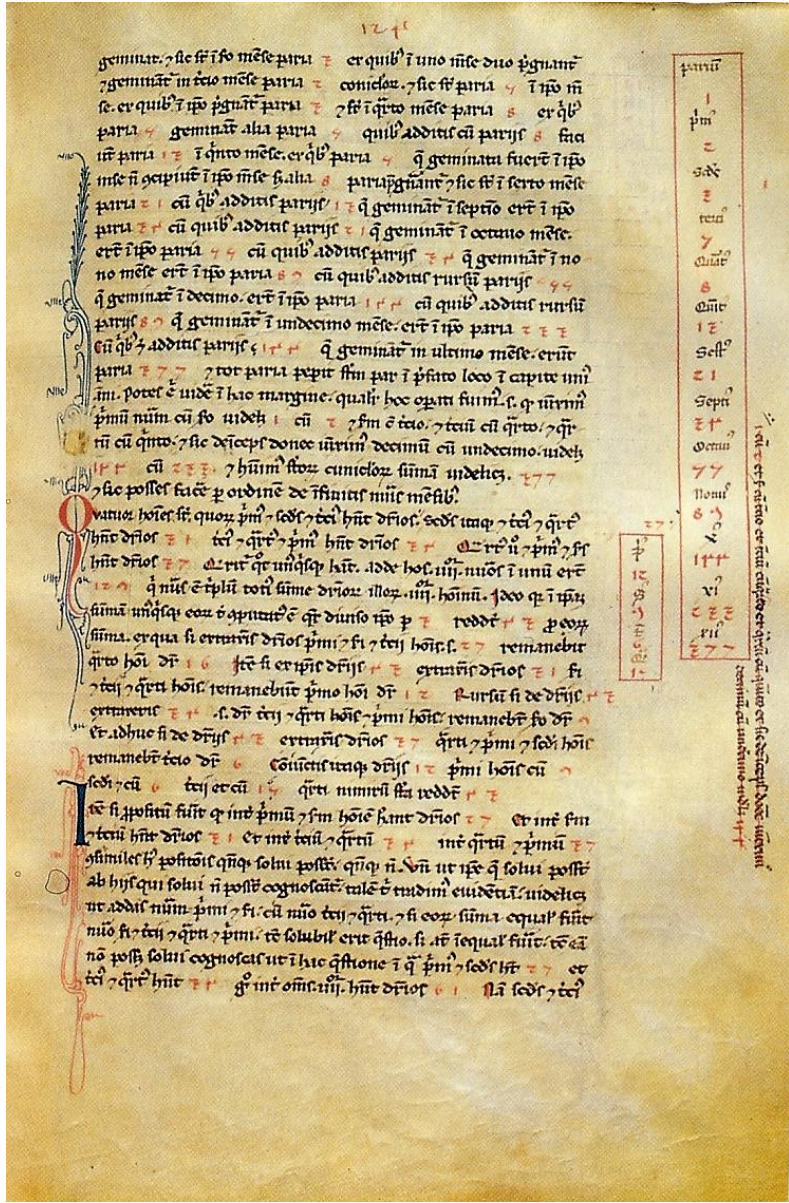


Leonardo Pisano Fibonacci
1170 - 1250

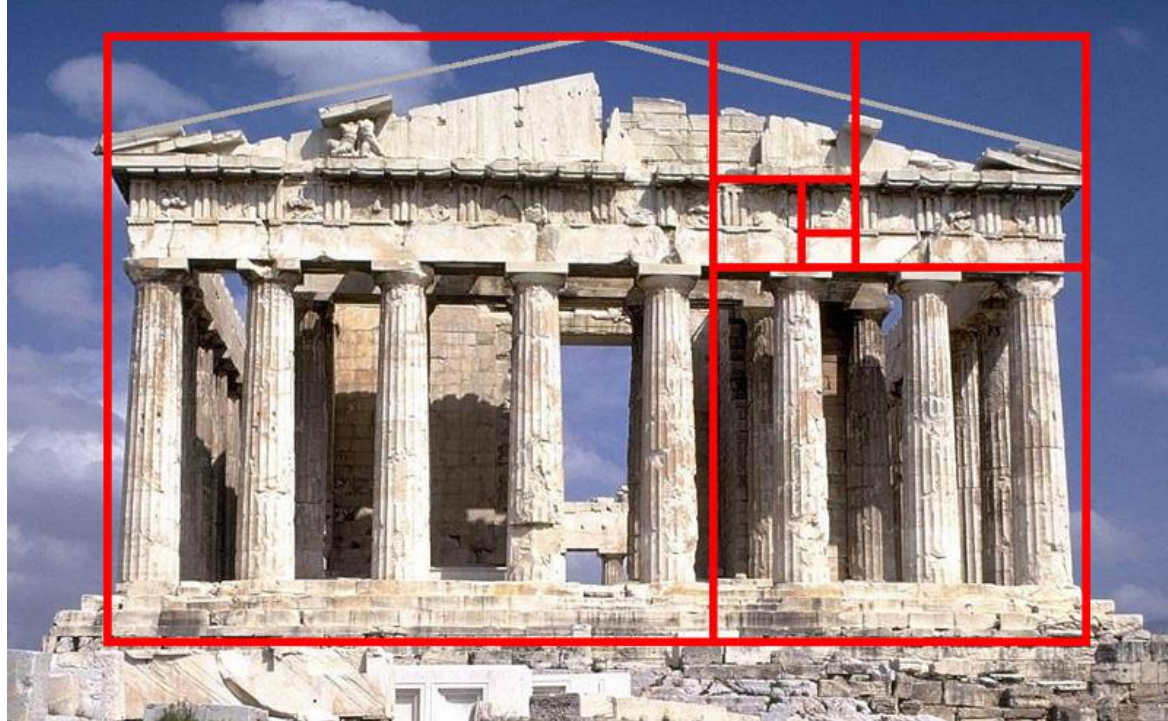
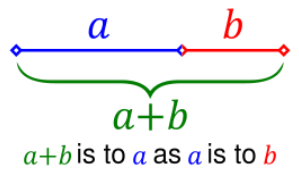
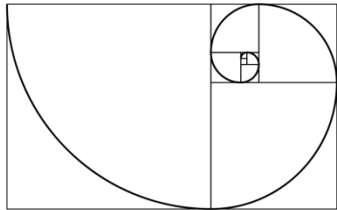
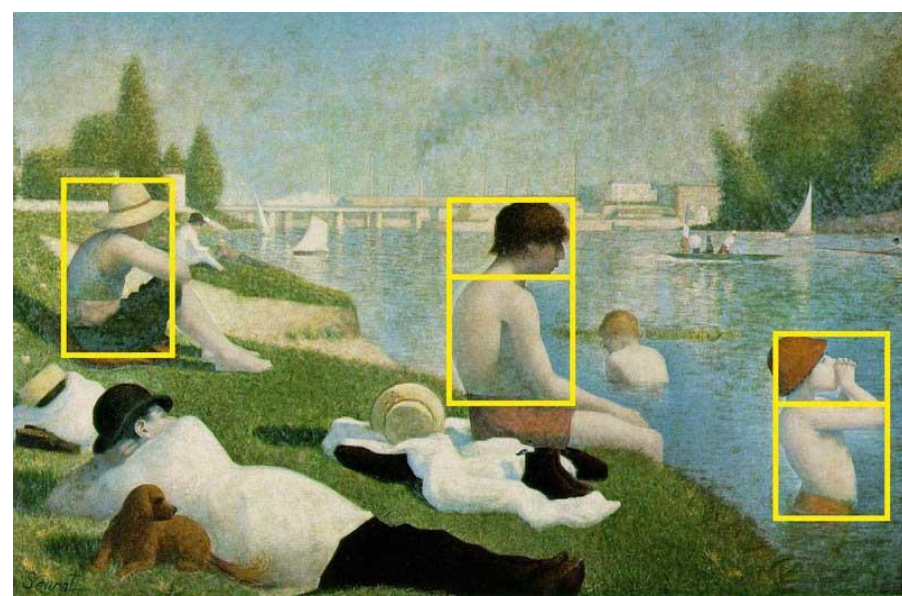


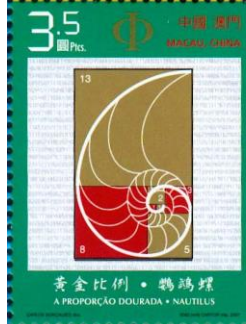
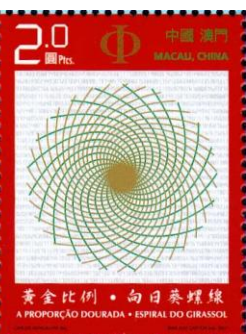
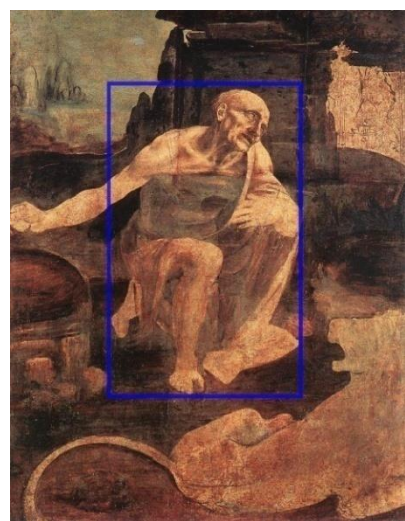
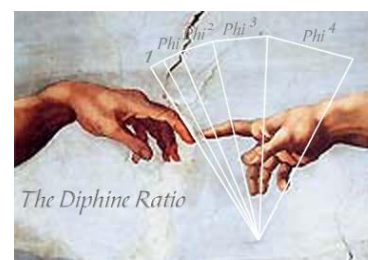
The Fibonacci Quarterly

Official Publication of The Fibonacci Association



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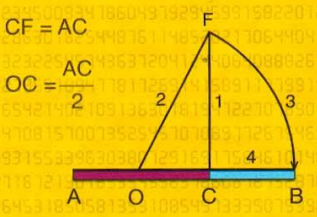
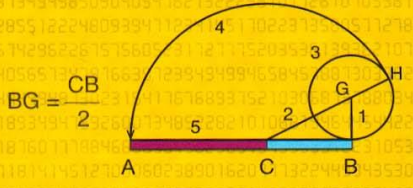
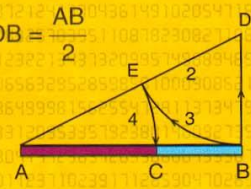




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科學與科技 - 黃金比例

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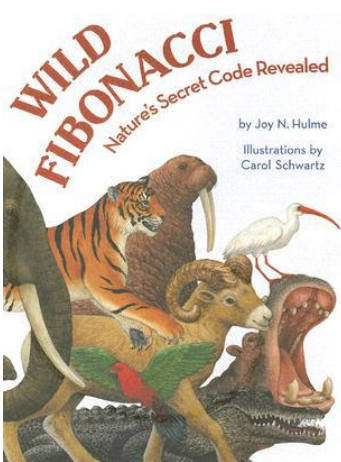
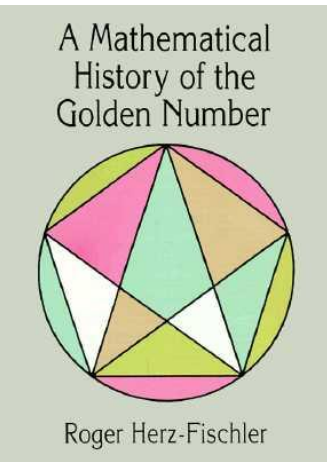
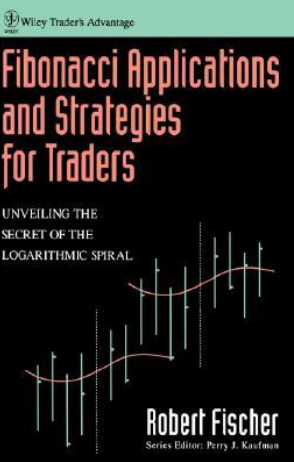
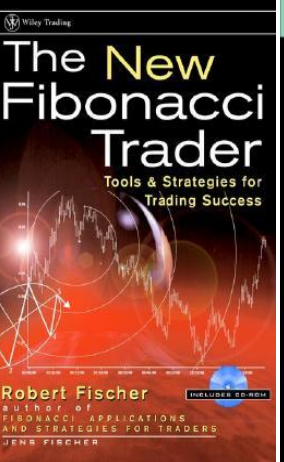
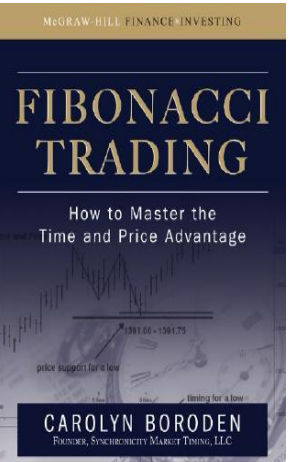
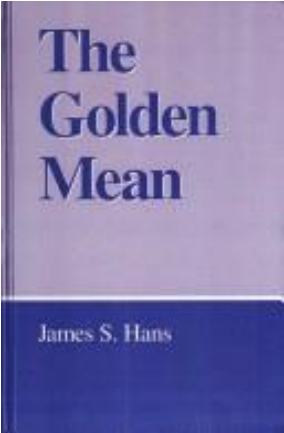
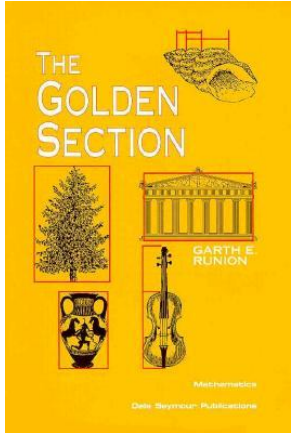
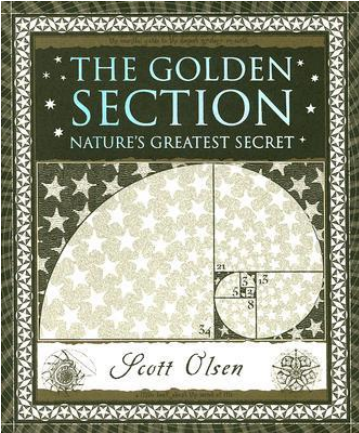
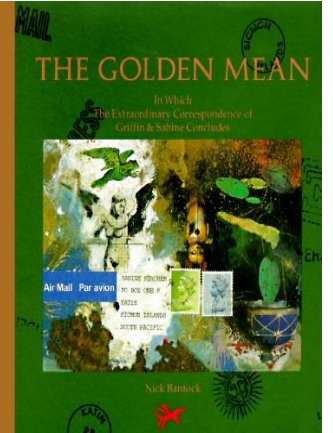
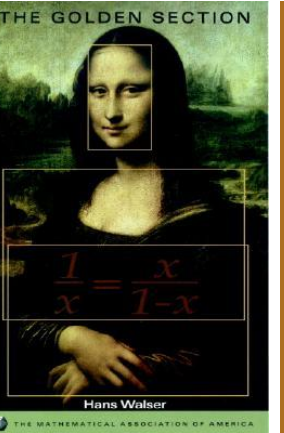
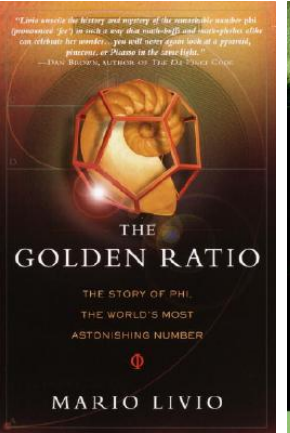
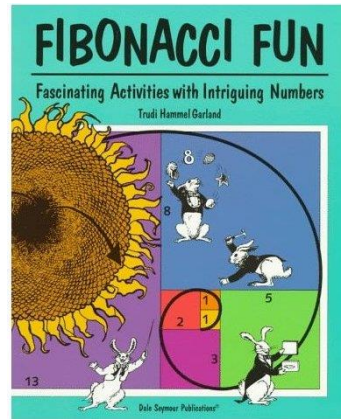
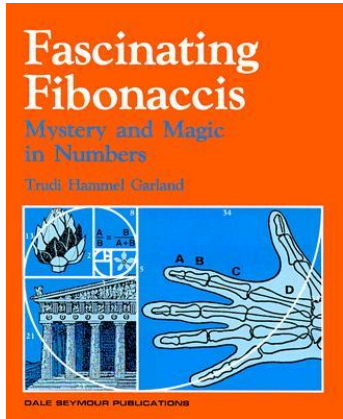
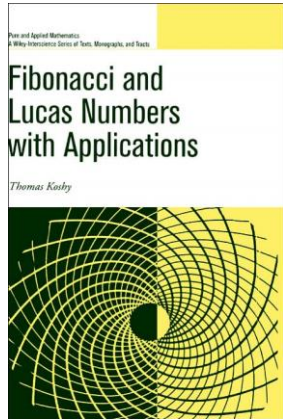
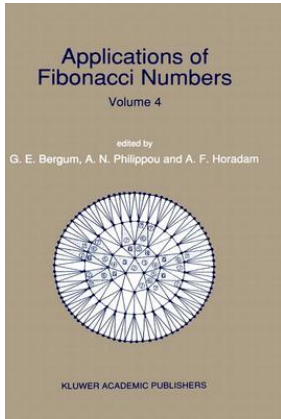
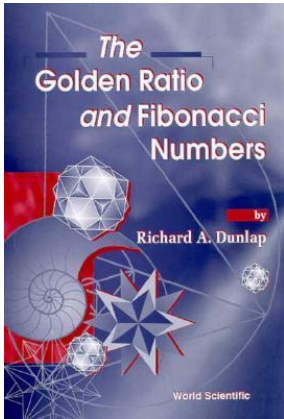
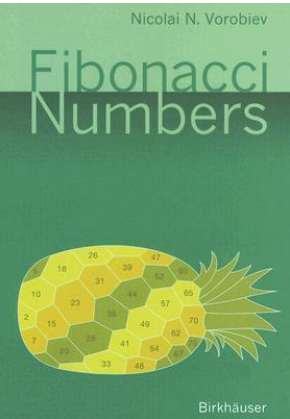


Ciência e Tecnologia - A Proporção Dourada



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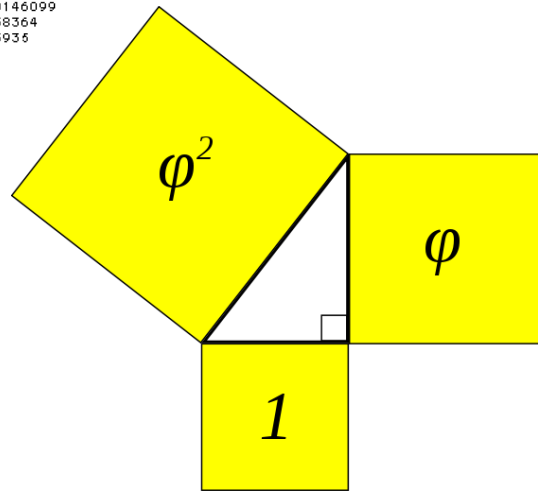
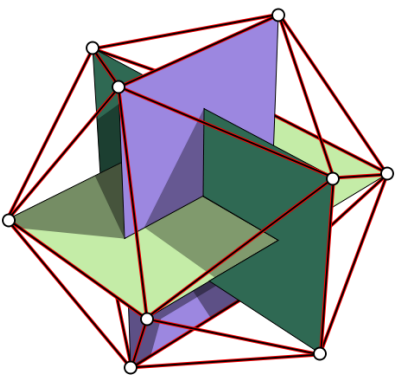
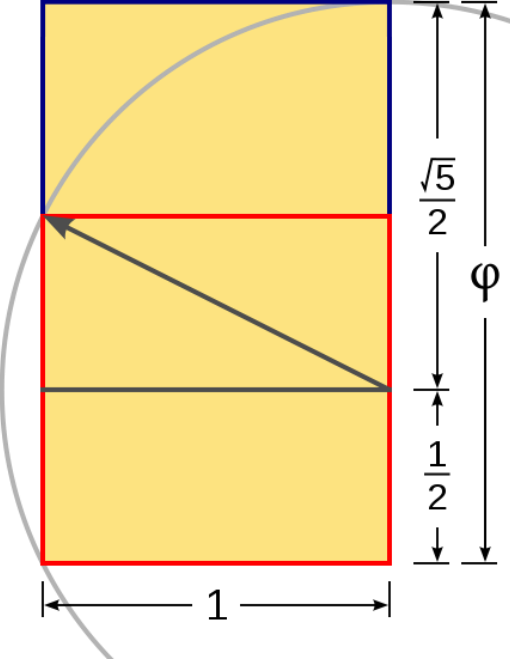
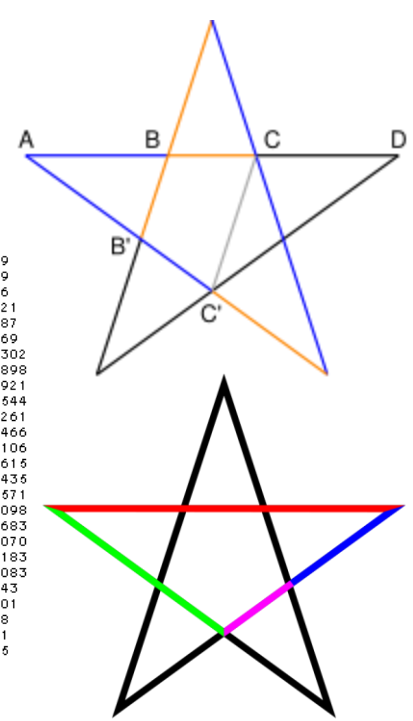




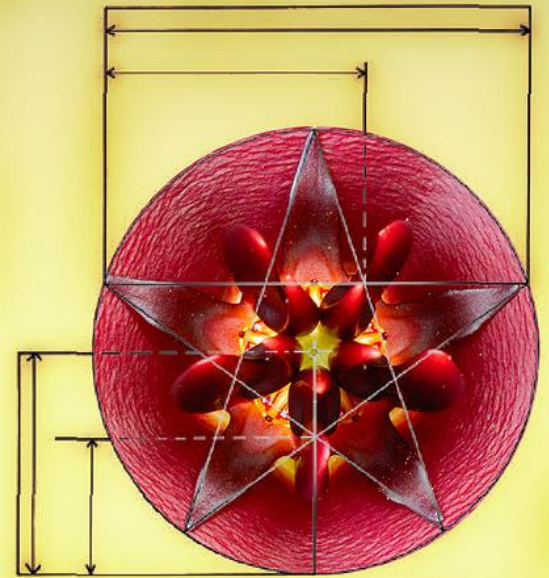
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651327106280323174069317334482343564631806801363108
649733360759966778712449058363675413289086
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4362652689448885
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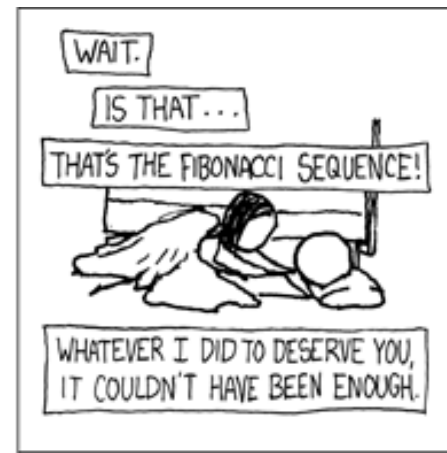
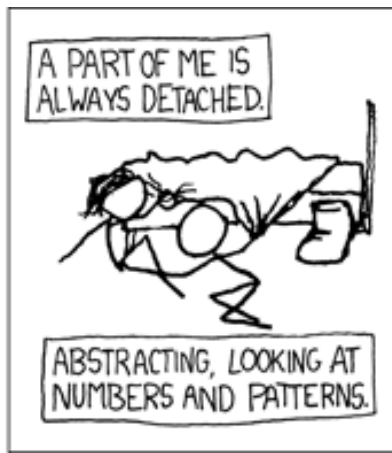
The Golden Ratio (1 : 1.618)







"This must be Fibonacci's."

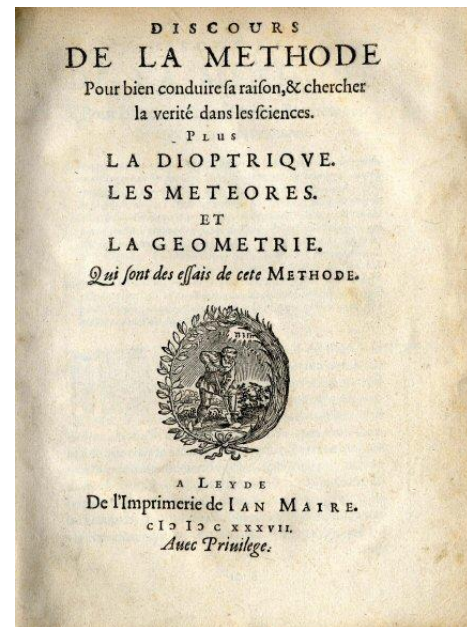
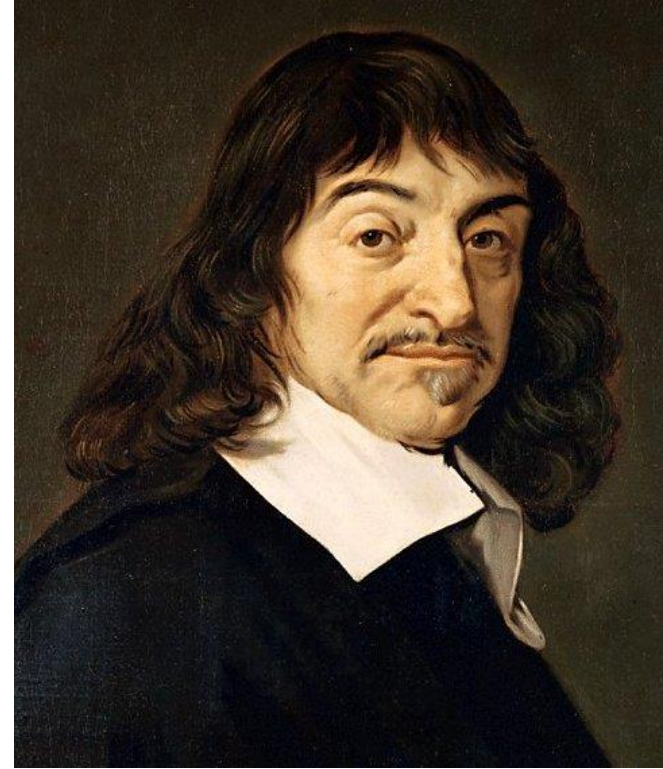


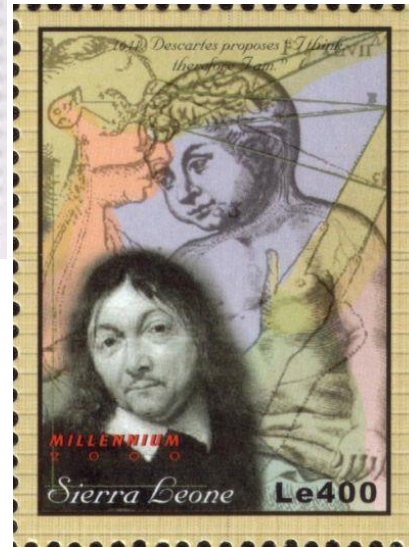
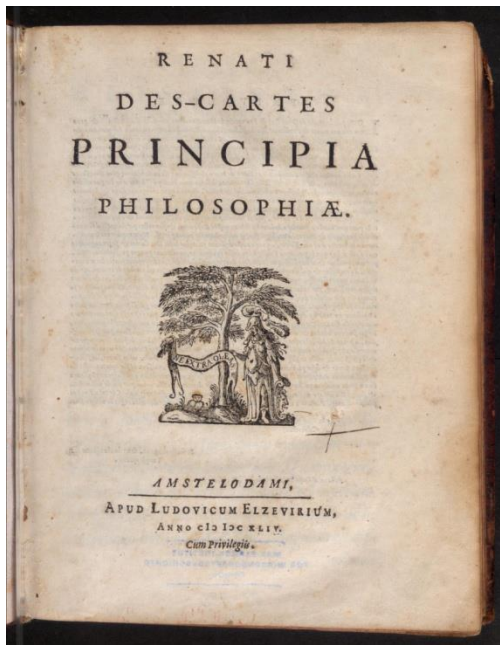
Historical Perspectives

René Descartes (1596-1650)

- Father of **modern philosophy**
- Invented **Cartesian coordinates**, **analytic geometry**, heuristics
- Characterized paradoxes & fallacies
- Discovered **momentum conservation**
- Authored “Principia Philosophiae”
- Pioneered methodological skepticism
- “**Cogito ergo sum**” - “Je pense, donc je suis ”
- “**Discours de la Méthode**” (1637) - one of the most influential works in modern science
- Pioneered the **scientific method** & revolution

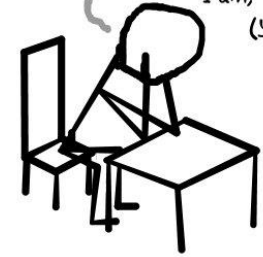
“For it is not enough to have a good mind:
one must use it well.” - Descartes





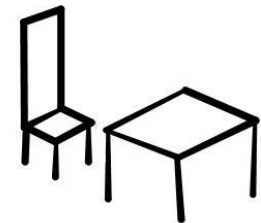
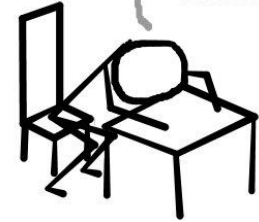


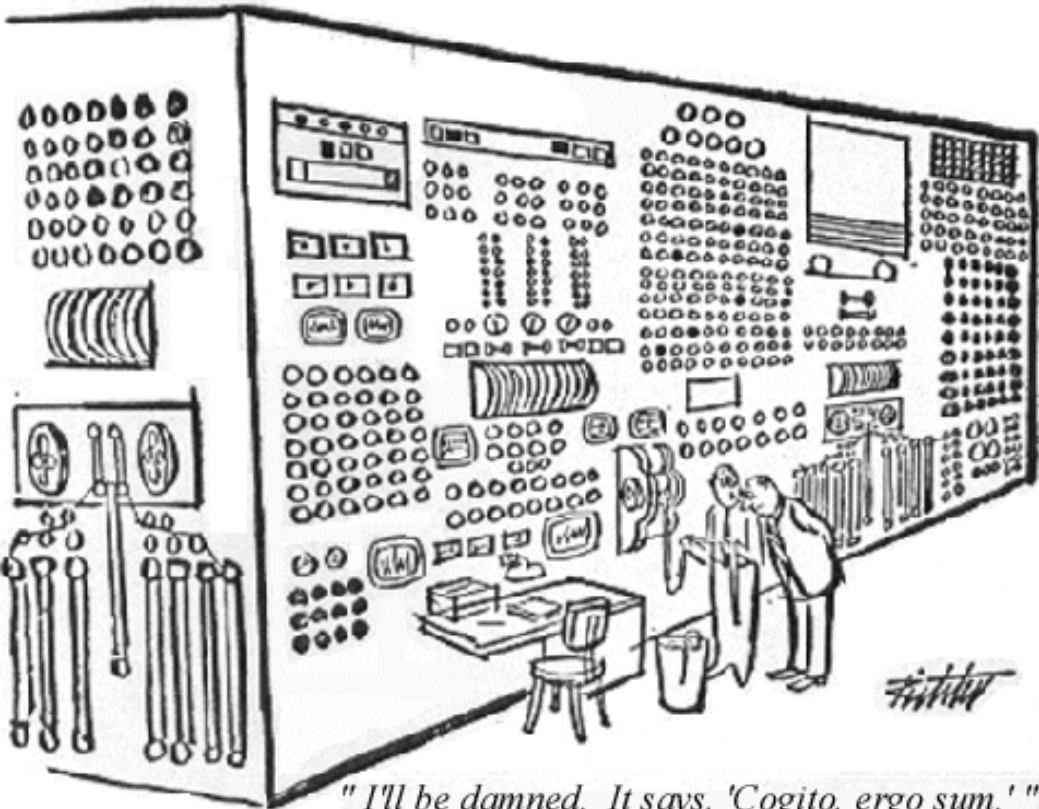
I think therefore I am.
 I am therefore I think... Ii think...
 I think, and therefore (yawn) -
 therefore I am.
 I am, therefore I
 (YAWN) think.
 I think....



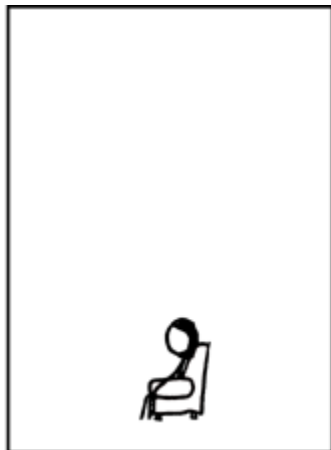
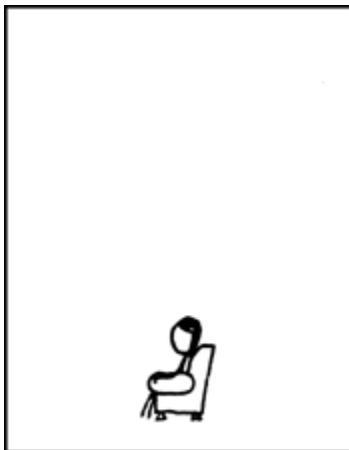
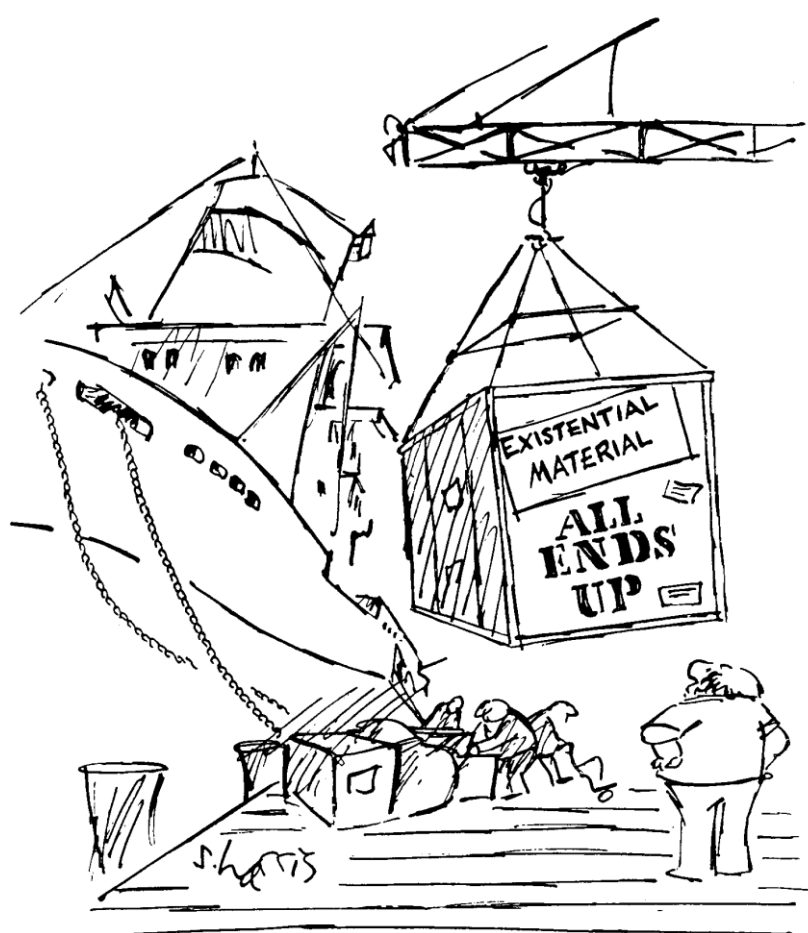
ZZZZzzzzz

ZZzzzzz.





"I'll be damned. It says, 'Cogito, ergo sum.'"



IF THE QUESTION OF WHAT IT ALL MEANS DOESN'T MEAN ANYTHING, WHY DO I KEEP COMING BACK TO IT?

SHE'S GETTING EXISTENTIAL AGAIN.

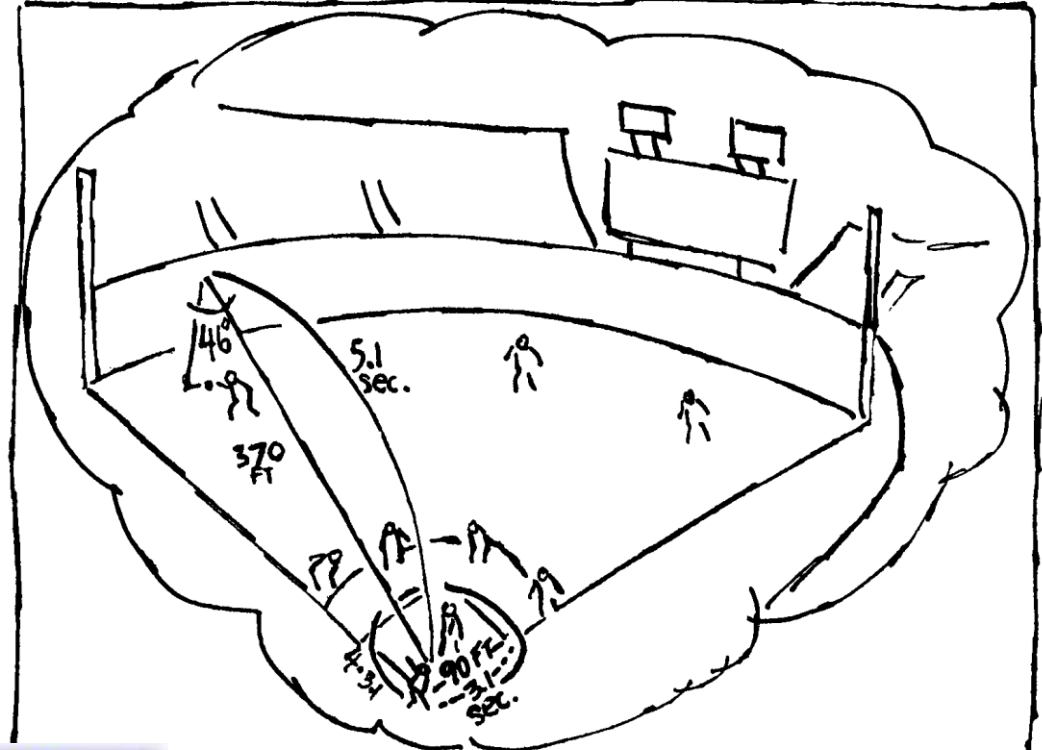
IT'S OKAY, I HAVE A SUPER SOAKER.



René Descartes 1596 - 1650



RENÉ DESCARTES EXPLAINS THE COORDINATE SYSTEM WHICH TIES TOGETHER ALGEBRA AND GEOMETRY

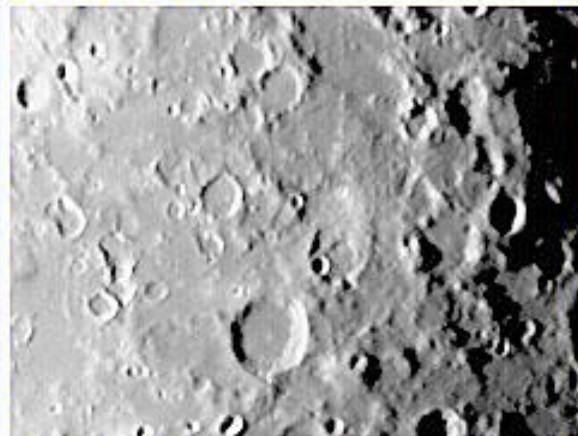


DESCARTES

48 km

98 / 04 / 16

D=254mm F/D=10



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20

B/W QuickCam

a.cidadao@mail.telepac.pt

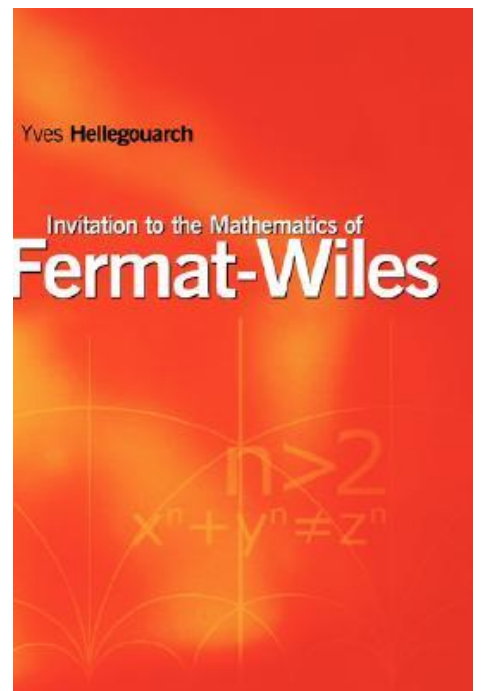
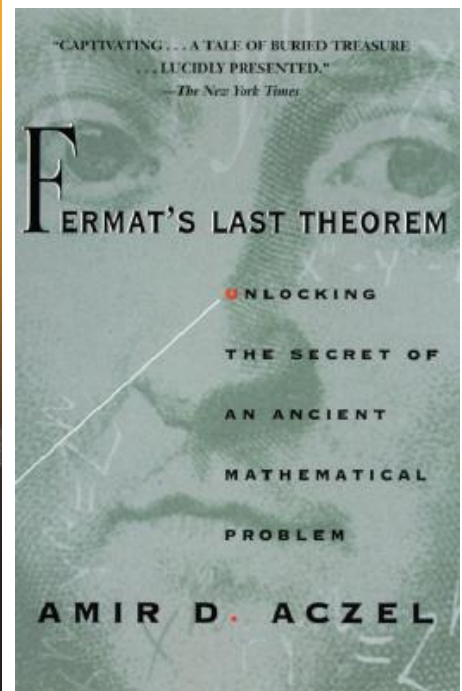
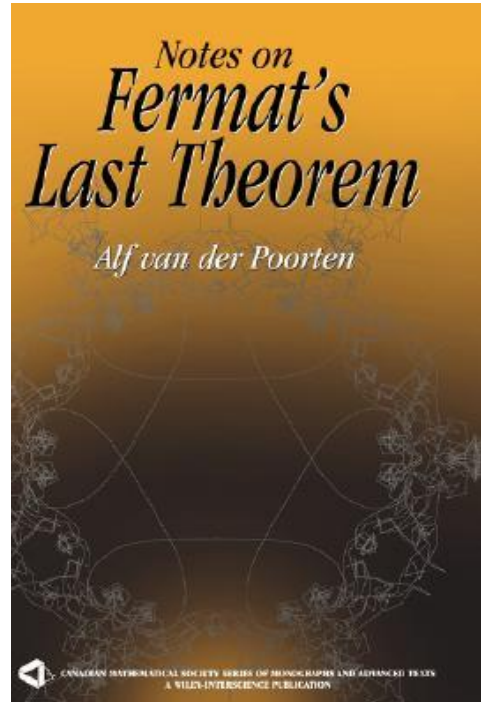
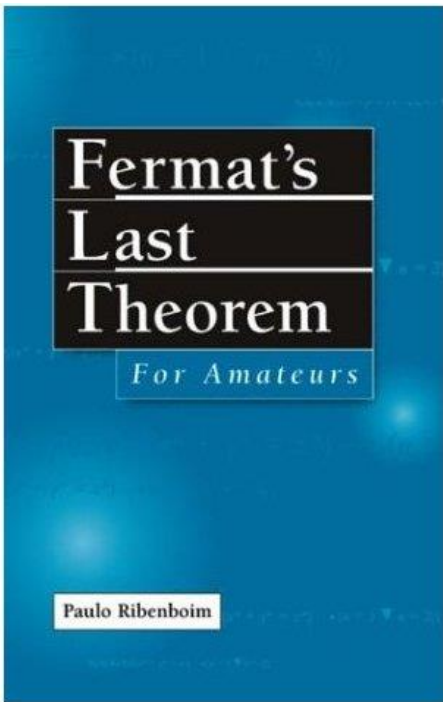
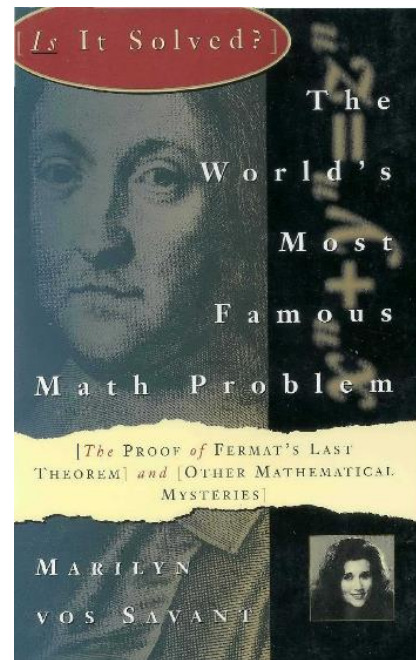
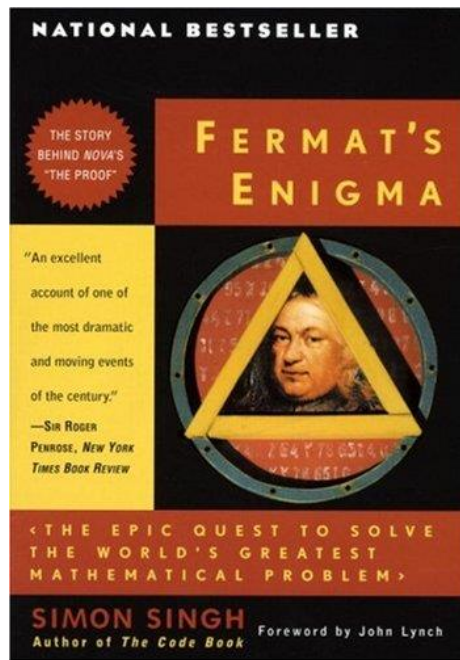


Historical Perspectives

Pierre de Fermat (1601-1665)

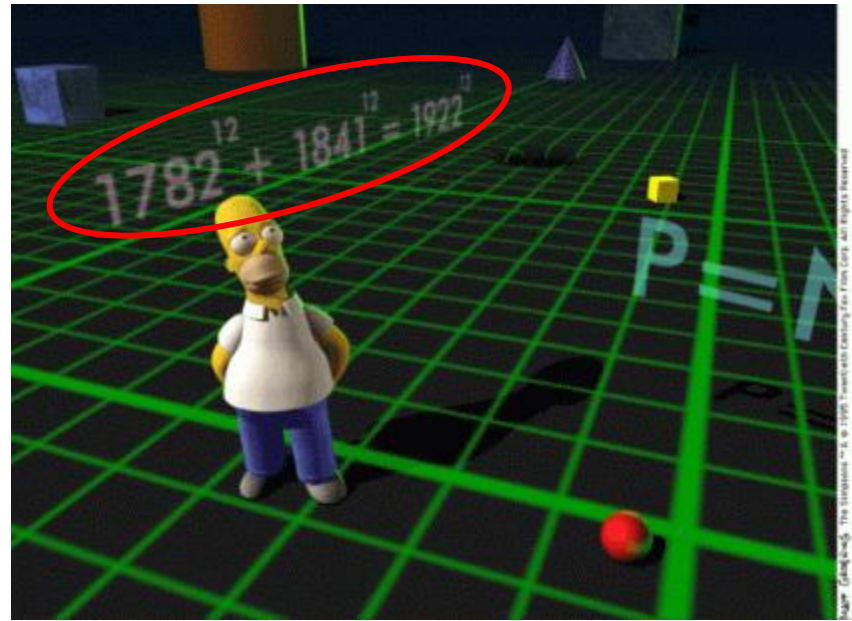
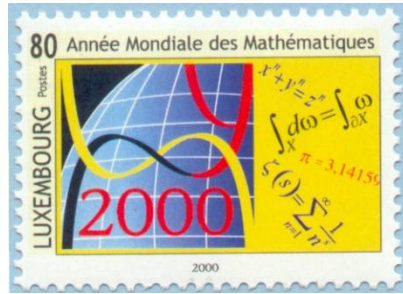
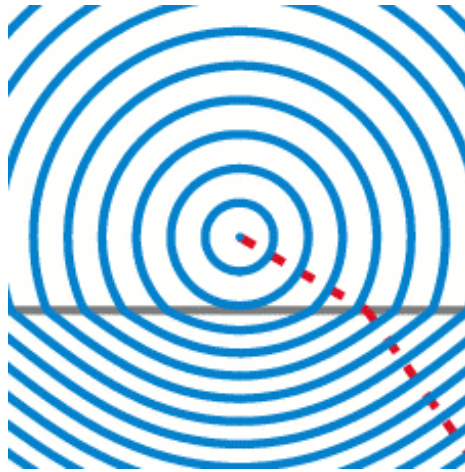
- Father of modern **number theory**
- Lawyer, Parlement of Toulouse
- Laid groundwork for calculus
- Contributions to optics, probability, and **analytic geometry**
- Fermat numbers, primes, perfect #'s
- Descartes' **Law of refraction**
- Reponsible for many open problems
- “**Fermat's Last Theorem**” (1637-1995)
- Recognized “principle of least action” and “principle of least time” in physics
- Influenced Newton and Leibniz



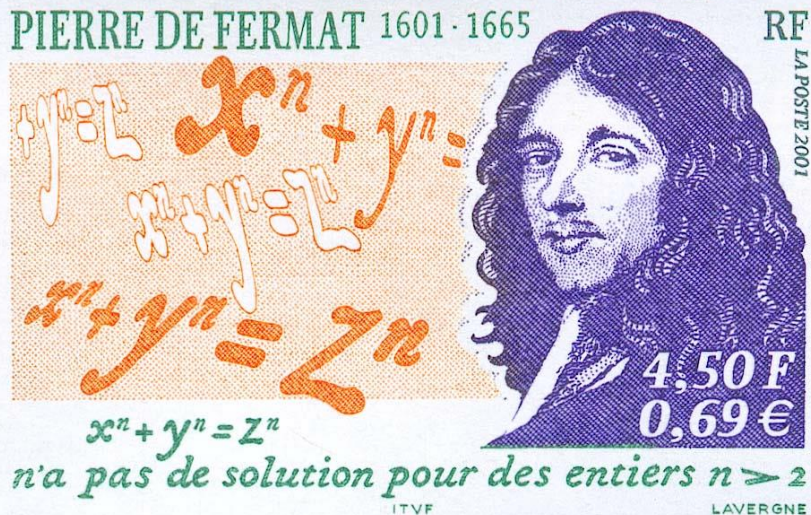


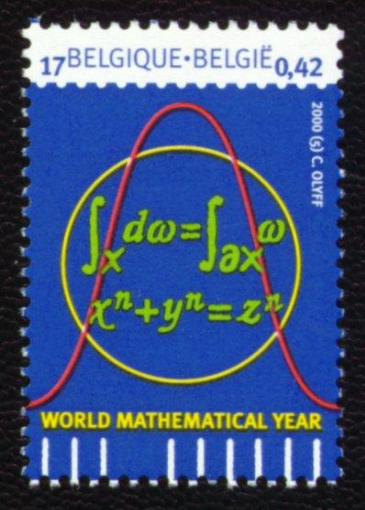


Pierre de Fermat
1601 - 1665



Fermat Prize for Mathematics Research





$X^n + Y^n = Z^n$
 has no non-zero integer solutions for x , y , and z when $n > 2$

SOMETIMES, LIFE IS JUST ONE GREAT NUMBER AFTER ANOTHER.



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A NEW MUSICAL

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JAMES MORGAN, ARTISTIC DIRECTOR CLAYTON PHILLIPS, MANAGING DIRECTOR PRESENTS **FERMAT'S LAST TANGO** A NEW MUSICAL
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 CHRIS THOMPSON • CHRISTIANNE TISDALE • CARRIE WILSHUSEN

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Music by Joshua Rosenblum
Book by Joanne Sydney Lessner
Lyrics by Lessner and Rosenblum



FERMAT'S *Last* TANGO

A NEW MUSICAL

THE YORK THEATRE COMPANY

*A Musical Fantasy inspired by Andrew Wiles
and his encounters with Fermat's Last Theorem*

"Rolling! Whimsical! Catchy & Clever!" - The New York Times

Followed by an Interview with Andrew Wiles

FERMAT'S LAST TANGO



A CMI production



In 1993 Andrew Wiles stunned the world when he announced a solution to "Fermat's Last Theorem," the famous unsolved mathematics problem set forth by Pierre de Fermat in 1637. In the musical *Fermat's Last Tango*, the fictional character Daniel Keane earns overnight acclaim when he presents his findings. However, fanfare soon gives way to doubt when the reincarnated Fermat discovers a hole in Keane's proof. The singular pursuit by Keane to correct this flaw results in a love triangle involving himself, his wife, and mathematics—the story of which is brought to life by Fermat and his immortal friends from the "AfterMath," namely: Pythagoras, Euclid, Newton, and Gauss. The musical is both a cheerful romp through history and a personal confrontation with destiny. It provides a testament to the extraordinary excitement of mathematics and its unparalleled beauty.

The Composer Joshua Rosenblum enjoyed mathematics while studying music at Yale along with the author, his wife Joanne Sydney Lessner. They both take an active role in the New York music community. This recording was captured by David Stern and his Emmy Award-winning crew during a performance at the York Theatre Company in New York City.



STARRING

Carl Friedrich Gauss / Reporter
Anna Keane
Pythagoras / Reporter
Pierre de Fermat
Daniel Keane
Euclid / Reporter
Sir Isaac Newton / Reporter

GILLES CHIASSON
EDWARDYNE COWAN
MITCHELL KANTOR
JONATHAN RABB
CHRIS THOMPSON
CHRISTIANNE TISDALE
CARRIE WILSHUSEN



Approximate Running Time:
100 minutes
Color/Not Rated/VHS/NTSC
Produced by The Clay Mathematics
Institute, Cambridge, MA
Arthur Jaffe, *Producer*
David Stern, *Director*
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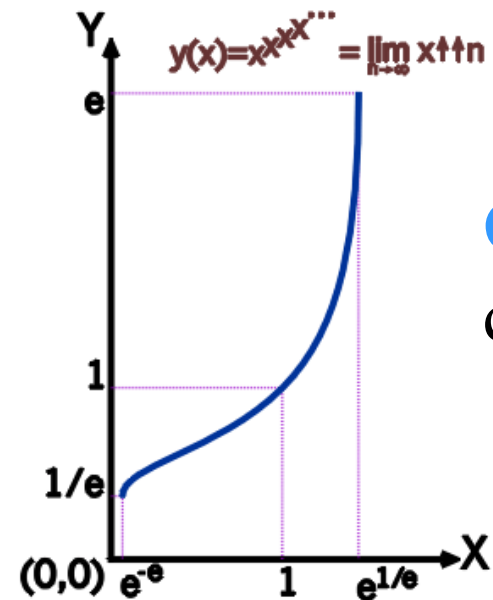
Problem: Solve the following equation for X:

$$X^{X^{X^{X^{\dots}}}} = 2 \Rightarrow X^2 = 2 \Rightarrow X = \sqrt{2}$$

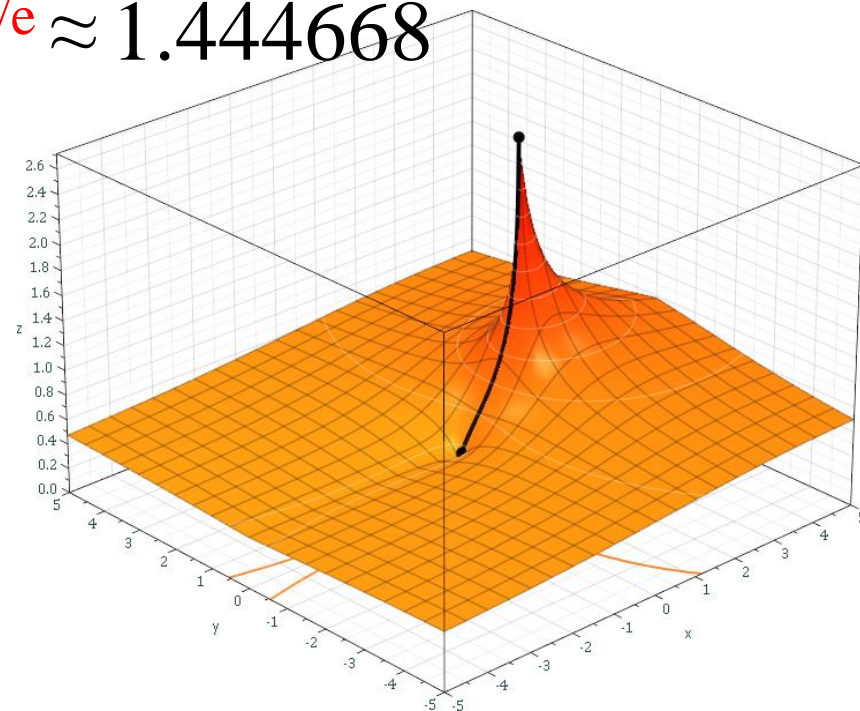
where the stack of exponentiated x's extends forever.

This “power tower” **converges** for:

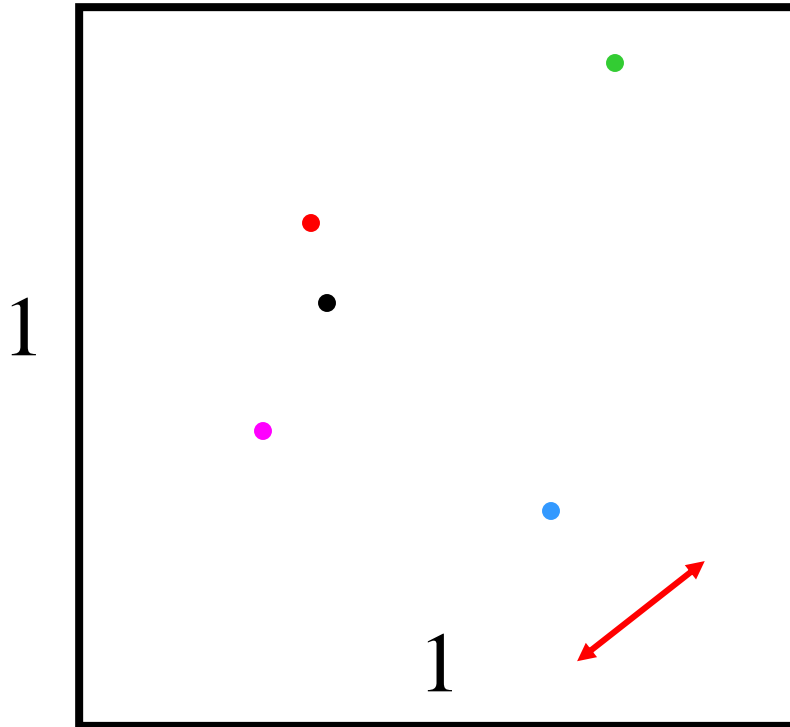
$$0.065988 \approx e^{-e} < X < e^{1/e} \approx 1.444668$$



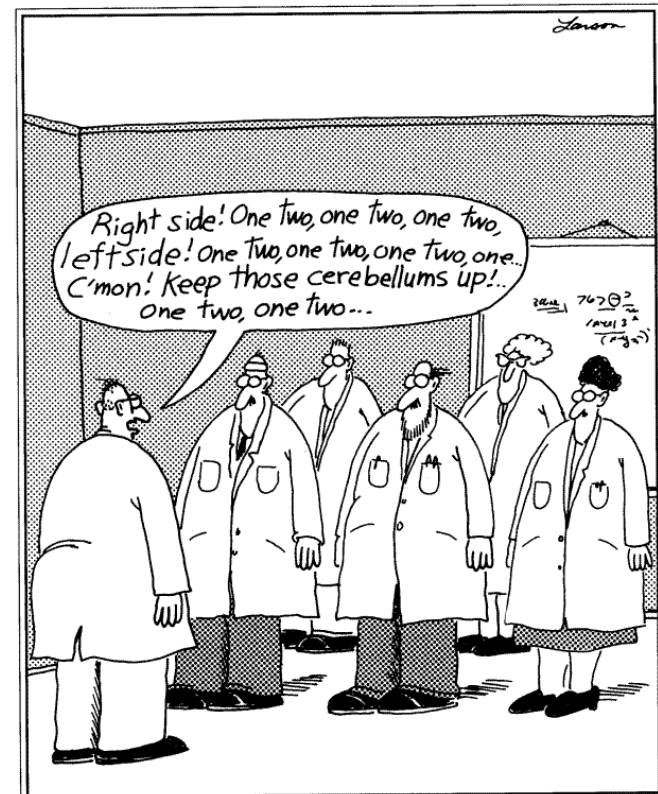
Generalization to
complex numbers:



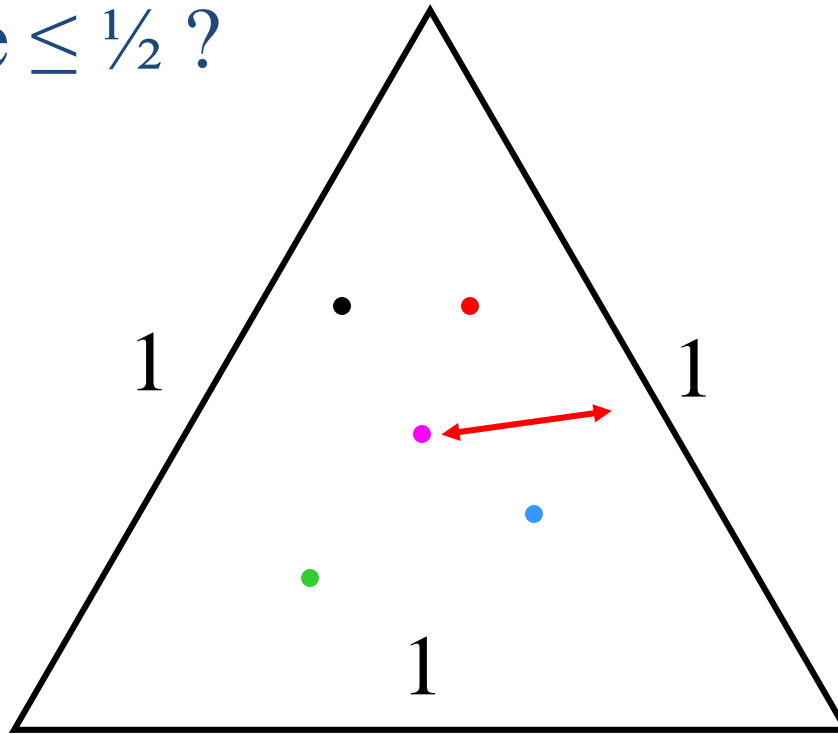
Problem: Given any five points in/on the unit square, is there always a pair with distance $\leq \frac{1}{\sqrt{2}}$?



- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Problem: Given any five points in/on the unit equilateral triangle, is there always a pair with distance $\leq \frac{1}{2}$?

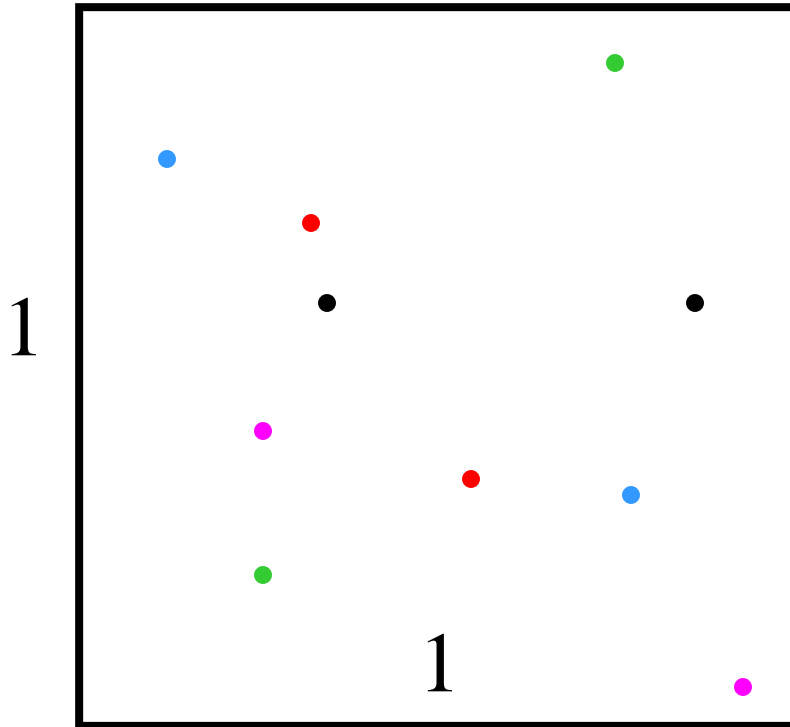


- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

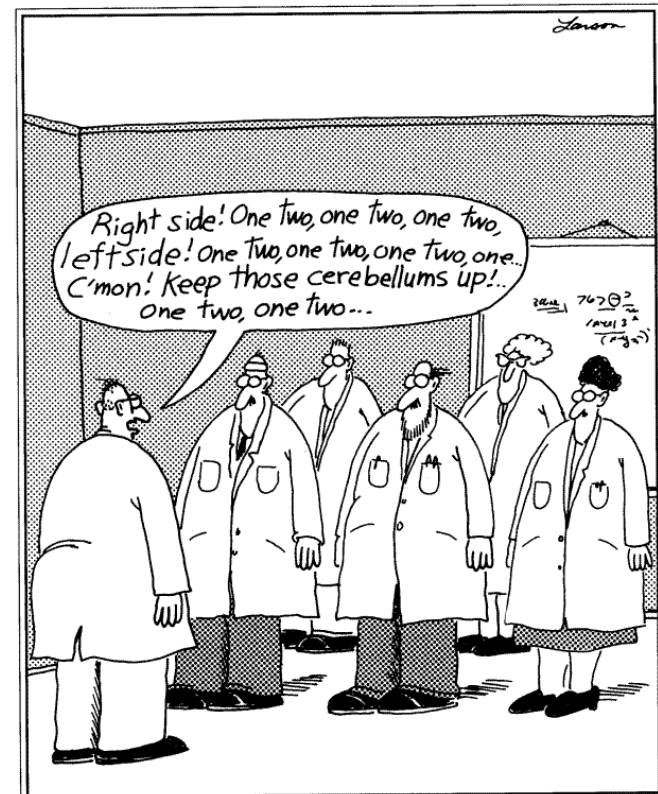


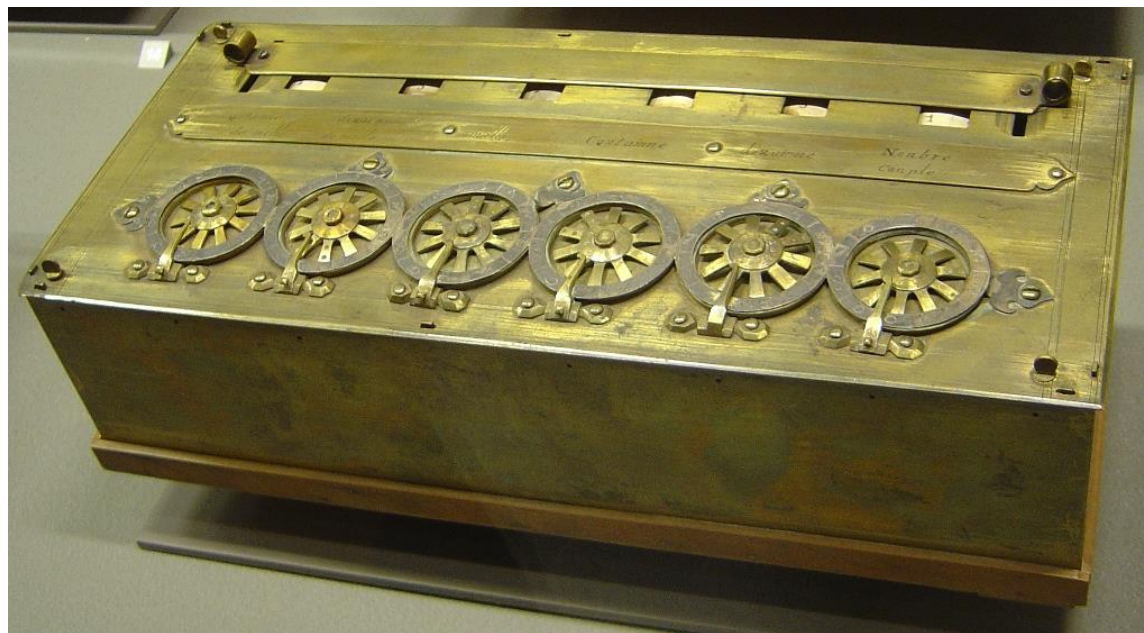
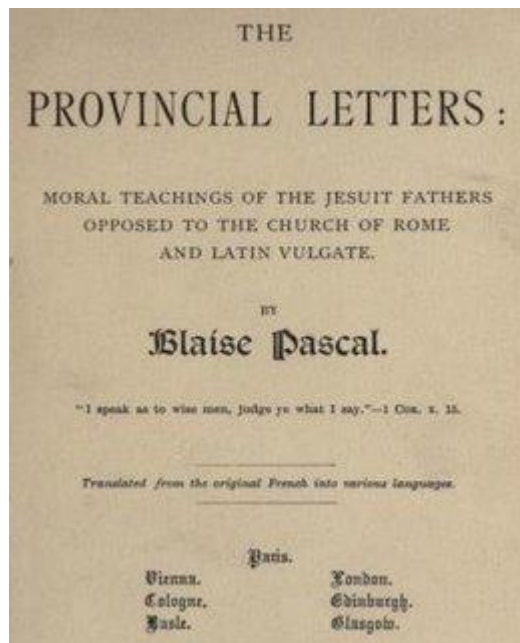
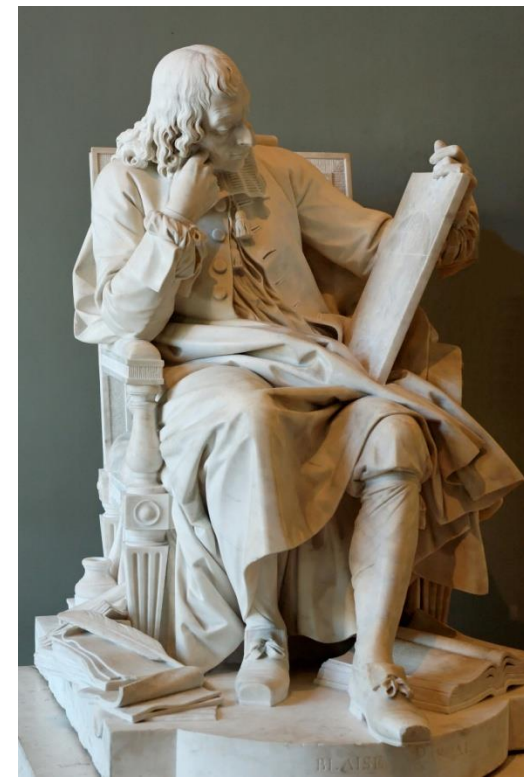
Math phobic's nightmare

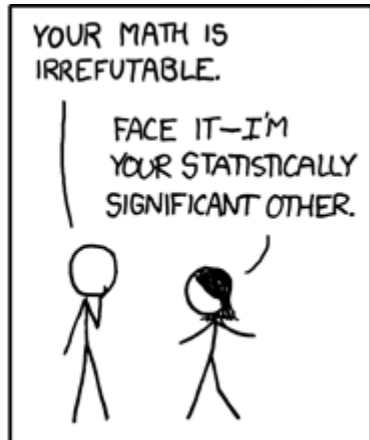
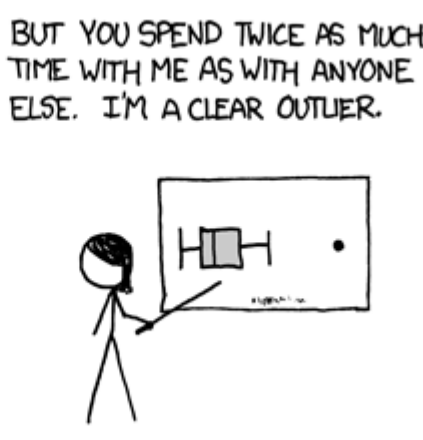
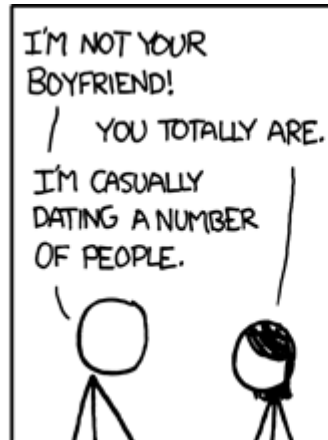
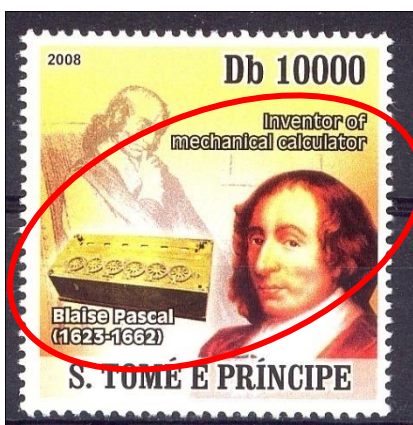
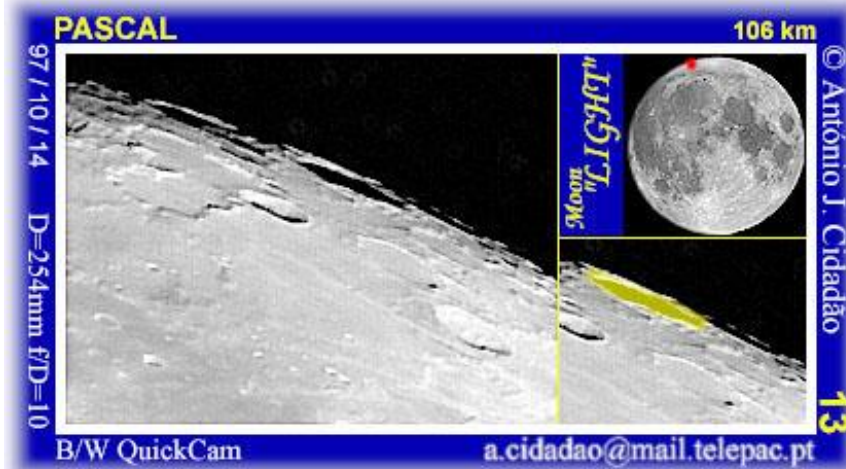
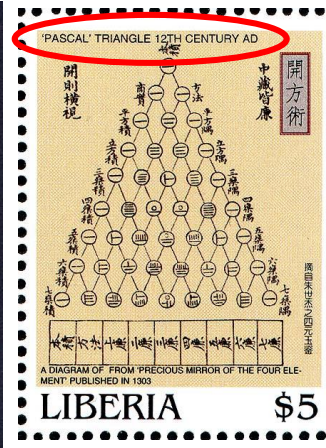
Problem: Given any ten points in/on the unit square, what is the maximum pairwise distance?



- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



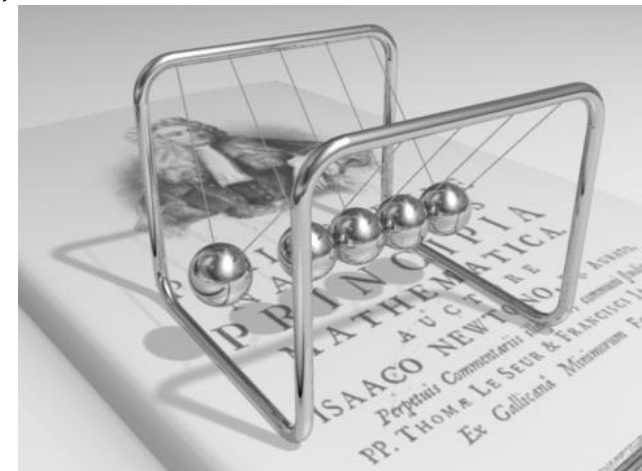
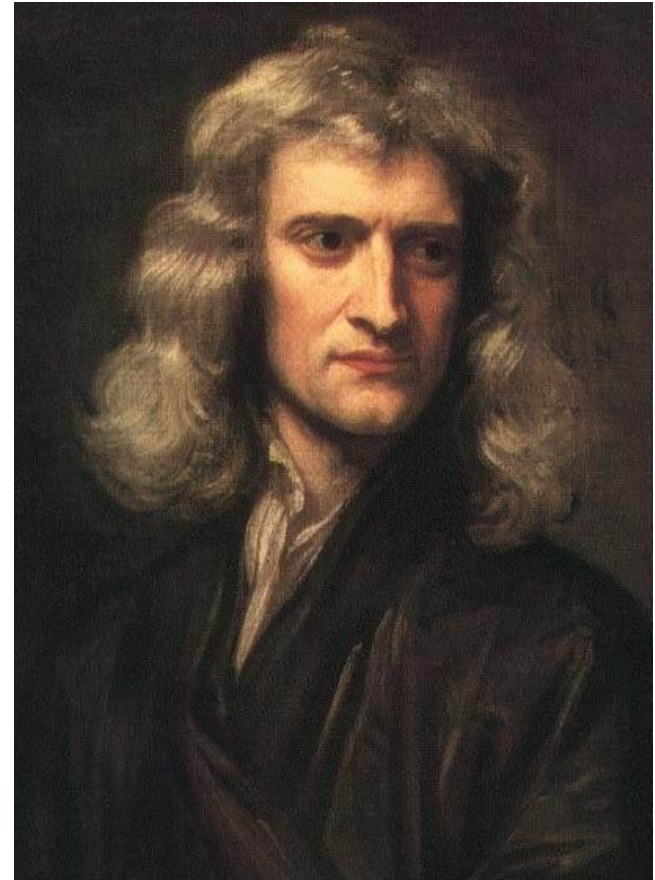




Historical Perspectives

Sir Isaac Newton (1643-1727)

- Mathematician, physicist, astronomer, philosopher, alchemist, theologian
- One of history's most influential people
- “**Principia Mathematica**” (1687)
- Invented **calculus**, theory of **gravitation**
- Founded “**Newtonian mechanics**”
- Discovered **laws of motion**, **inertia**
- “**Newtonian fluid**”, “**Newtonian Universe**”
- Advanced the Scientific Revolution
- Developed practical **reflecting telescope**, **theory of color**, “**Newton's method**”
- SI unit of force: **newton**



NO 16-566

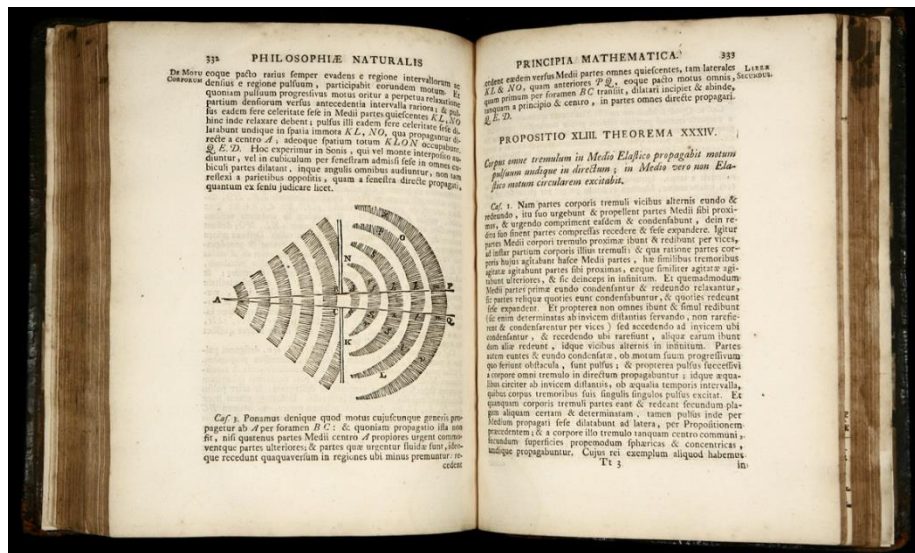
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore ^{Autore} J. S. NEWTON ^{Equite honorato,} Trin. Coll. Cantab. Soc. Matheseos
^{Professore} ^{Lucasiano,} & Societatis Regiæ Sodali.
^{et Societatis Regiæ Societatis præsido.}

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,

Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud
plures Bibliopolas. Anno MDCLXXXVII.



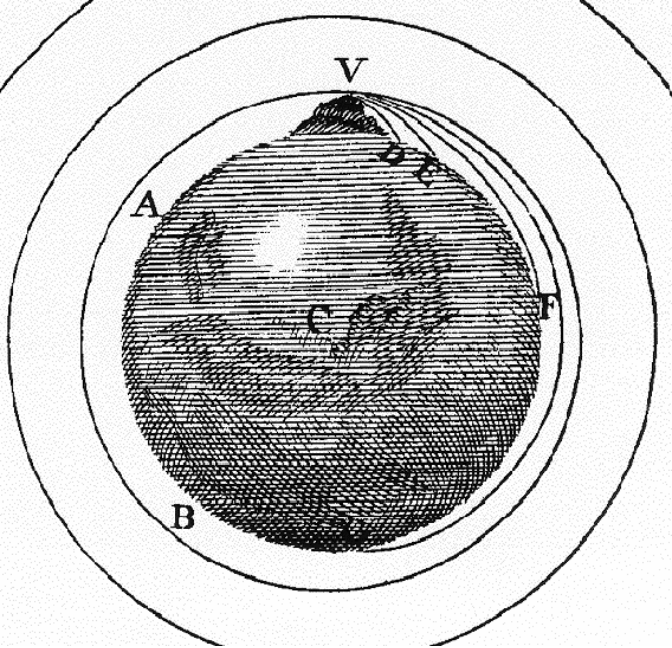
333 PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA 333

De Motu eoque pulso raris semper evadens & regione intervallorum se
 densius & regione pulsum, participabit eorundem motum. Et
 quoniam pulsum progressivus motus erit a perpetua relaxatio-
 nis partium distans versus antecedentia intervalla raris & pul-
 sis eadem fere celeritate fere in Medii partes quiescentes & pul-
 sine inde relaxare debent; pulsus illi eadem fere celeritate fere di-
 recte a centro A, adeoque spatum totum K L O N occupabunt de-
 sig. & D. Hoc experitur in Sonis: qui vel motu Interpositio au-
 duntur, vel in cubicalium per fenestram admitti fere in omnes cu-
 biculi partes dilatant. Inque anglis omnibus audiantur, non tam
 reflexi a parietibus oppositis, quam a fenestris directe propaga-
 quant ex sensu iudicare licet.

PRINCIPIA MATHEMATICA 333
 cedent eadem versus Medii partes omnes quiescentes, tam laterales
 K L & N O quam anteriores P Q, eoque pulso motus omnis, Sicquod
 que trahunt per foramen B C trahit, dilatari incipiet & abinde
 usque a principio & centro, in partes omnes directe propagari.
 Et c.

PROPOSITIO XLIII. THEOREMA XXXIV.
 Corpus omne tremulum in Medio Elastico propagabit motum
 pulsum antequam in directionem, in Medio vero non Ela-
 stico motum circulares excutiat.

Cas. 1. Nam partes corporis tremuli vicibus alternis eundo se
 relaxando, in suo urgentur & propelluntur partes Medii sibi proxi-
 mas, & urgendo comprimunt eadem & condensantur, dein re-
 laxata sua sunt partes complicas recedere & fere expandere. Igitur
 partes Medii corpori tremulo proximæ ibunt & redibunt per vices,
 ad illam partem corporis illius tremuli & qua ratione partes cor-
 poris huius agitantur huius Medii partes, his vicibus tremulis
 agitantur partes sibi proximæ, eoque similiter agitantur
 vicibus ulteriore, & sic deinceps in infinitum. Et quoniam huiusmodi
 Medii partes prime eundo condensantur & redeundo relaxantur,
 & partes reliquæ quoties eunt condensantur, & quoties relaxantur
 fere excedunt. Et propterea non omnes ibunt & finalis redibunt
 (Et enim determinatis ab invicem dilatantur servando, non rarefactis
 & condensantur per vices) sed accedendo ad vicem ubi
 condensantur, & recedendo ubi rarefactis, aliquæ eorum ibunt
 tum aliter redeunt, & vicibus alternis in infinitum. Partes
 autem eundo & redeundo condensantur, ob motum suum progressivum
 quo huiusmodi oblativa, sunt pulsus: & propterea pulsus successivi
 in tempore omni tremulo in directionem propagantur: & aliquæ aqua-
 libus vicibus ab invicem dilatantur, ob aequalia temporis intervalla,
 quibus corpus tremulosum suis vicibus singulis pulsus excitat. Et
 quoniam corpus tremuli partes eas & rediant fecundum pla-
 num aliquam certam & determinatam, tamen pulsus inde per
 medium propagari fere dilatantur ad latera, per Propositionem
 precedentem, & a corpore illo tremulo tanquam centro communi
 vicibus sphericis propagantur sphericæ & concentricæ,
 & vicibus propagantur, quæ uti exemplum aliquod habemus
 in



Arithmetica Universalis;
 SIVE
 DE COMPOSITIONE
 ET
 RESOLUTIONE
 ARITHMETICA
 LIBER.

Cui accessit
 HALLEIANA
*Aequationum Radices Arithmetice
 inveniendi methodus.*

In Usum Juventutis Academicæ.

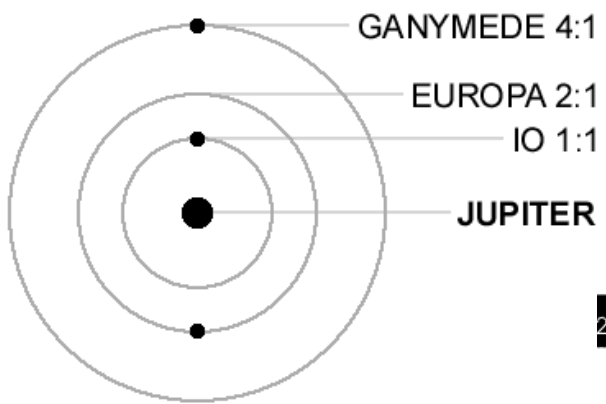
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 INFLEXIONS and COLOURS
 OF
LIGHT.

ALSO
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 OF THE
 SPECIES and MAGNITUDE
 OF
Curvilinear Figures.

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NEWTON

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12

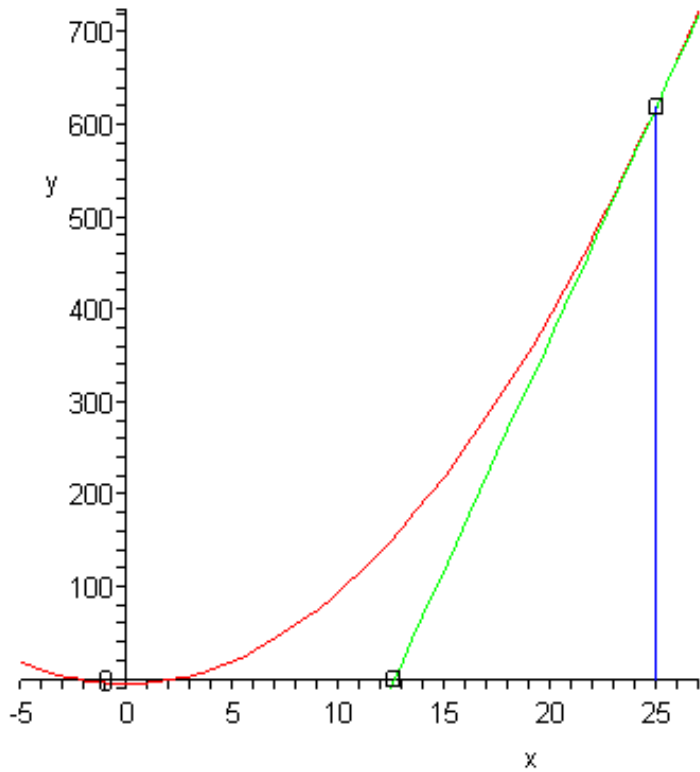
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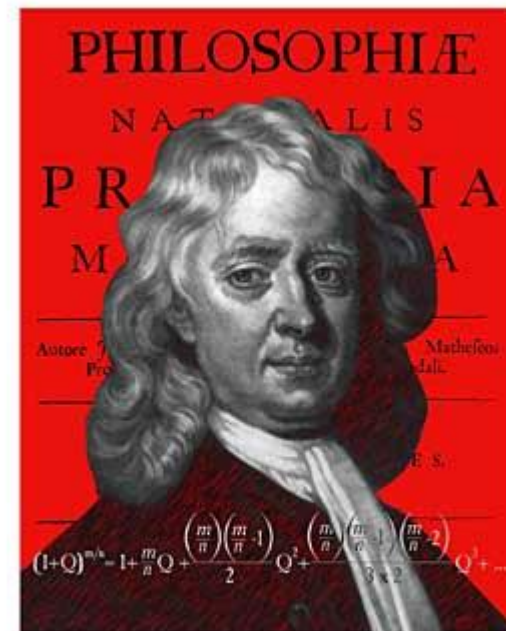
B/W QuickCam

a.cidadao@mail.telepac.pt

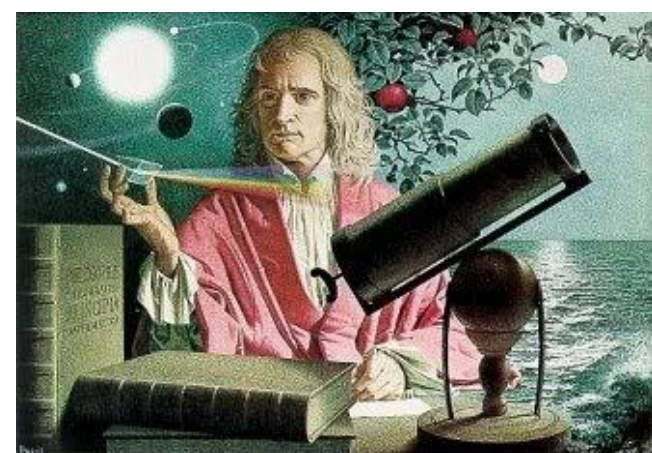
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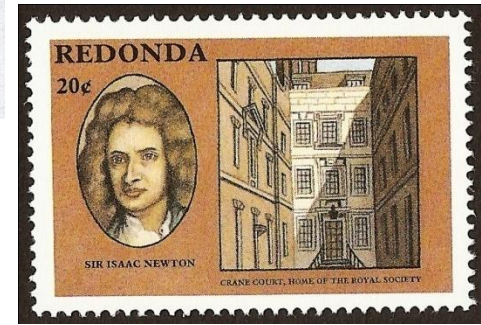
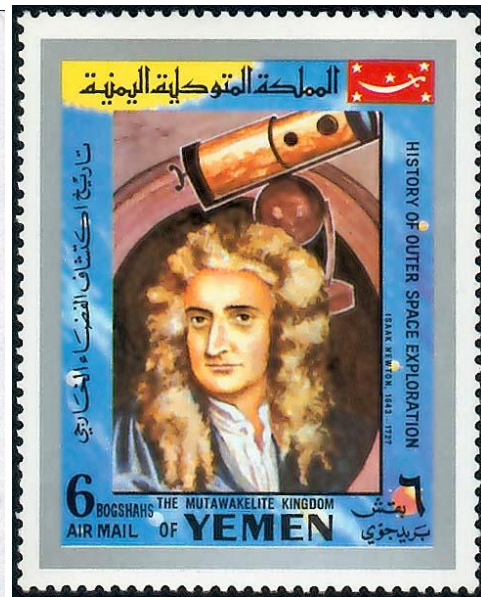
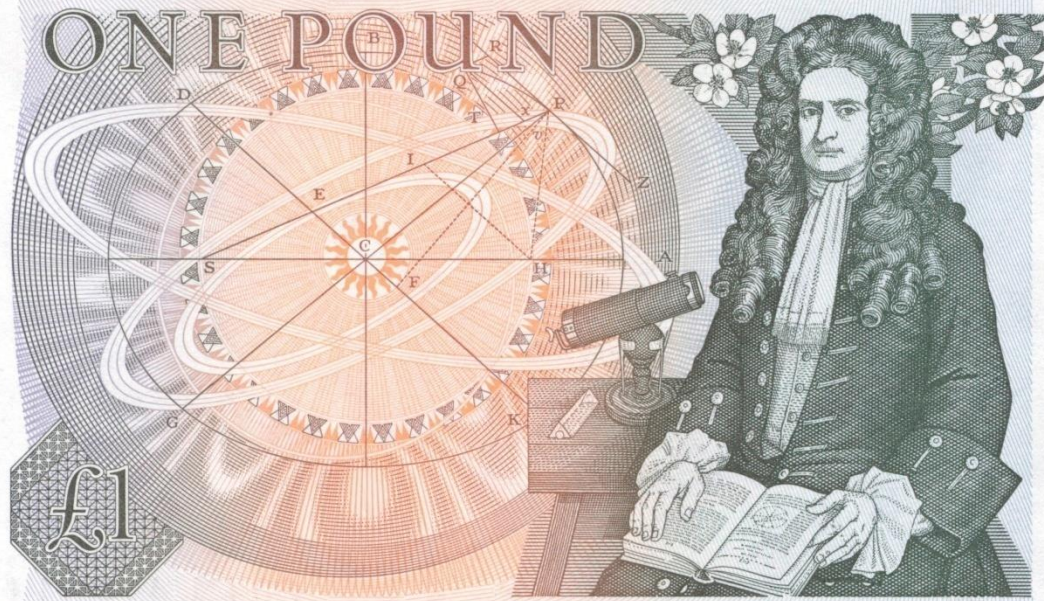


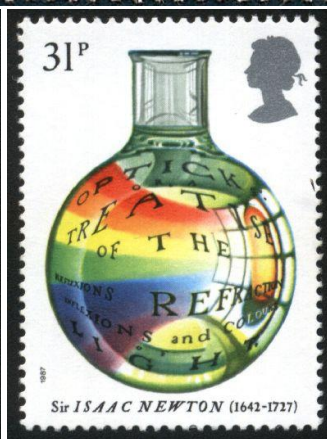
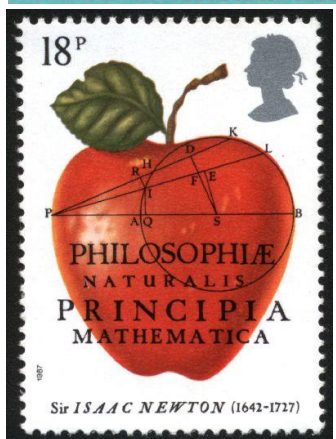
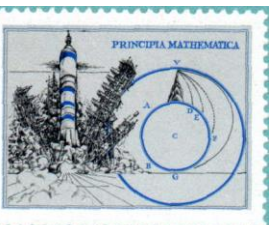
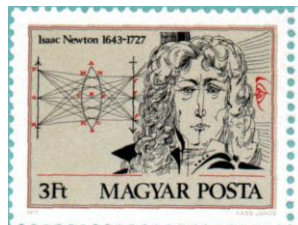
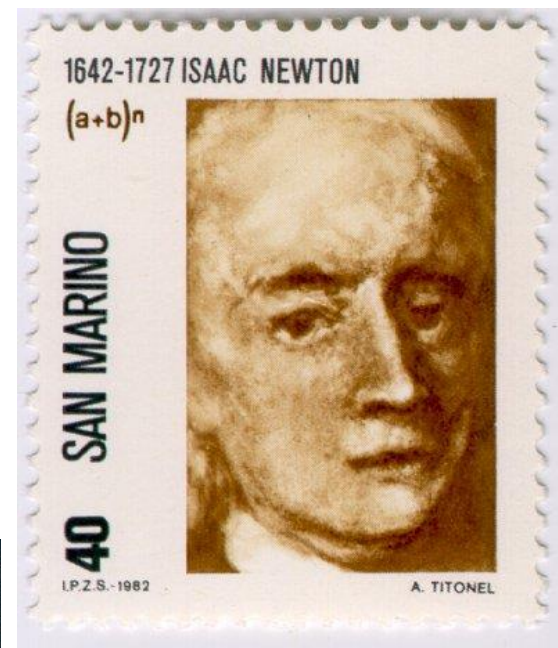
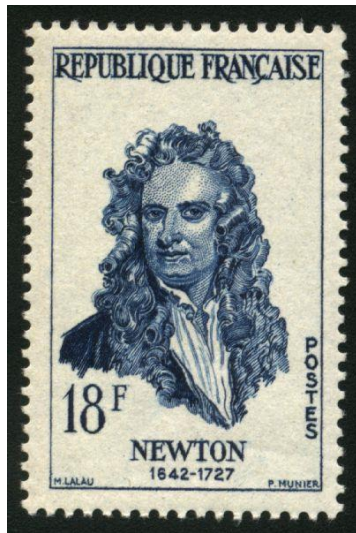


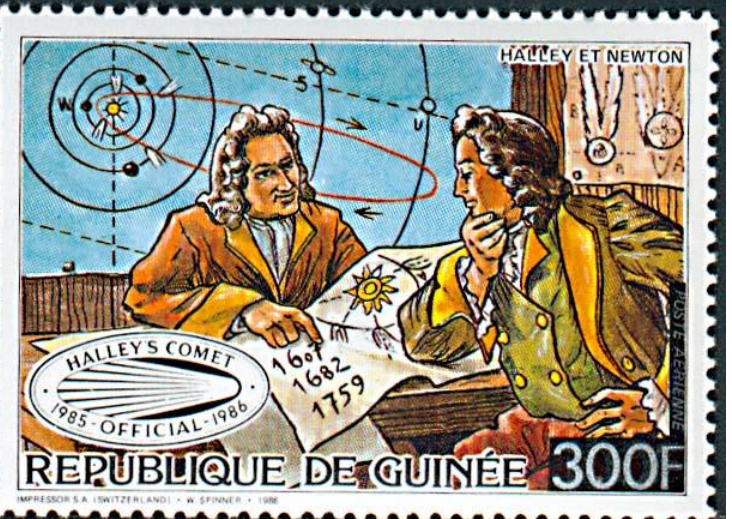
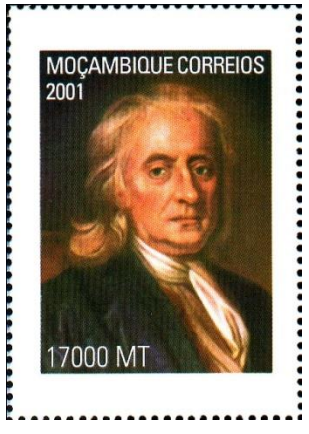
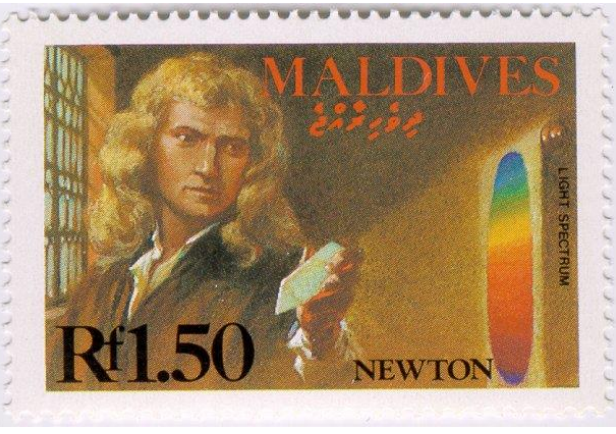
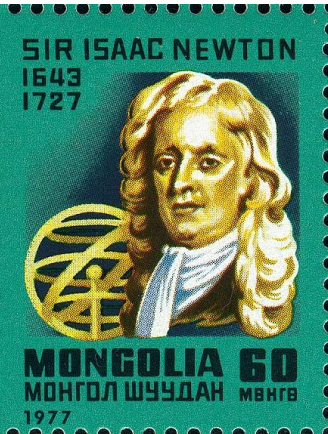
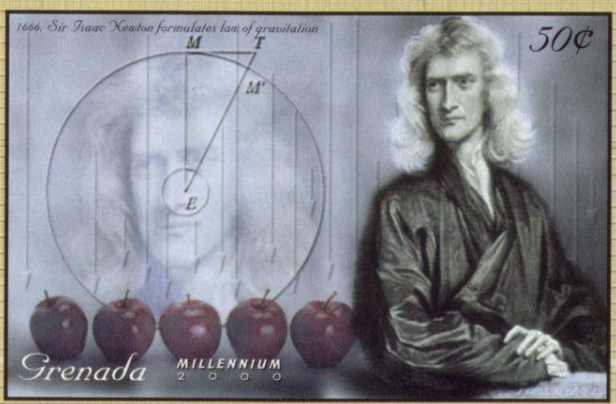


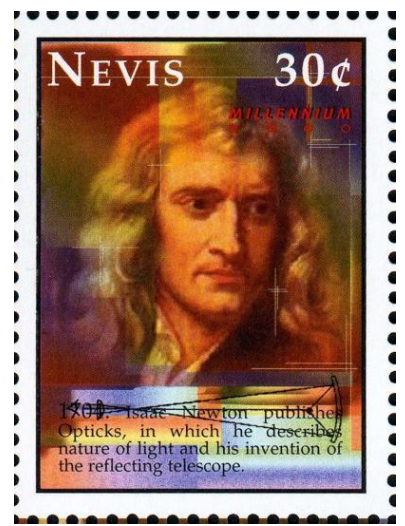
[Sir Isaac Newton 1642 (1643 New Style Calendar) - 1727]





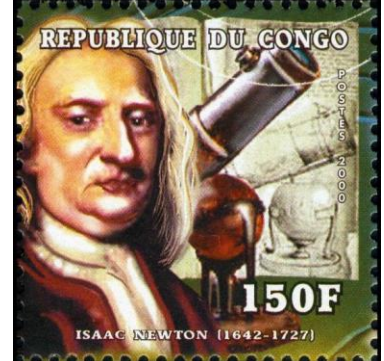
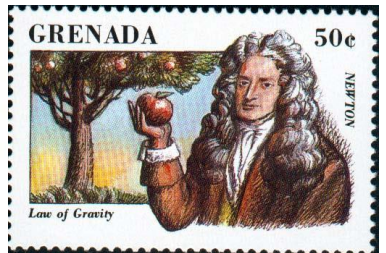
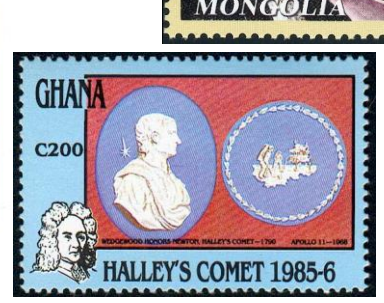


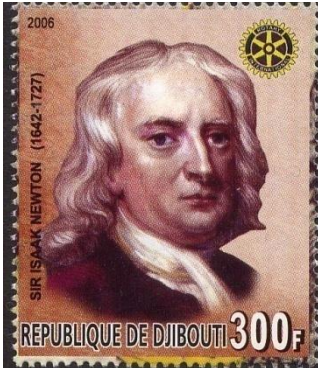
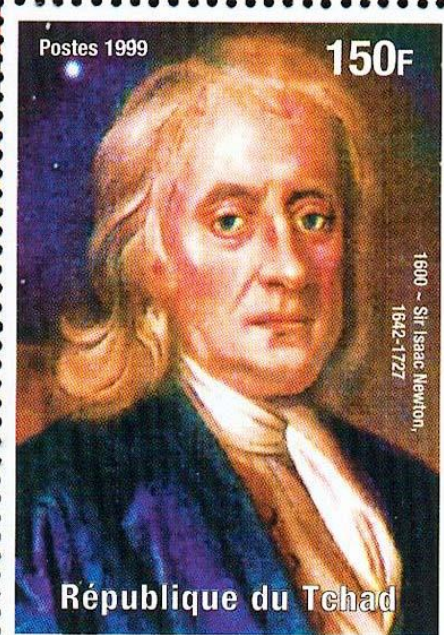
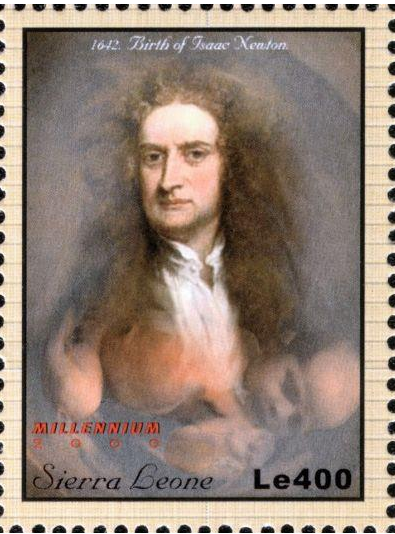
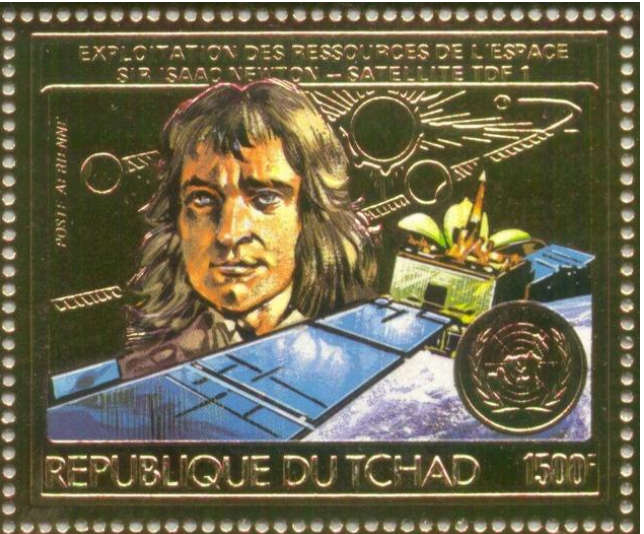
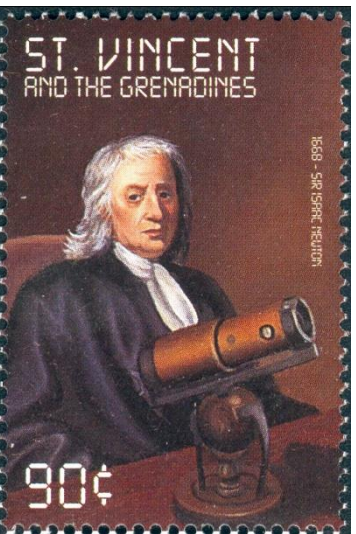
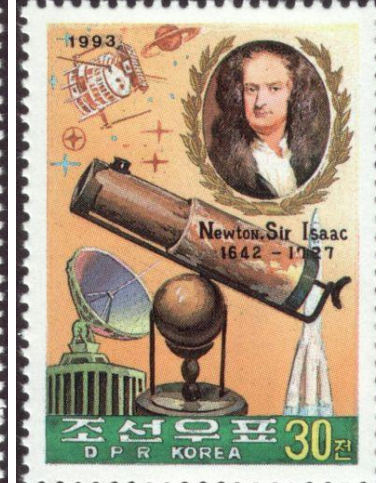
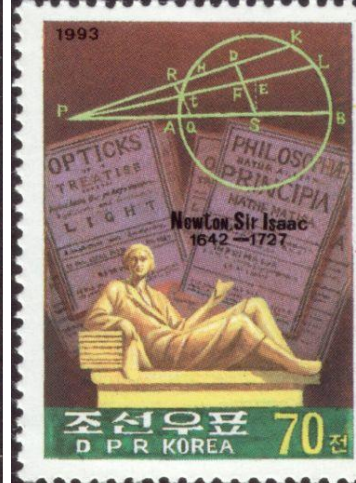
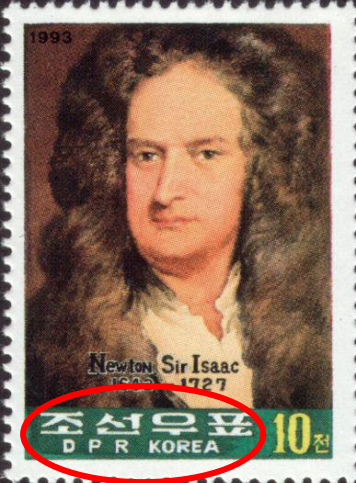


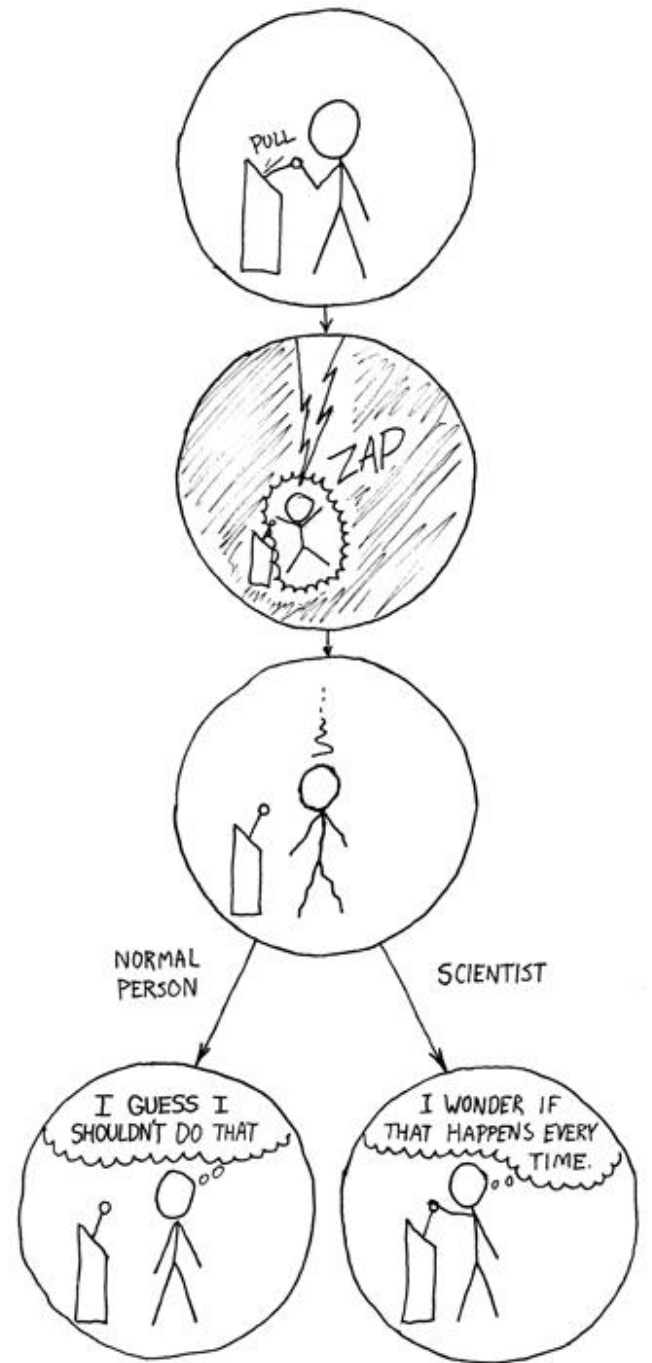


300-летие опубликования И. Ньютоном "Математических начал натуральной философии"

Исаак Ньютон



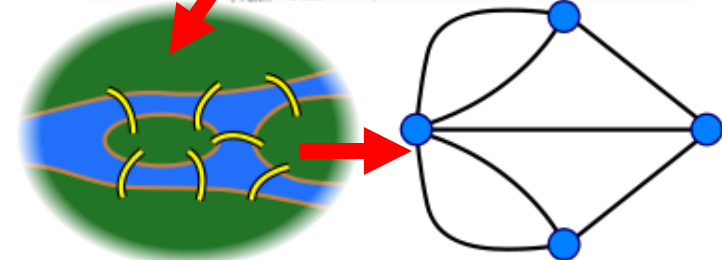
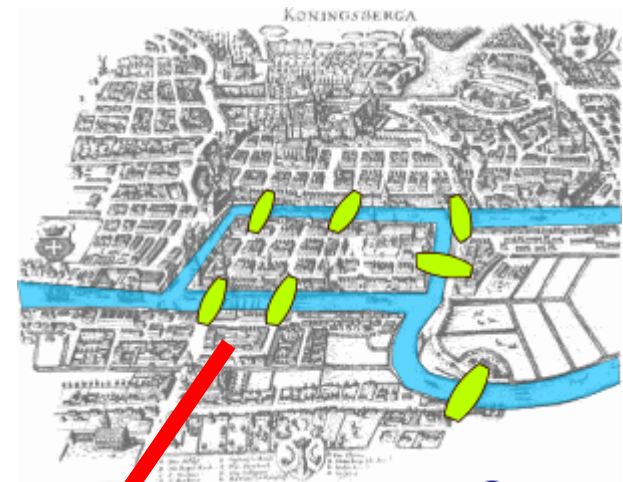
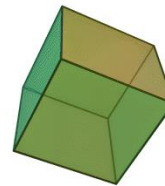




Historical Perspectives

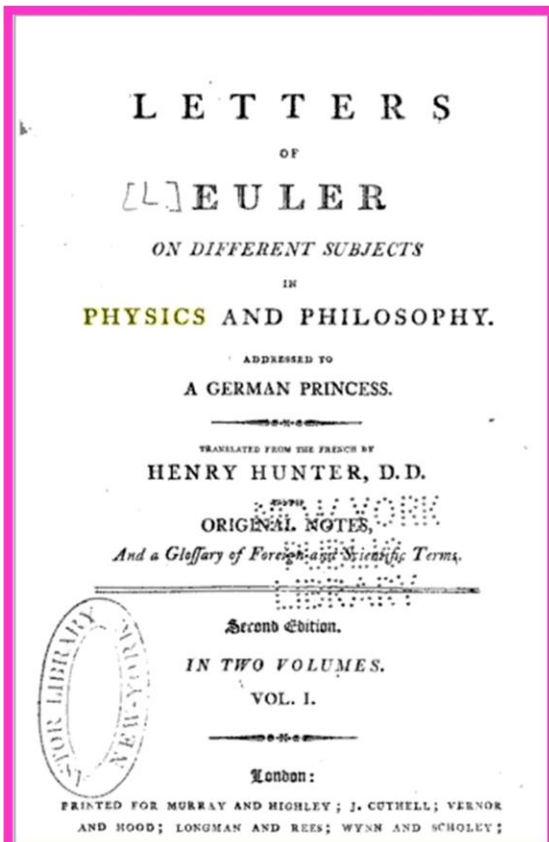
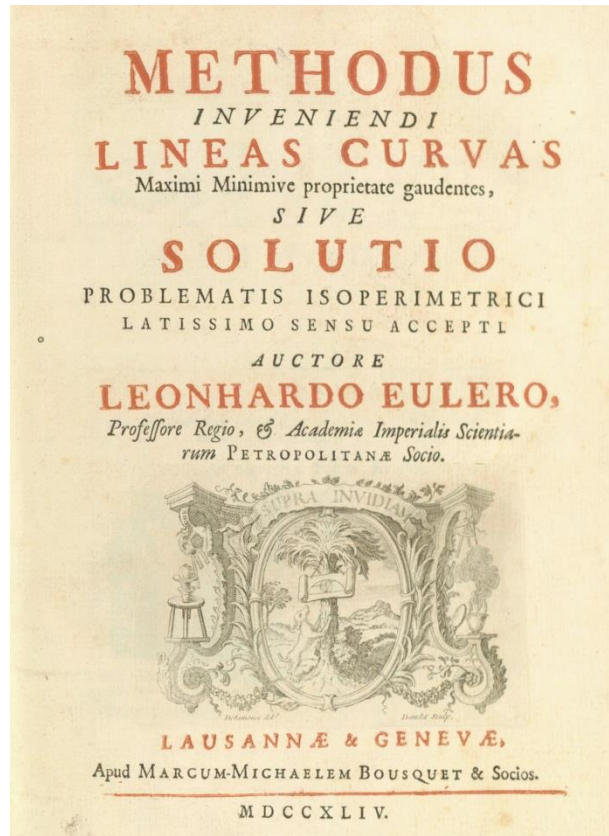
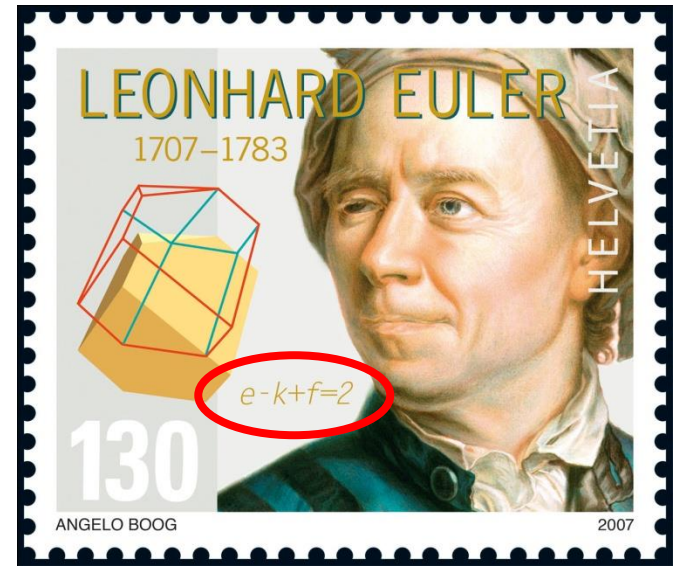
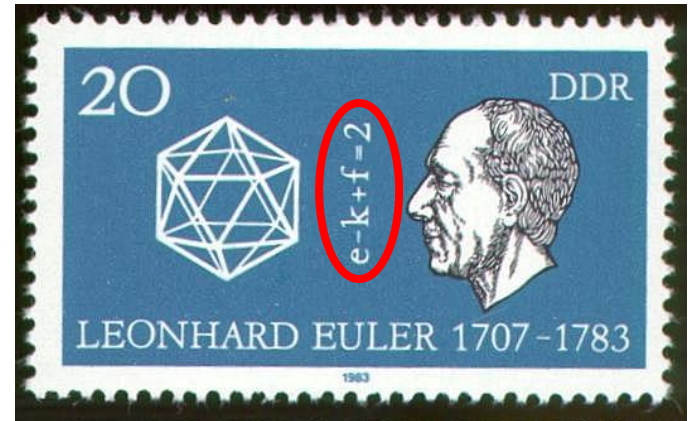
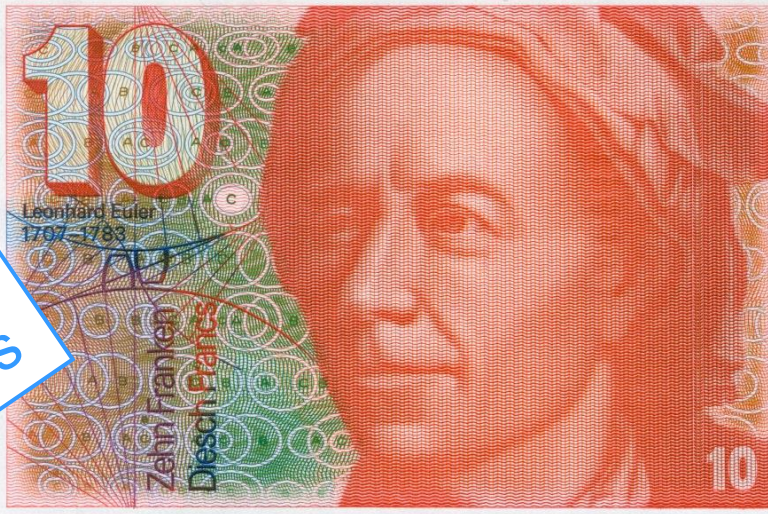
Leonhard Euler (1707–1783)

- Invented graph theory
- “**Bridges of Königsberg**”, Prussia
- Eulerian tour
- Euler’s formula: $V + F = E + 2$
- Euler’s number: e
- Euler’s identity: $e^{i\pi} + 1 = 0$
- Major contributions to analysis, algebra, calculus, number theory, topology, optics, fluid dynamics, mechanics, astronomy, education



SCHWEIZERISCHE NATIONALBANK
BANCA NAZIUNALA SVIZRA

Swiss
Francs



$$e^{i\pi} + 1 = 0$$

$$e^{iu} = \cos(u) + i \sin(u)$$

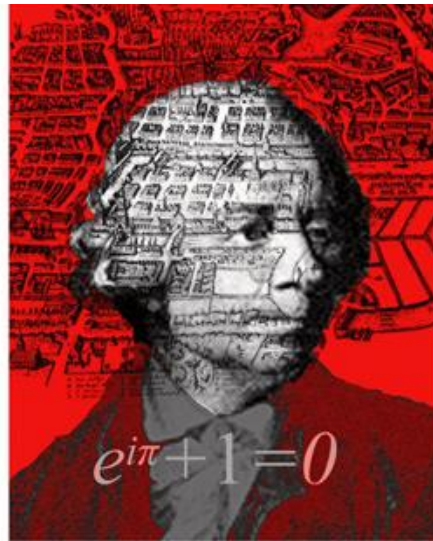
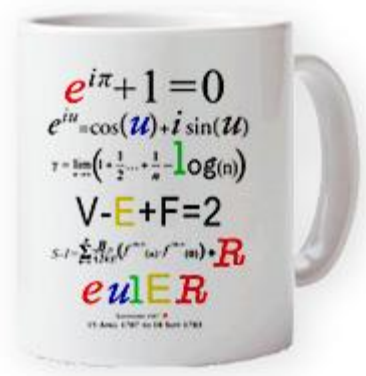
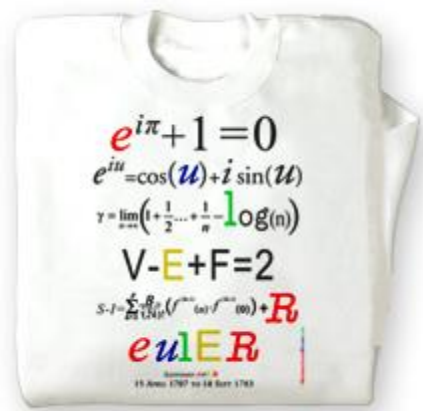
$$\gamma = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} \dots + \frac{1}{n} - \log(n) \right)$$

$$V - E + F = 2$$

$$S - I = \sum_{k=1}^p \gamma \frac{B_{2k}}{(2k)!} (f^{(2k-1)}(n) - f^{(2k-1)}(0)) + R$$

euler
 LEONHARD **euler**
 15 APRIL 1707 TO 18 SEPT 1783

MEMBERSHIP@EULER.COM



Leonhard Euler
1707 - 1783

EULER 28 km / 2240 m

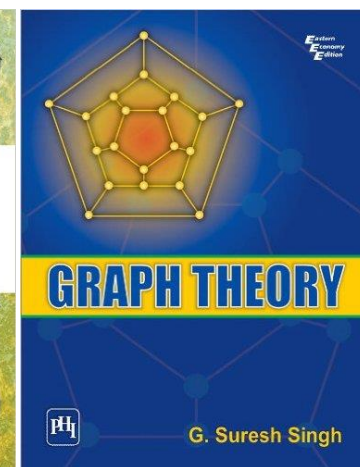
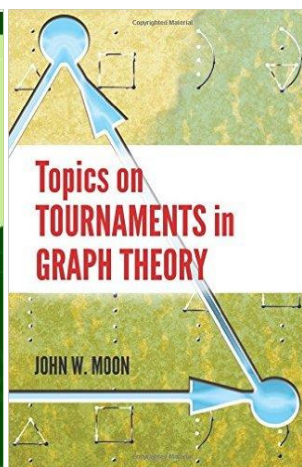
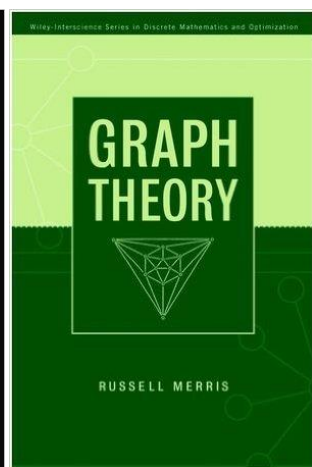
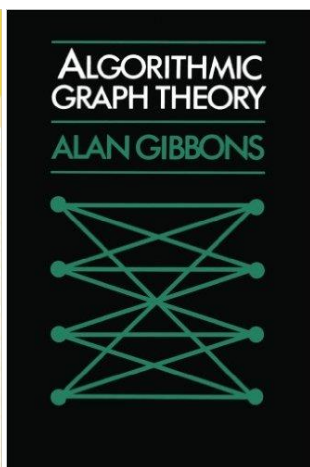
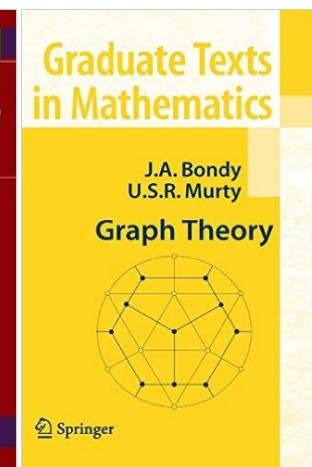
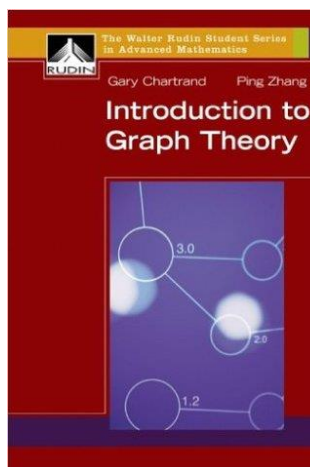
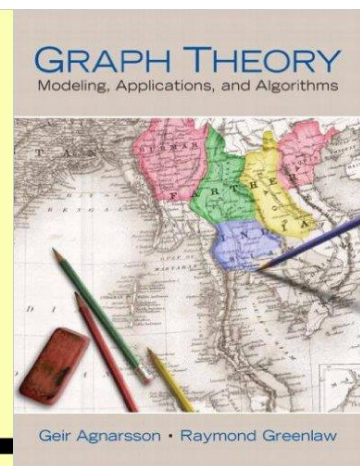
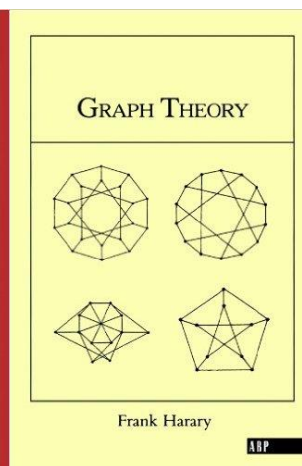
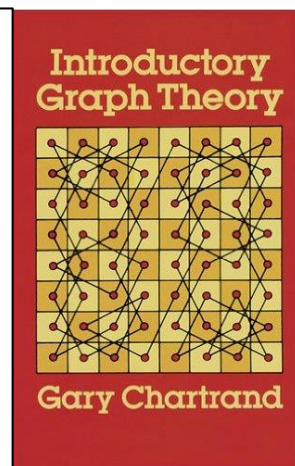
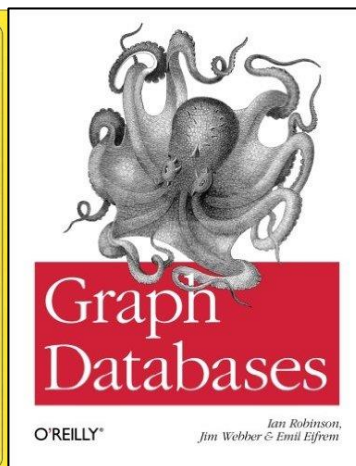
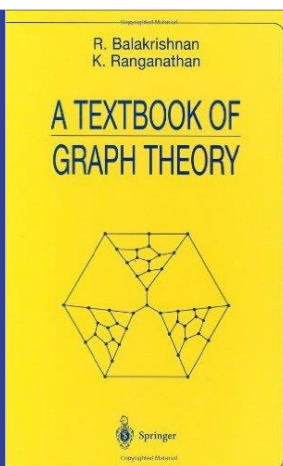
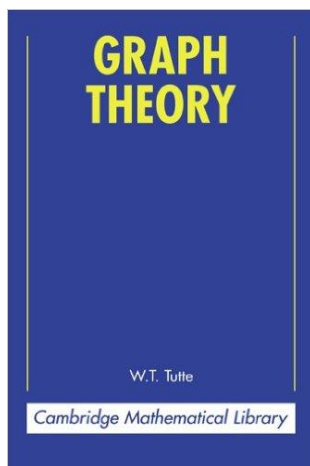
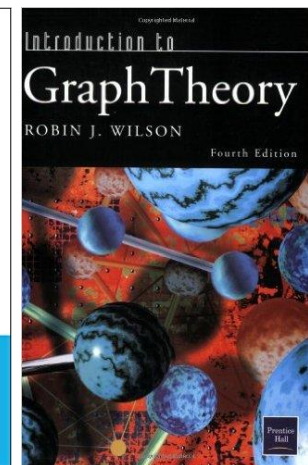
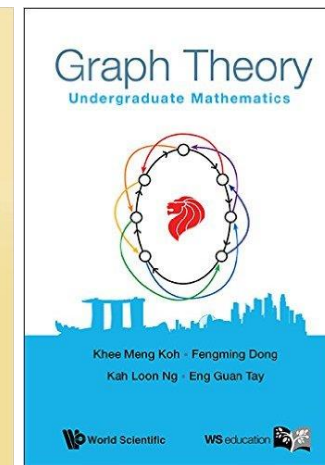
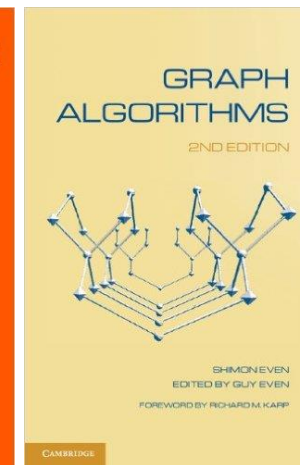
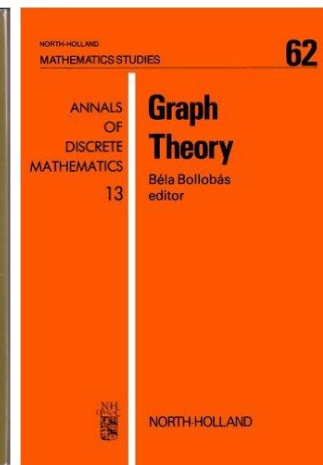
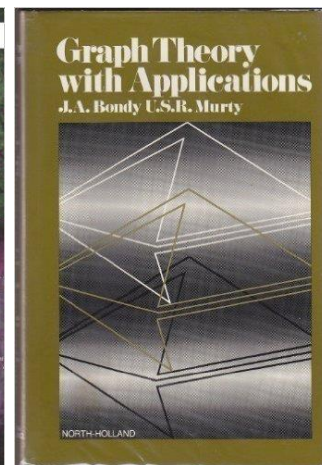
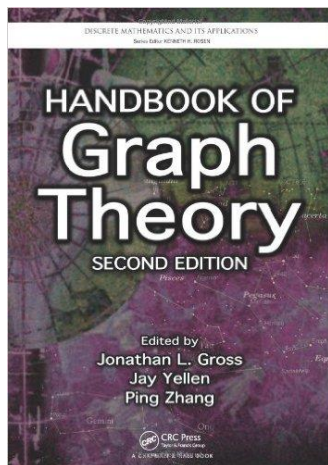
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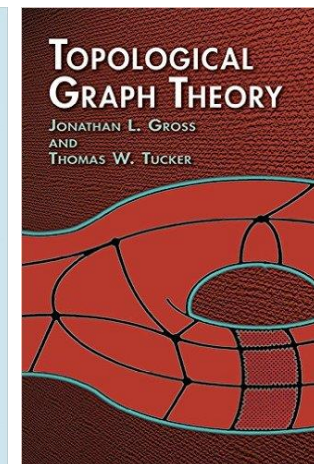
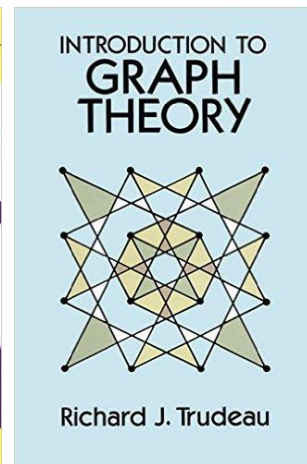
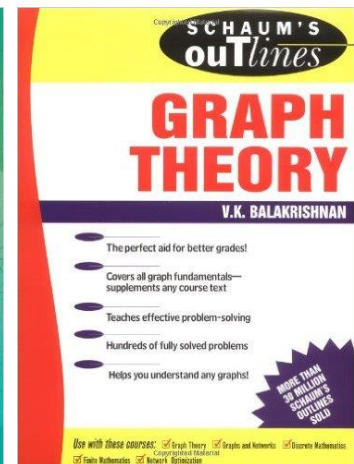
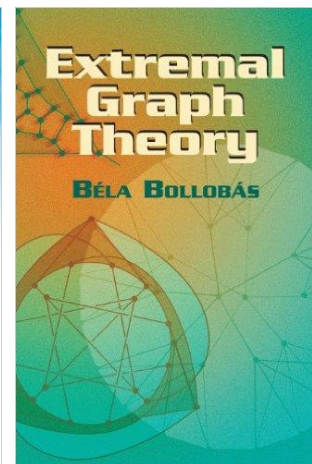
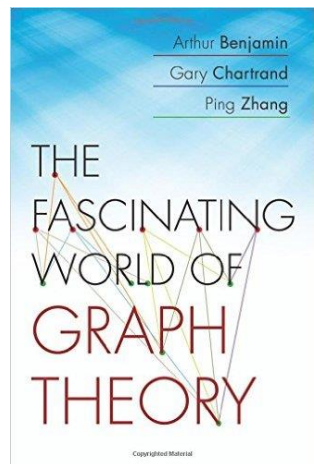
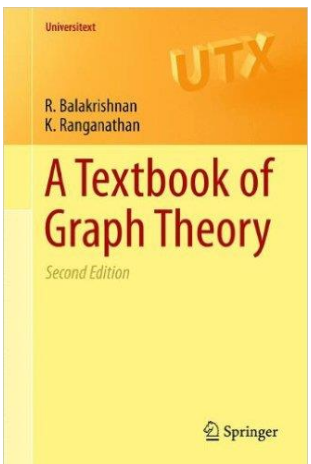
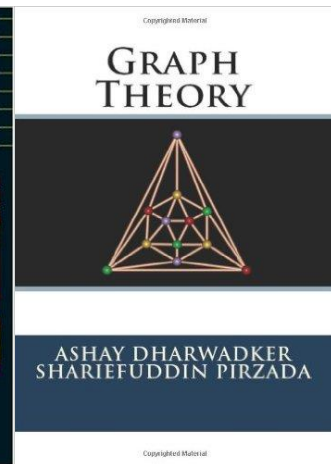
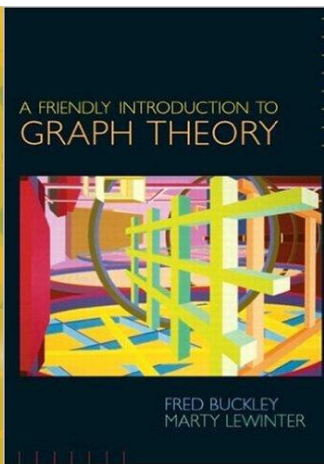
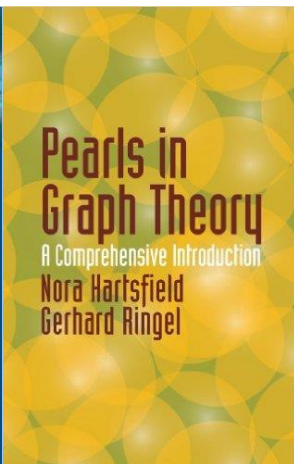
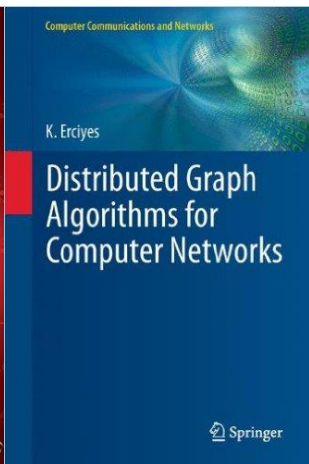
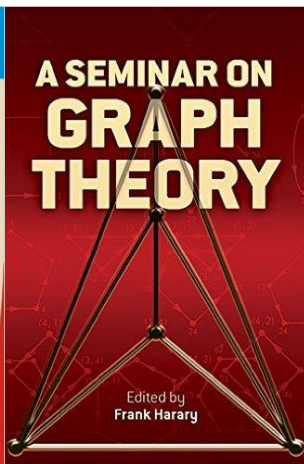
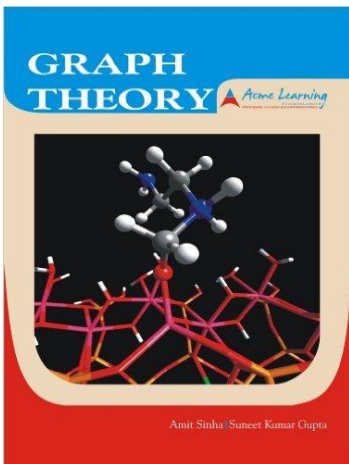
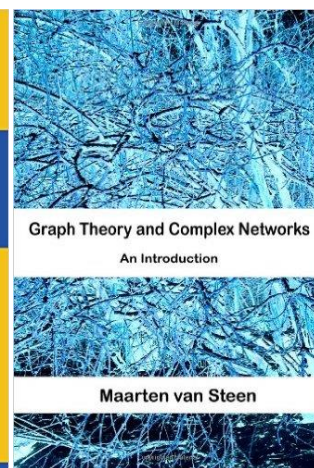
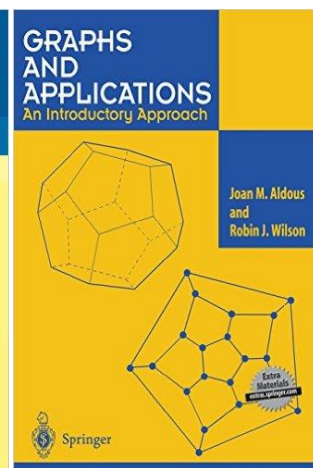
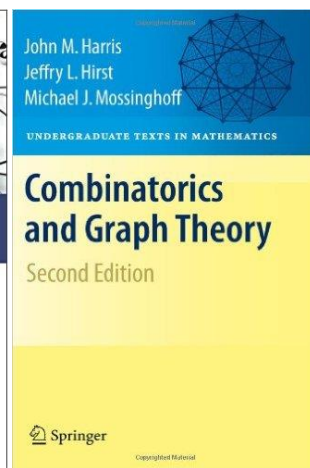
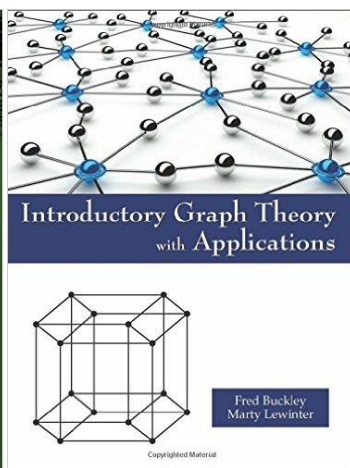
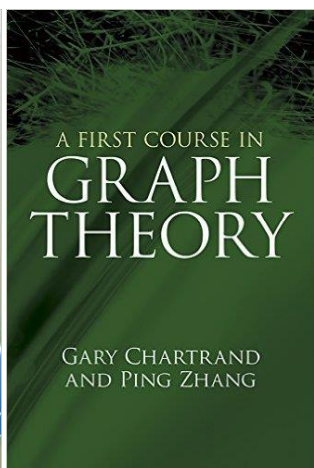
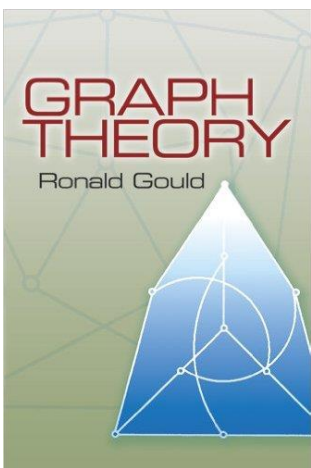
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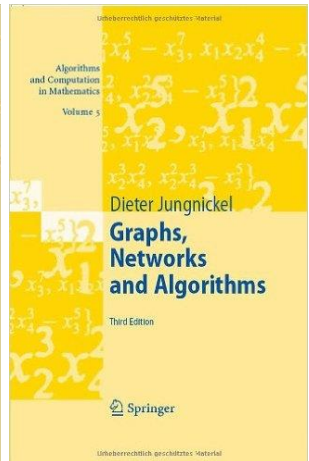
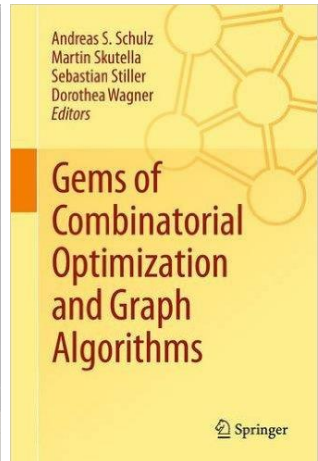
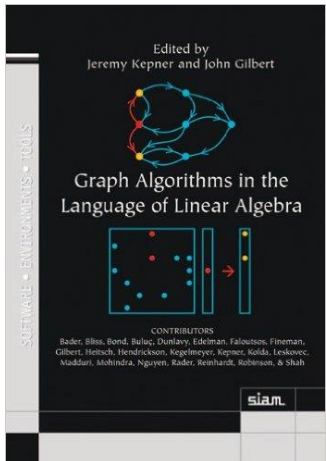
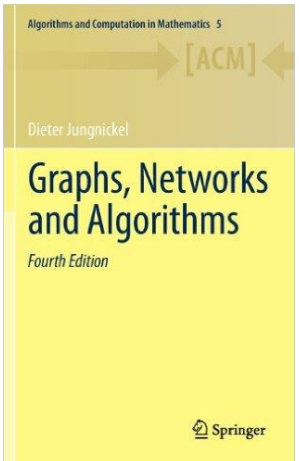
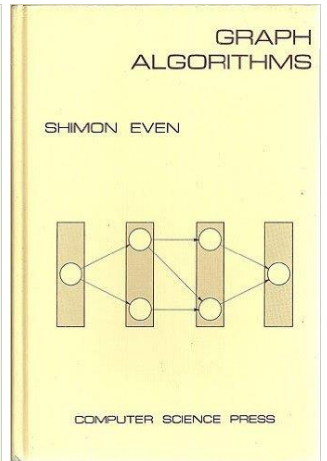
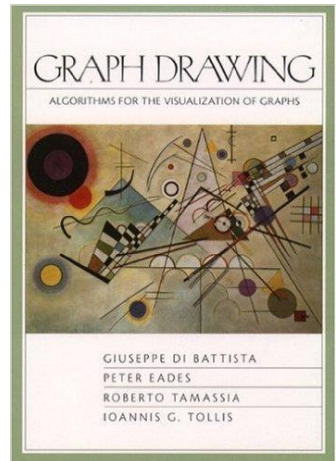
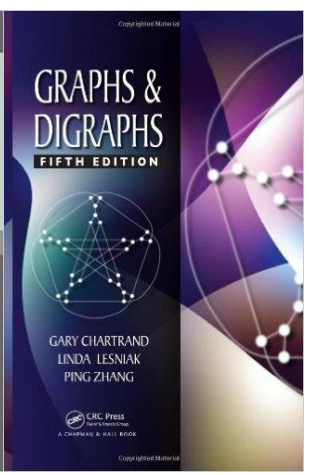
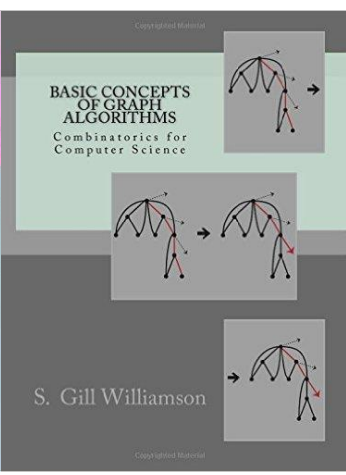
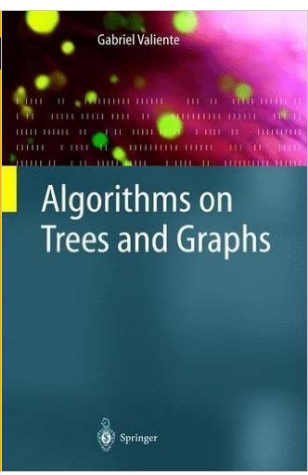
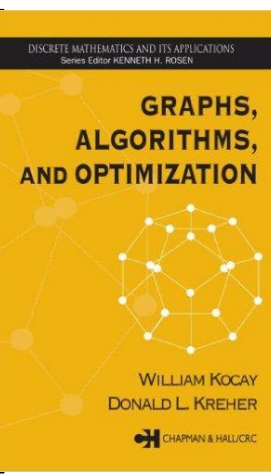
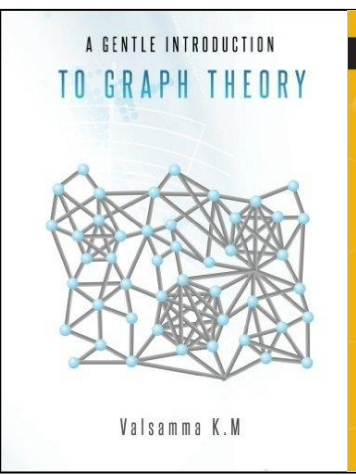
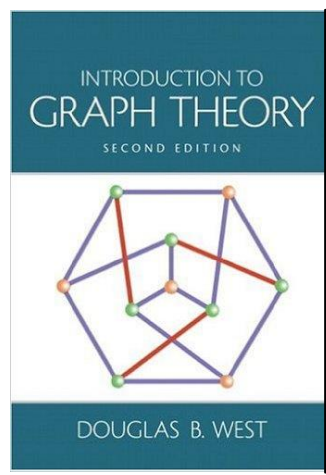
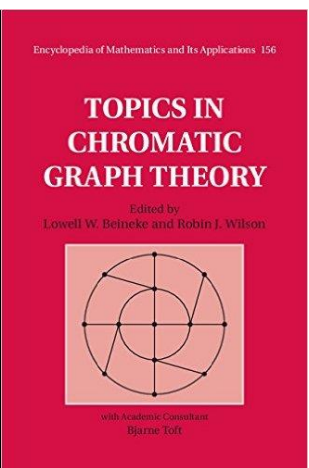
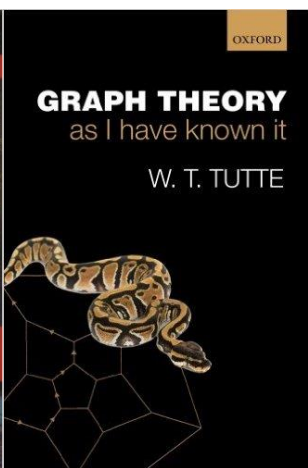
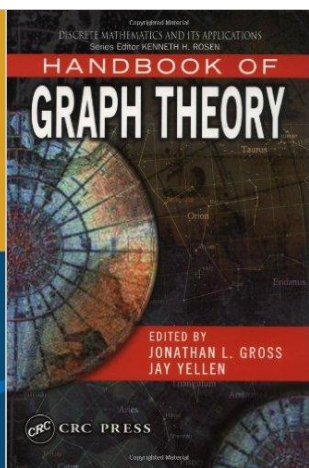
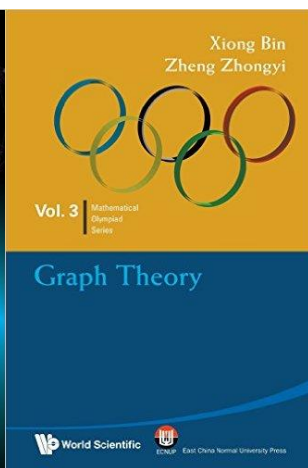
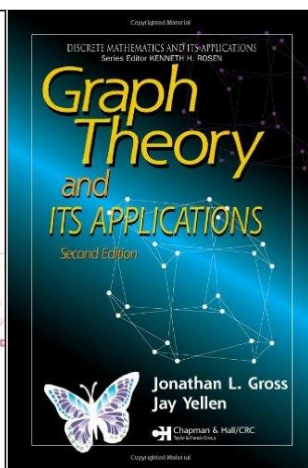
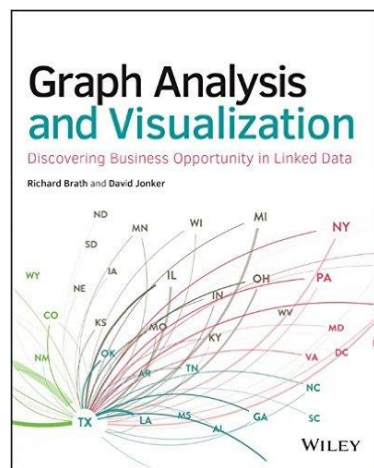
B/W QuickCam a.cidadao@mail.telepac.pt

Moon Light









SEVENTH CRACOW **CONFERENCE ON GRAPH THEORY** "RYTRO '14"

September 14-19, 2014
Rytro, Poland

The meeting is next in the series of former Cracow Conferences on Graph Theory organized in [Niedzica](#) (1990), [Zgorzelisko](#) (1994), [Kazimierz Dolny](#) (1997), [Czorsztyn](#) (2002), [Ustroń](#) (2006) and [Zgorzelisko](#) (2010).

Selected papers presented at the conference will be published in a Special Issue of [Discrete Mathematics](#) dedicated to the 7th Cracow Conference on Graph Theory. Already six Special Issues of DM were devoted to our conferences (volumes: [121](#), [164](#), [236](#), [307/11-12](#), [309/22](#), [312/14](#)).

Invited speakers:

[Ralph Faudree](#), University of Memphis, USA

[Linear Forests on Hamiltonian Cycles](#)

[András Gyárfás](#), Hungarian Academy of Sciences, Budapest, Hungary

[Vertex covers by monochromatic pieces - results and problems](#)

[Wilfried Imrich](#), Montanuniversität Leoben, Austria

[Graph Products and Symmetry Breaking in Graphs](#)

[Ken-ichi Kawarabayashi](#), National Institute of Informatics, Tokyo, Japan

[Coloring graphs with some forbidden or restricted configurations](#)

[Jan Kratochvíl](#), Charles University, Prague, Czech Republic

[Extending Partial Geometric Representations of Graphs](#)

[Dieter Rautenbach](#), Universität Ulm, Germany

Related events:



Cycles and
Colourings



5th Polish
Combinatorial
Conference



17th Workshop on
Hereditary Graph
Properties



INTERNATIONAL CONFERENCE ON **GRAPH THEORY AND ITS APPLICATIONS**

December 16-19, 2015

Amrita School of Engineering, Coimbatore, India

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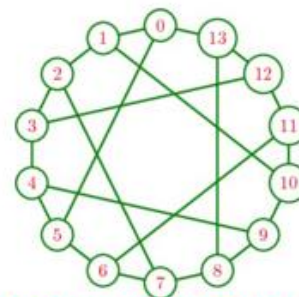
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About The Conference

This will be a Four-day Conference in Graph Theory, Graph Algorithms and its applications. It will be focusing on the subareas in graph theory that has applications in Optimization, Computing Techniques, VLSI Design and Testing, Image Processing, and Network Communications. The goal of this conference is to bring top researchers in these areas to Amrita to foster collaboration and to expose students to important problems in the growing field. The conference is expected to stimulate joint work among researchers from India and abroad and attract research students and postdoctoral fellows who work in graph theory. The Conference will cover a broad range of topics in Graph Theory. The topics include, but are not limited to:

- Graph Theory
- Algebraic Graph Theory
- Algorithms and Computing Techniques
- Graph Optimization
- VLSI Design and Testing
- Image Processing
- Networks
- Communications and Control Theory



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VENUE :Amrita School of Engineering, Coimbatore, India

Academic Cooperations :



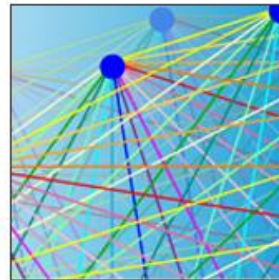
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28th Cumberland Conference on Combinatorics, **Graph Theory** & Computing

May 15-17, 2015

(12:30p on May 15 to 12:30p on May 17th)

University of South Carolina

Columbia, SC 29208

Dedicated to László Székely's 60th Birthday!**Join us for dinner on Friday, May 15th, at The Flying Saucer!**[For more details and a map, please go here.](#)

The Cumberland Conference on Combinatorics, Graph Theory, and Computing is an annual conference that brings together internationally known researchers, industrial mathematicians and computer scientists, and southeastern university and college professors, post-docs, graduate and undergraduate students, to discuss the latest advances in discrete mathematics and computer science. Particular efforts are made to include faculty and students from smaller institutions throughout the region. Each year, a different university in the Cumberland region hosts the conference, usually in the month of May, immediately after the end of the spring semester. This year, the University of South Carolina is pleased to host the Twenty-Eighth Cumberland Conference, dedicated to **László Székely's 60th Birthday**, on its Columbia campus, from May 15 – 17, 2015. We are excited to announce that the following leading researchers have agreed to participate as plenary speakers.

Modern Trends in Algebraic Graph Theory

Program

Registration

Call for Abstracts

List of Participants

Accommodations

Travel

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Conference Photos

Slides of all Talks

MODERN TRENDS IN ALGEBRAIC GRAPH THEORY

An International Conference June 2-5, 2014

*Villanova University
Villanova, Pennsylvania*

Modern Trends in Algebraic Graph Theory - AFTERMATH

First, I wish to express my deepest gratitude to those who helped to make MTAGT a reality..

Generous financial support was provided by the National Science Foundation, Villanova (VU) College of Arts and Sciences, VU Office of Research and Graduate Programs, VU Office of Research and Sponsored Projects, VU Office of the Vice President of Academic Affairs, and VU Office of the President.

Staffing support was provided by Marie O'Brien, Lorraine McGraw, Doug Norton, Najib Nadi, Taylor Berrang, Carrie Caswell, Carolyn Romano, Joseph Reiter, and Pat Woldar.

An indispensable role was played by the Office of Conference Services. In particular, I wish to mention Ron Diment and Stefanie Austinat. I also wish to thank Elisa Wiley and Clete Rickert for web support.

Last but not least, I wish to thank those who attended MTAGT. When all is said and done, the success of a conference depends integrally on the qualifications of its participants.

We had a wonderfully strong and diverse group. More than half of the 110 participants traveled to Villanova from 20 different nations. Over 20% of the participants were female, and roughly 25% were graduate students/recent PhDs. We are most proud of these demographics.

The conference presentations were truly inspired. I am most pleased to now report their online availability:

[<video recordings of plenary talks>](#)

[<Slides of all talks>](#)

Mathematics alone does not make a successful mathematics conference. It is a desirable (if not imperative) to promote healthy multicultural relations, and unobstructed lines of communication between participants. As

EUROCOMB 2015

Bergen

European Conference on Combinatorics, Graph Theory and Applications

August 31 — September 4, 2015

Maria Chudnovsky, Princeton
Amin Coja-Oghlan, Goethe Univ. Frankfurt
Zdeněk Dvořák, Charles Univ. Prague
Pavol Hell, Simon Fraser Univ.
Subhash Khot, Courant Inst. Math. Sci.
Daniel Lokshтанov, Univ. Bergen
Francisco Santos, Univ. Cantabria
Van Vu, Yale Univ.

Helge Tverberg session
(chairs Jiří Matoušek & Jaroslav Nešetřil):
Imre Bárány, Hungarian Acad. Sci.
Gil Kalai, Hebrew Univ. Jerusalem
Günter Ziegler, Freie Universität Berlin

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Balázs Szegedy, Hungarian Acad. Sci.

Organizers:
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Pål Grønås Drange (co-chair)
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Electronic Journal of Graph Theory and Applications (EJGTA)

The Electronic Journal of Graph Theory and Applications (EJGTA) is a refereed journal devoted to all areas of modern graph theory together with applications to other fields of mathematics, computer science and other sciences. The journal is published by the Indonesian Combinatorial Society (InaComBS), Graph Theory and Applications (GTA) Research Group - The University of Newcastle - Australia, and Faculty of Mathematics and Natural Sciences - Institut Teknologi Bandung (ITB) Indonesia. Subscription to EJGTA is free. Full-text access to all papers is available for free.

All research articles as well as surveys and articles of more general interest are welcome. All papers will be refereed in the normal manner of mathematical journals to maintain the highest standards.

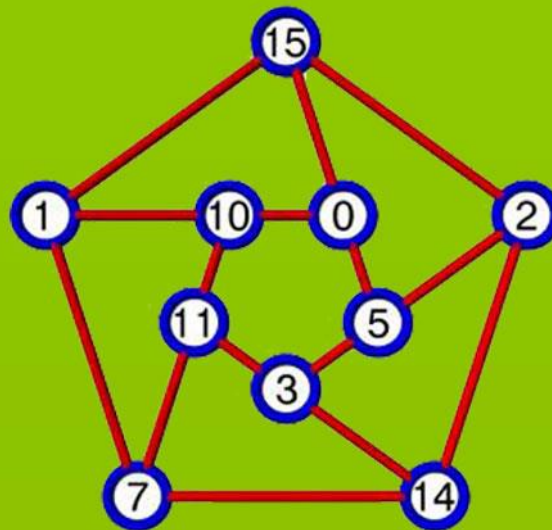
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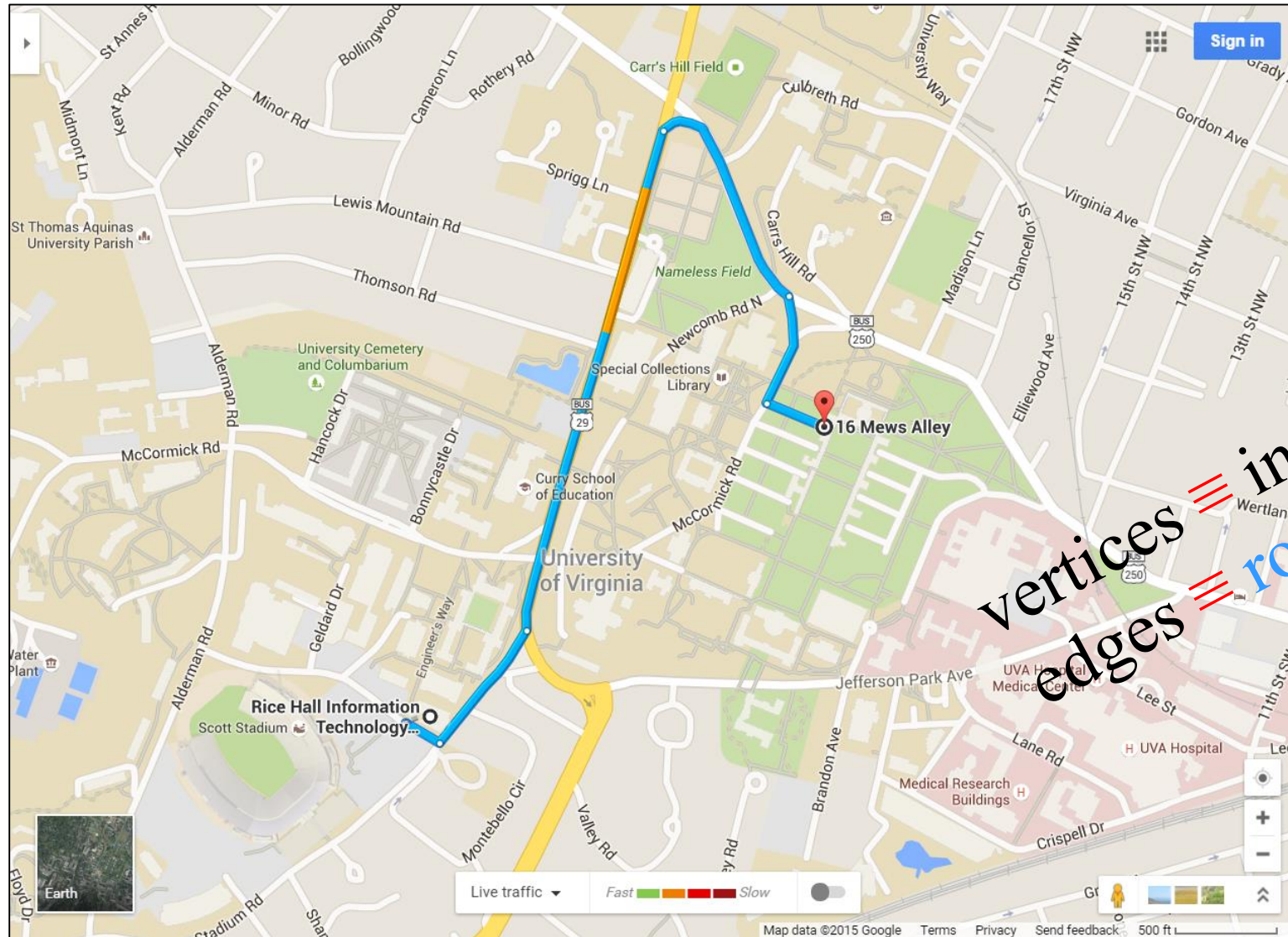
G Marimuthu, India

Applications of Graphs

- Geographical information / GPS systems



Leonhard Euler
1707–1783



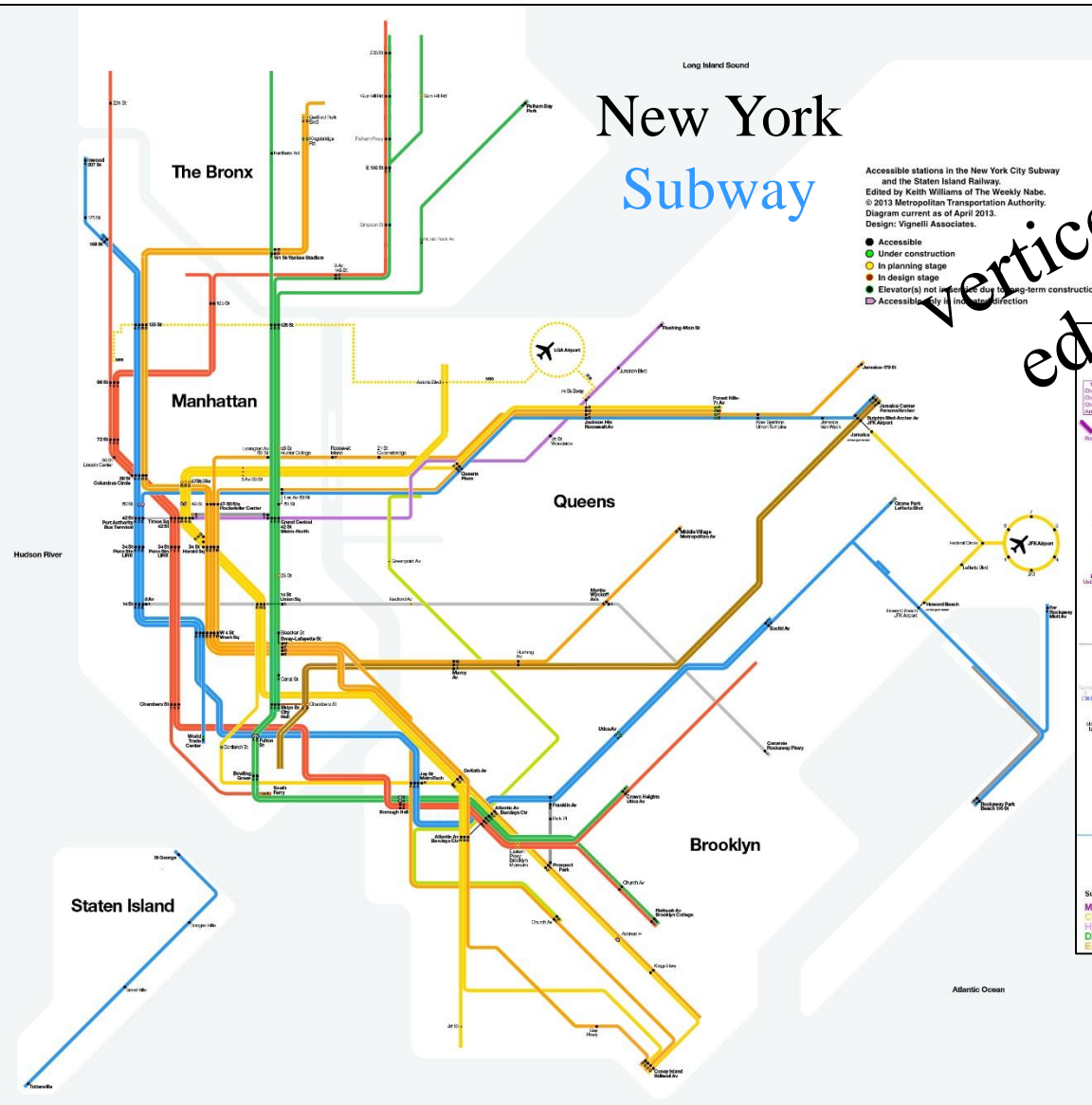
vertices intersections
edges roads / streets

Applications of Graphs

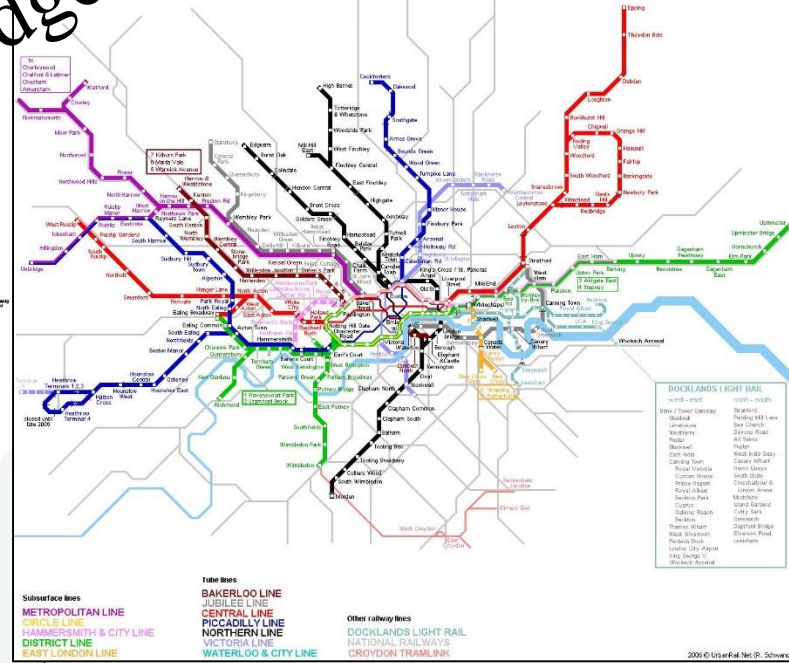


Leonhard Euler
1707–1783

- Subway maps



vertices ≡ stations
 edges ≡ tracks



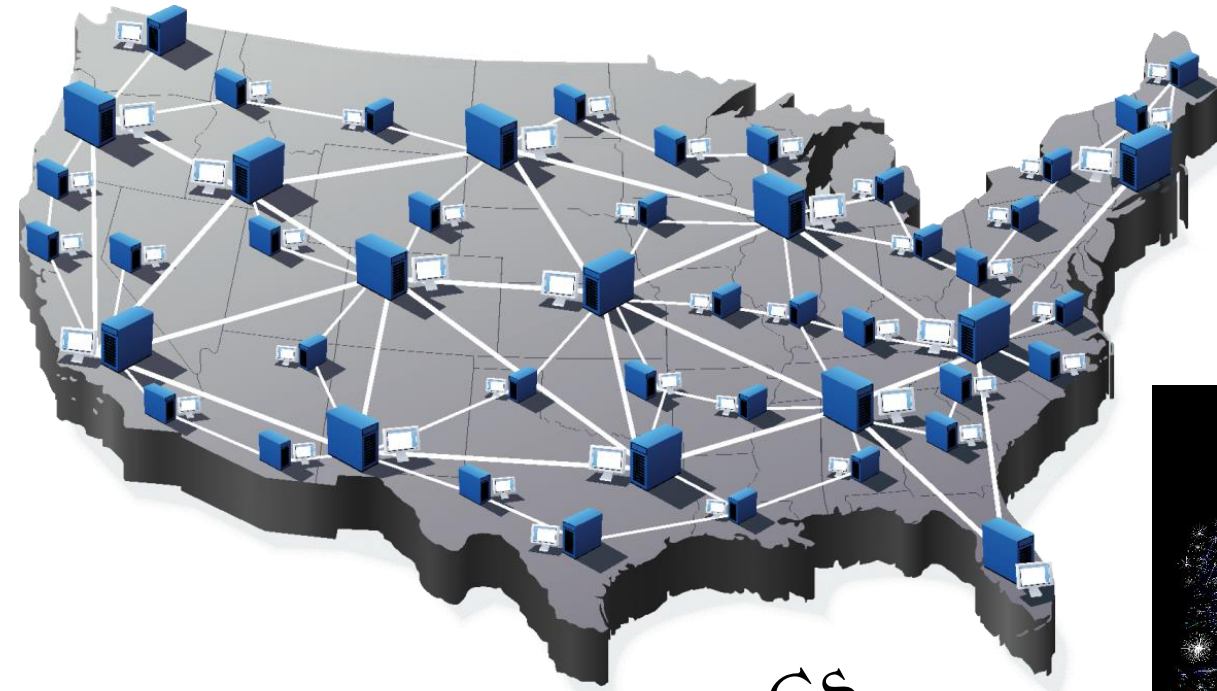
London underground

Applications of Graphs

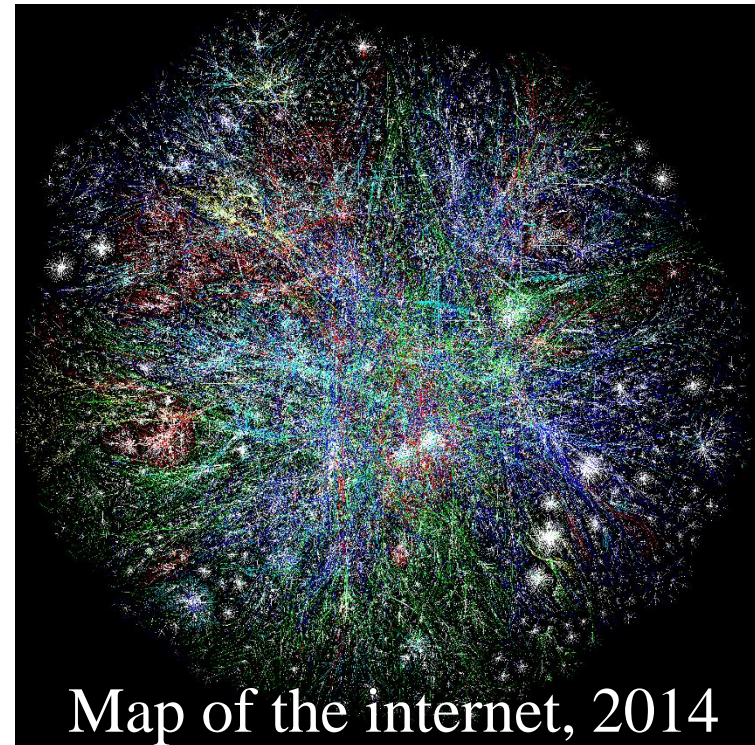
- Computer networks



Leonhard Euler
1707–1783



vertices \equiv routers / PCs
edges \equiv fiber / links



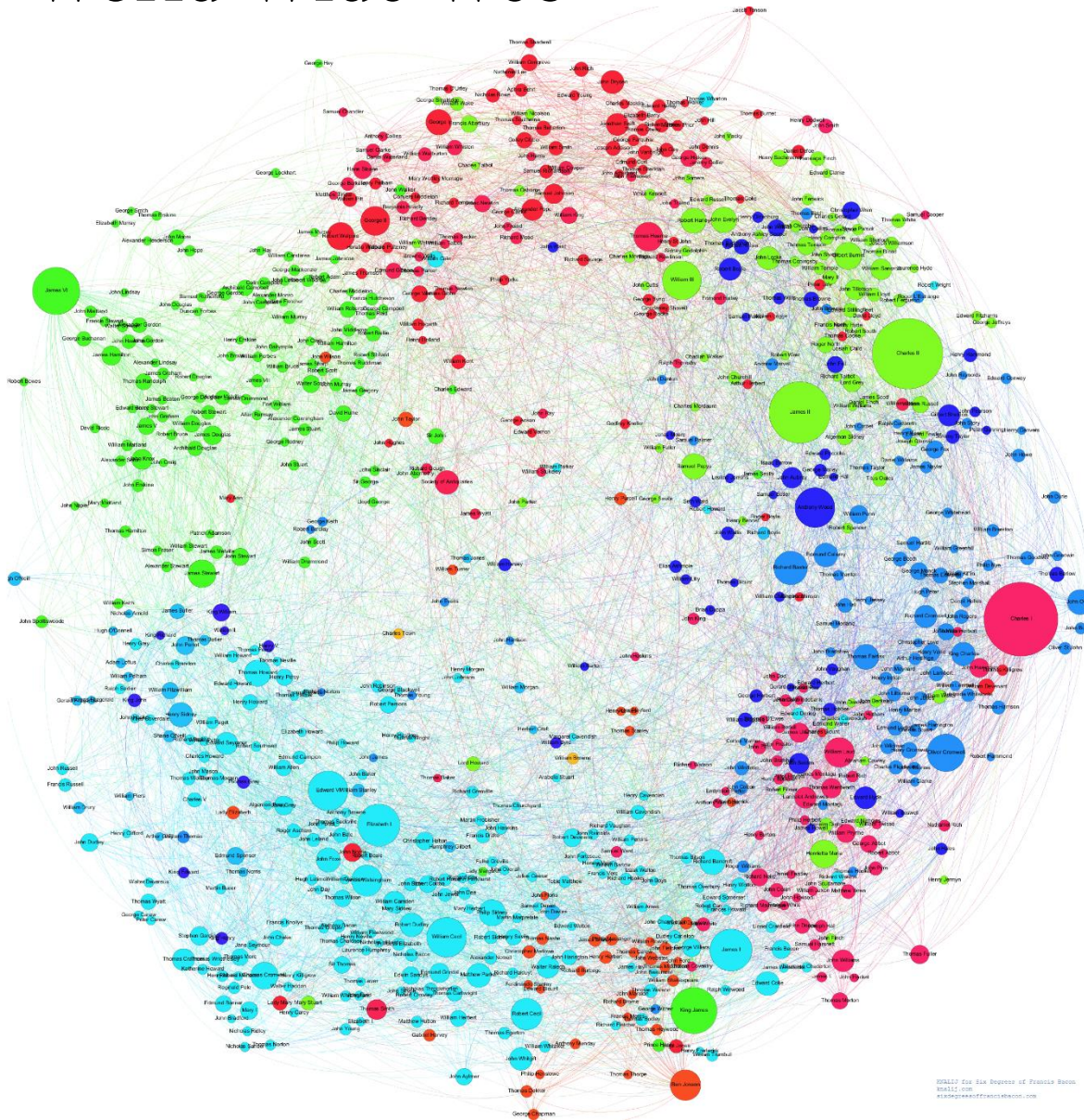
Map of the internet, 2014

Applications of Graphs



Leonhard Euler
1707–1783

- World Wide Web



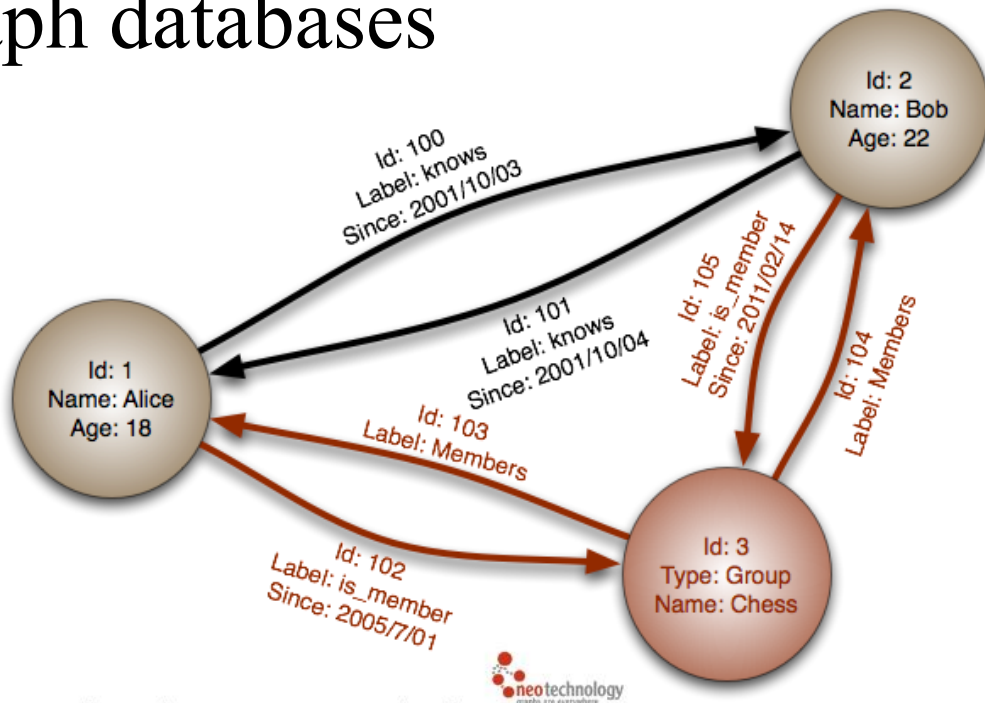
vertices \equiv pages
edges \equiv URLs

Applications of Graphs

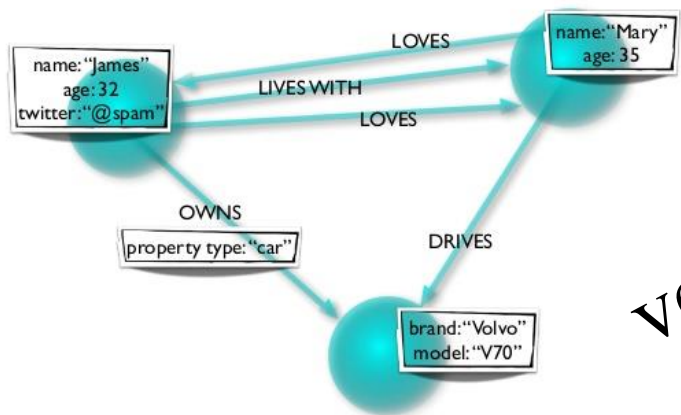


Leonhard Euler
1707–1783

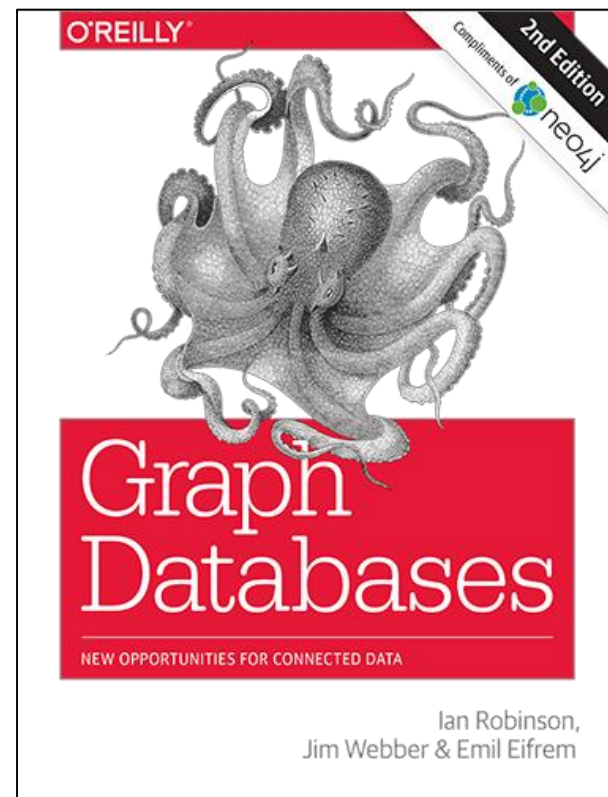
- Graph databases



Graph data model



vertices \equiv records
edges \equiv relations

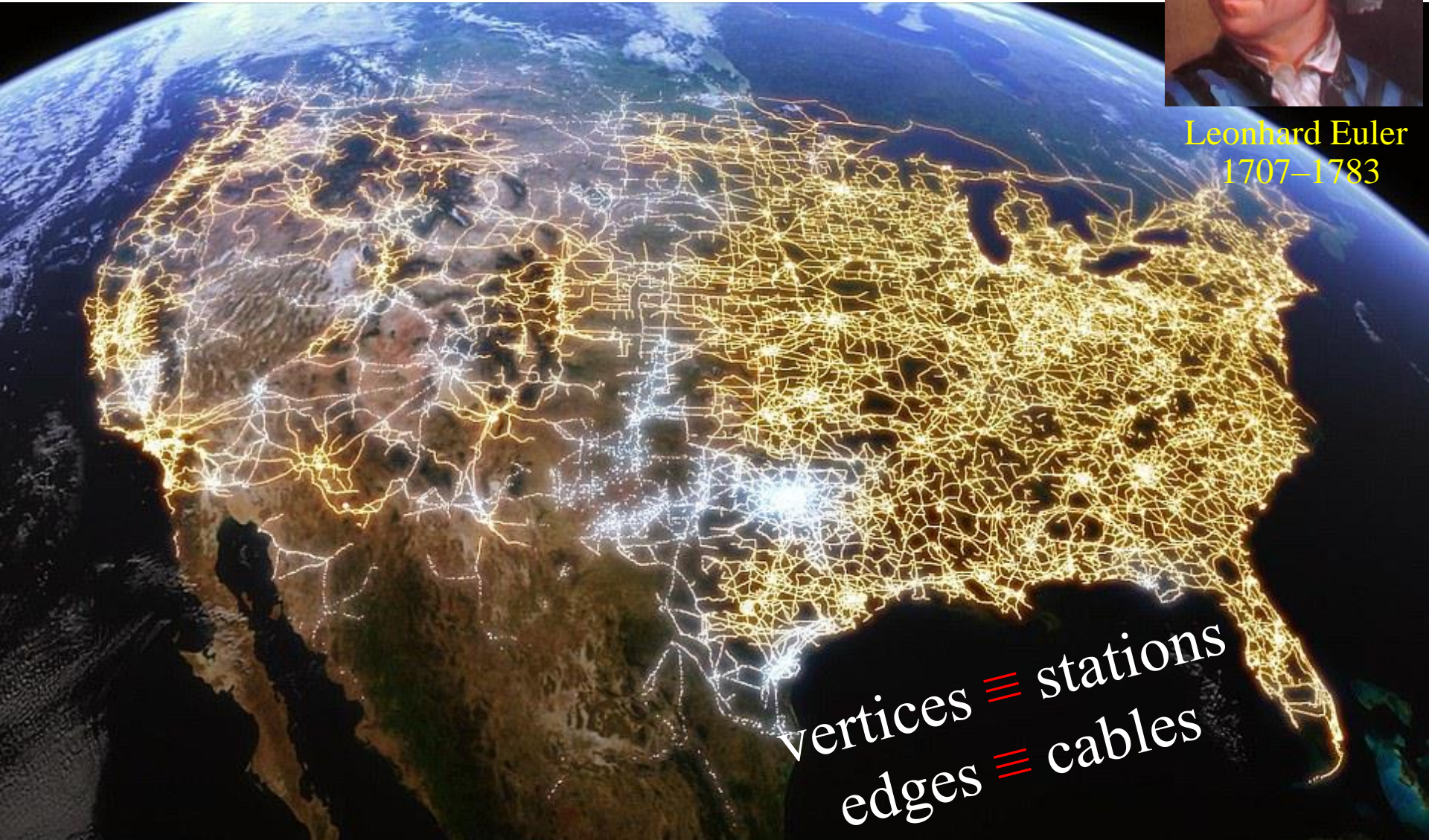


Applications of Graphs

- Electrical grids



Leonhard Euler
1707–1783



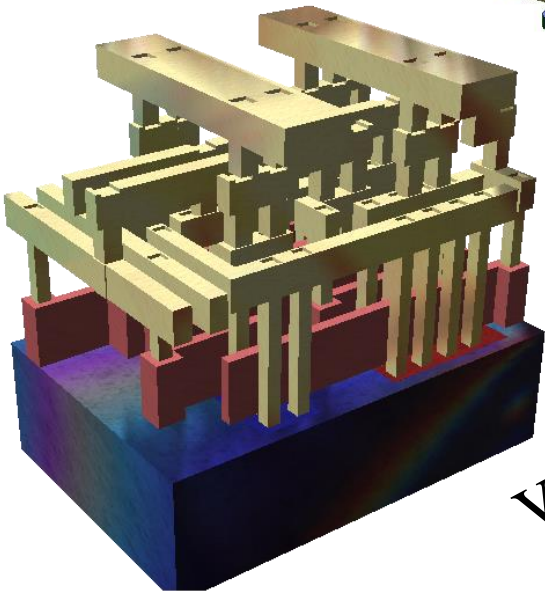
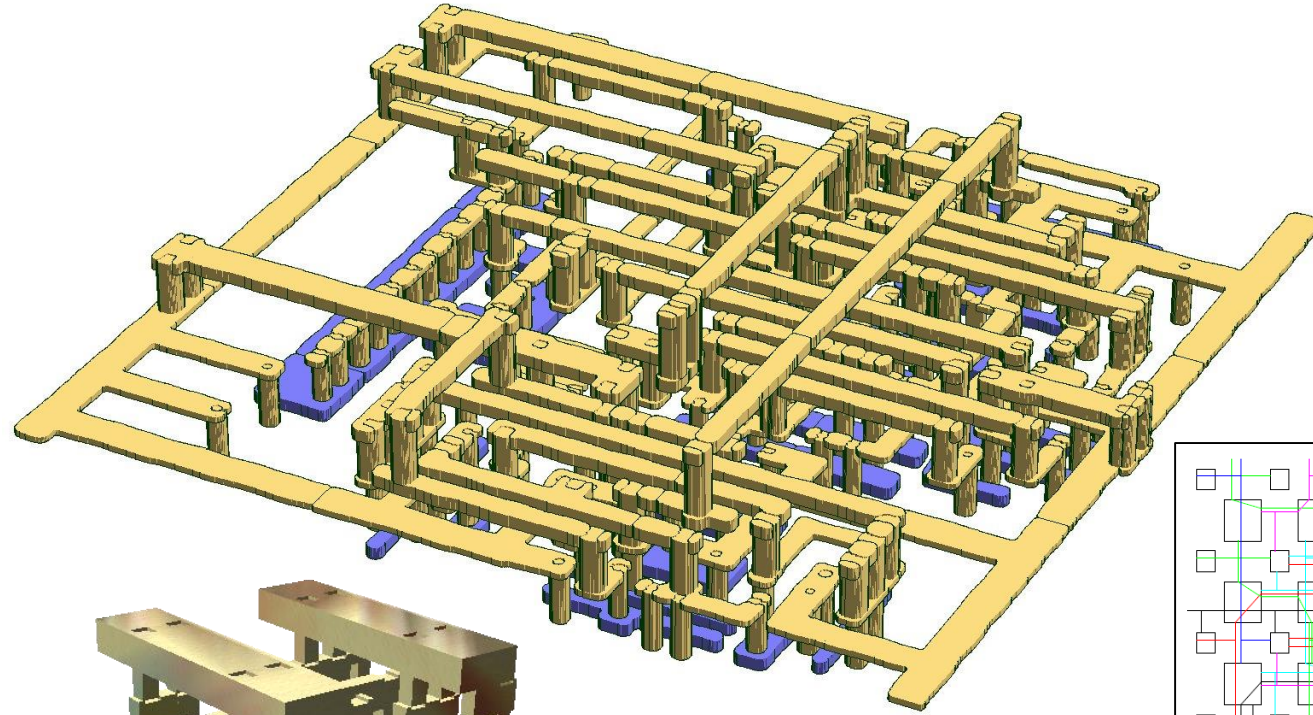
vertices \equiv stations
edges \equiv cables

Applications of Graphs

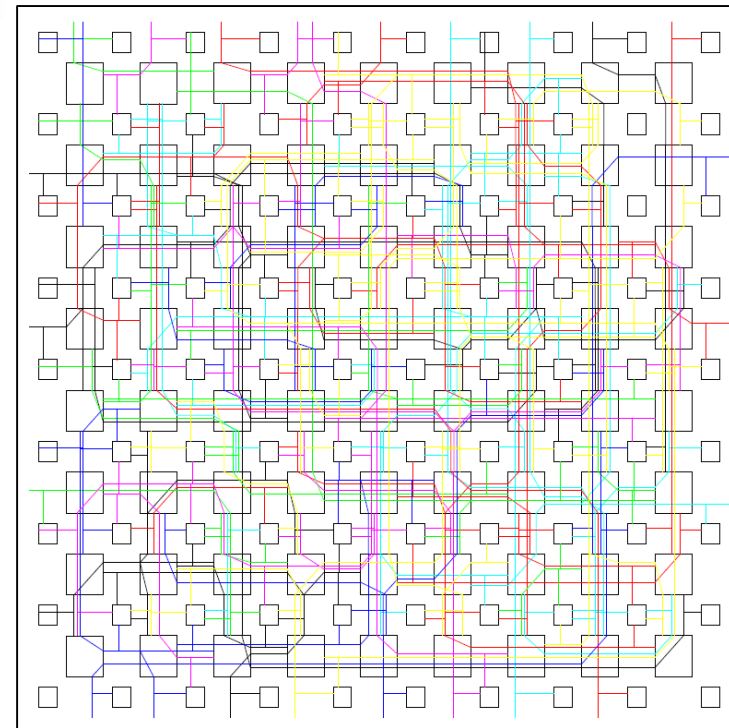


Leonhard Euler
1707–1783

- Integrated circuit design (VLSI chips)



vertices \equiv transistors
edges \equiv wires



Applications of Graphs

- CAD / building HVAC design



Leonhard Euler
1707–1783



New Faculty building of University of Ljubljana, Slovenia
Courtesy of Ljubljana university incubator technology transfer - Certus

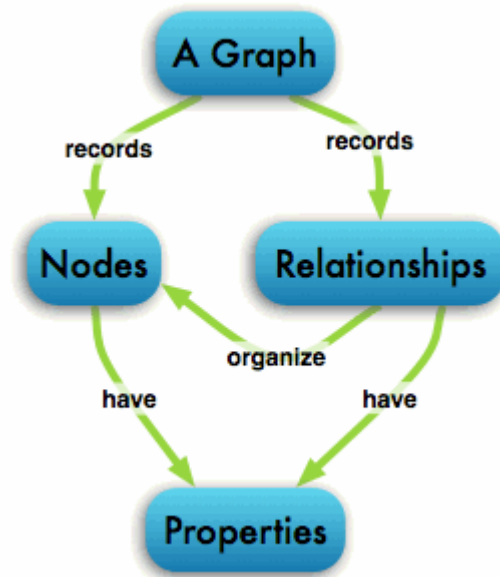
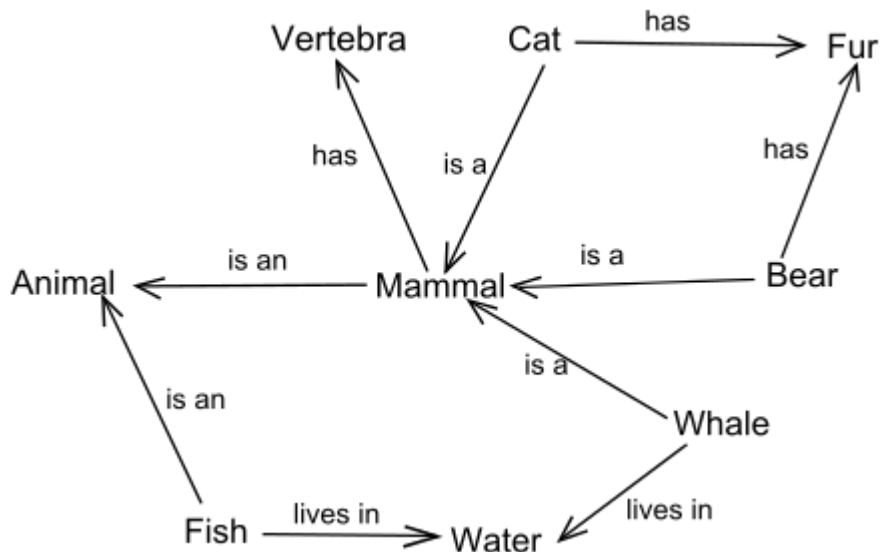
vertices \equiv connectors
edges \equiv ducts

Applications of Graphs

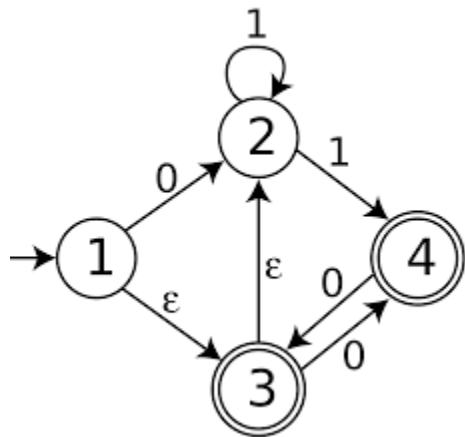


Leonhard Euler
1707–1783

- Semantic nets



- Finite automata



vertices \equiv states
edges \equiv transitions

vertices \equiv objects
edges \equiv relations

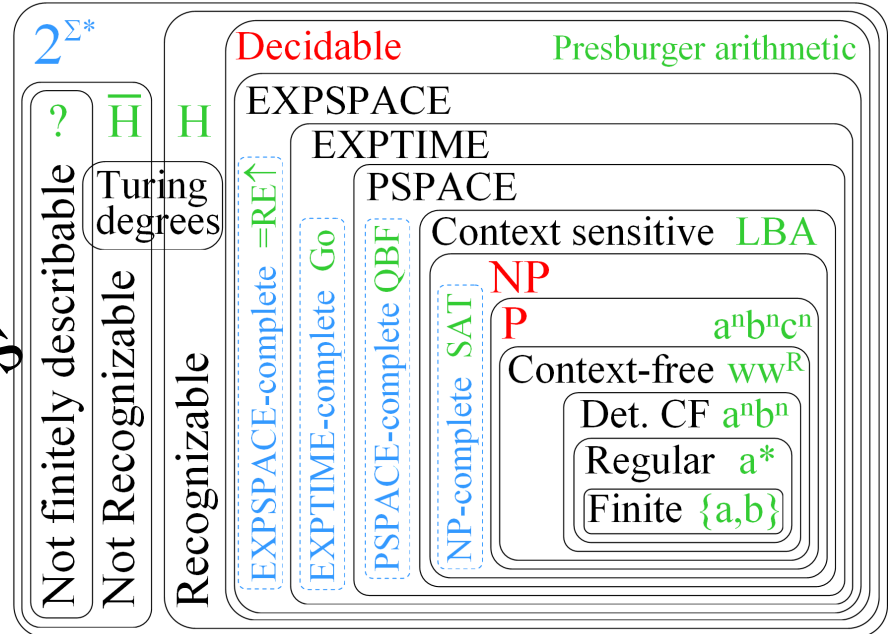
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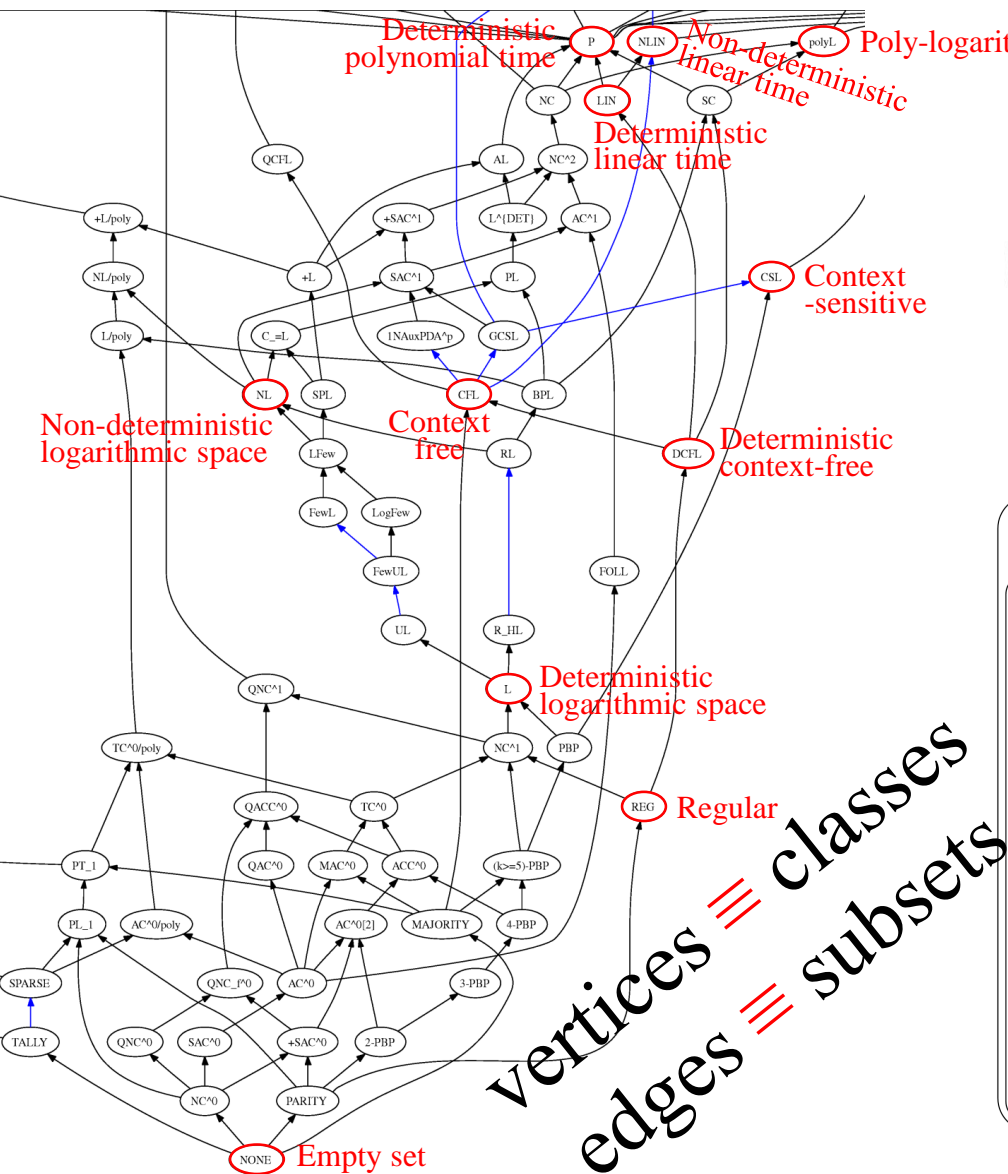
Leonhard Euler
1707–1783



The Extended Chomsky Hierarchy



- Time / space complexity classes



vertices == classes
edges == subsets

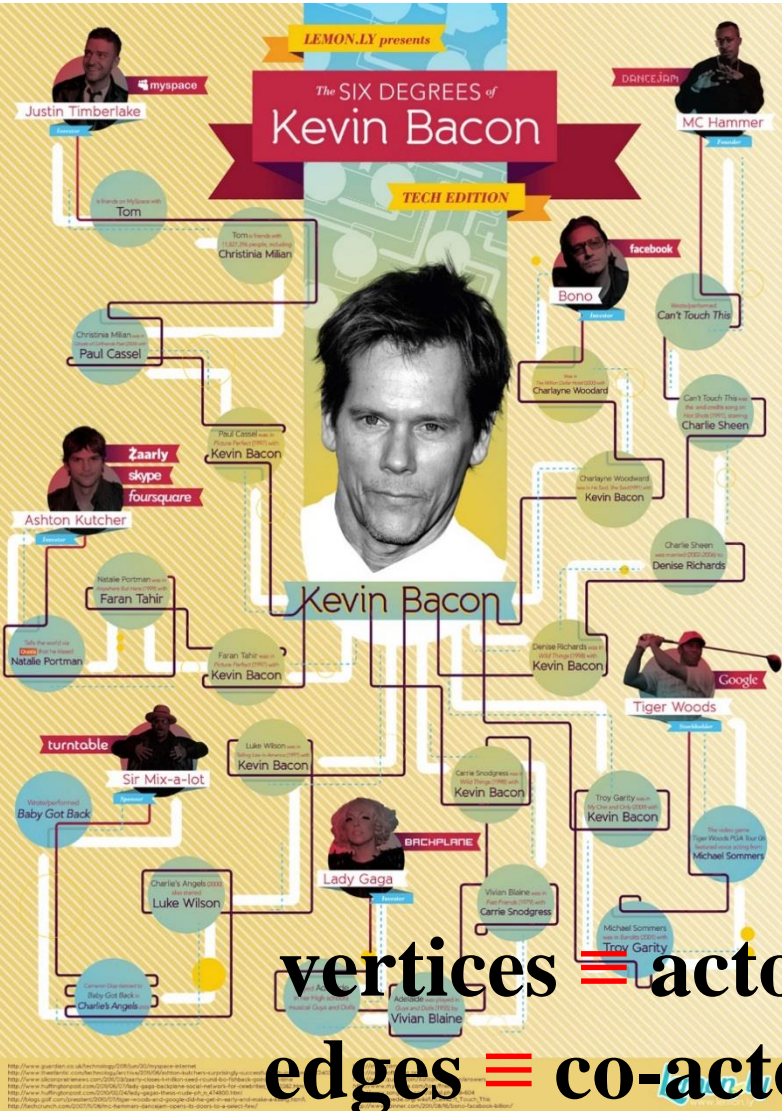
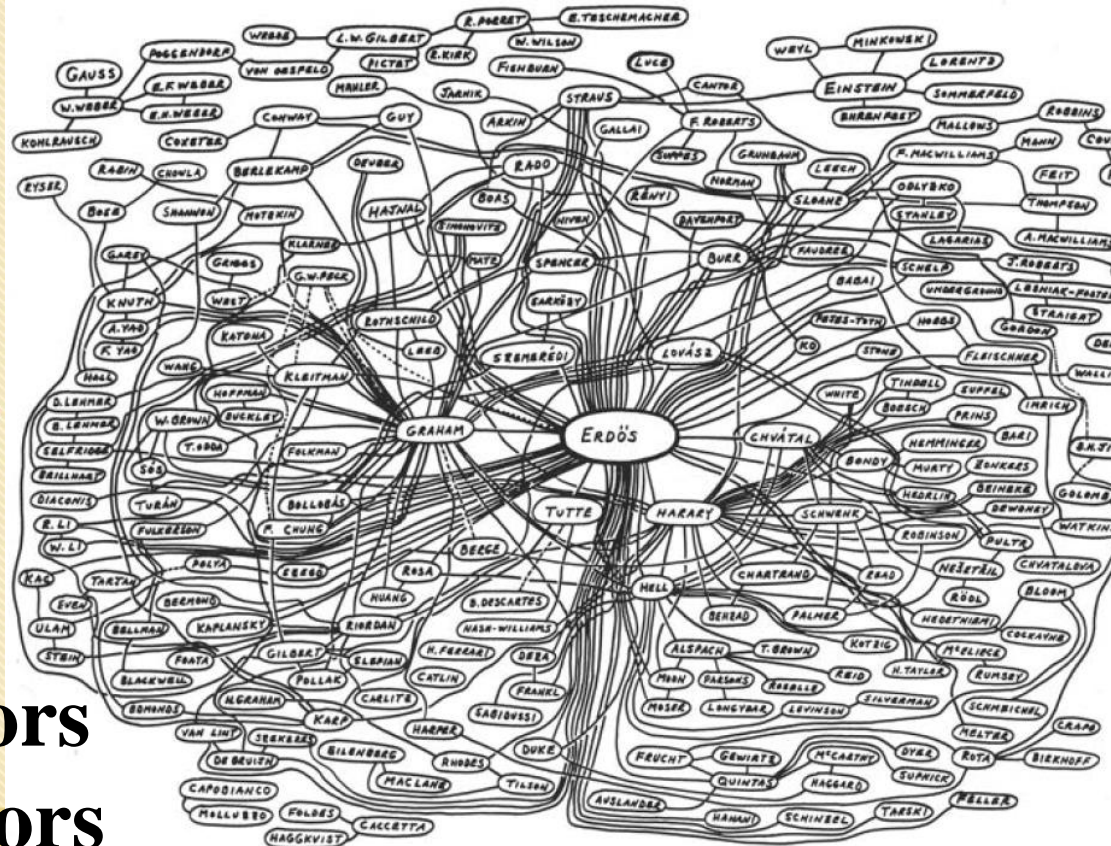
Applications of Graphs



Leonhard Euler
1707–1783

- Erdős numbers - “6 degrees” of separation

vertices \equiv authors
edges \equiv co-authors

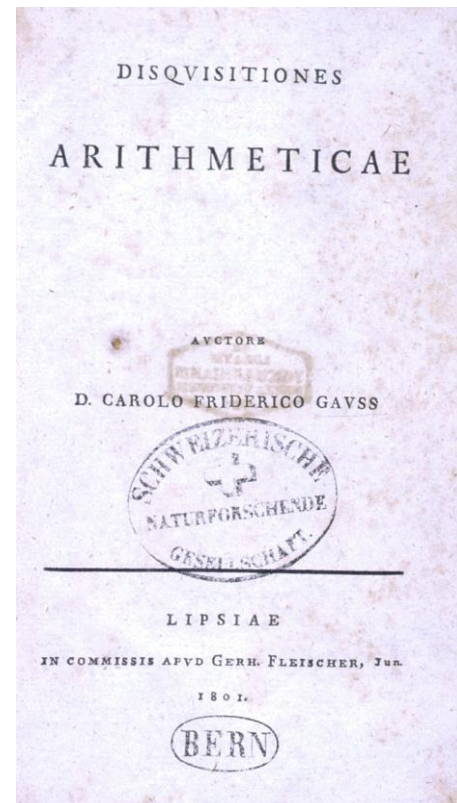


vertices \equiv actors
edges \equiv co-actors

Historical Perspectives

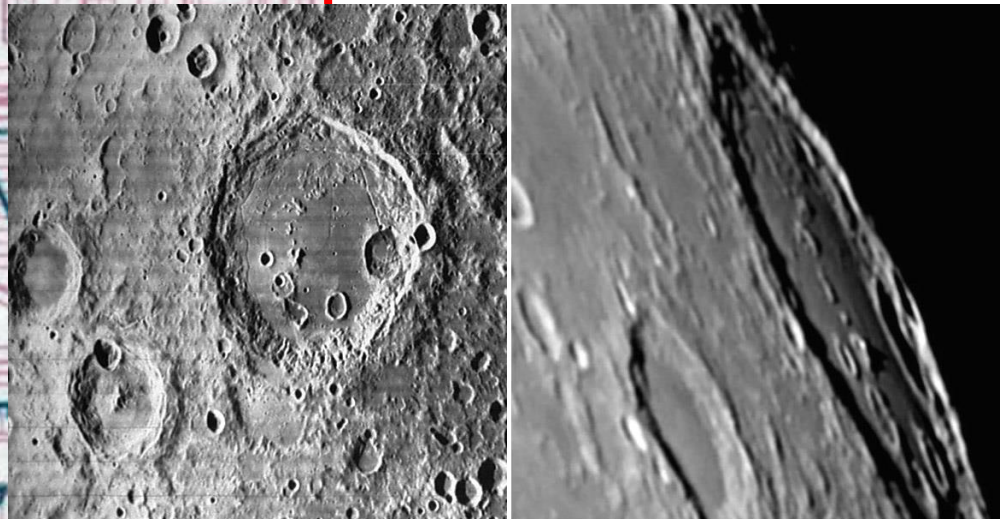
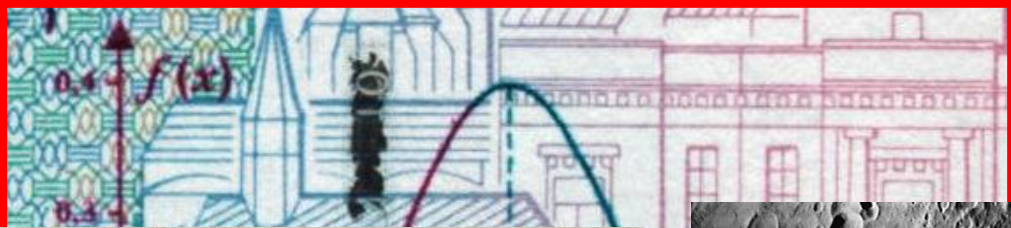
Carl Friedrich Gauss (1777–1855)

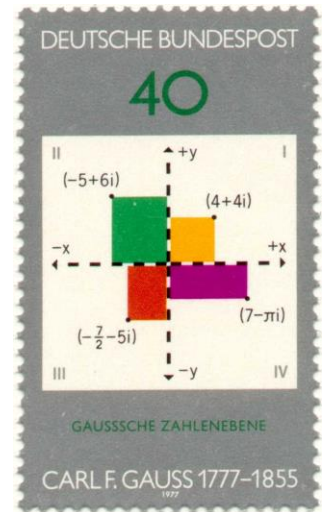
- “Prince of Mathematics”
- Founded modern number theory
- Authored “**Disquisitiones Arithmeticae**”
- Fundamental Theorem of Algebra
- Major contributions to astronomy, optics, electromagnetism, statistics, geometry
- **Gaussian distribution, Gaussian elimination, Gaussian noise, Gaussian integers & primes, Gauss’ Law, Gauss’ constant, “degaussing”**
- SI unit of magnetic field strength: **gauss**
- Students: Dedekind, Riemann, Bessel



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GAUSS 177 km

97/10/16 D=254mm F/D=10

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Moon LIGHT



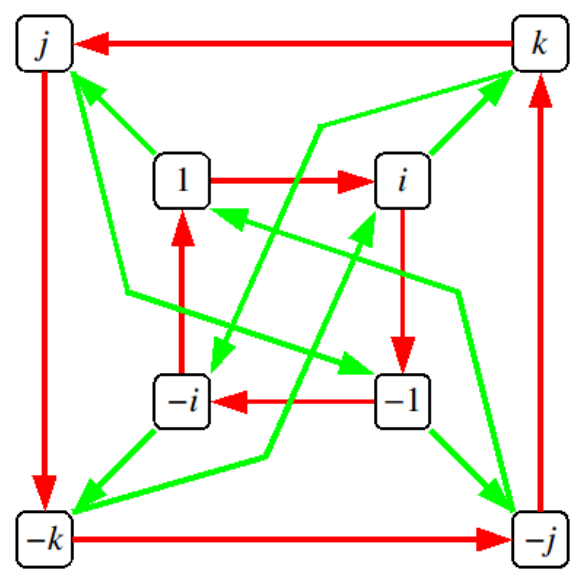
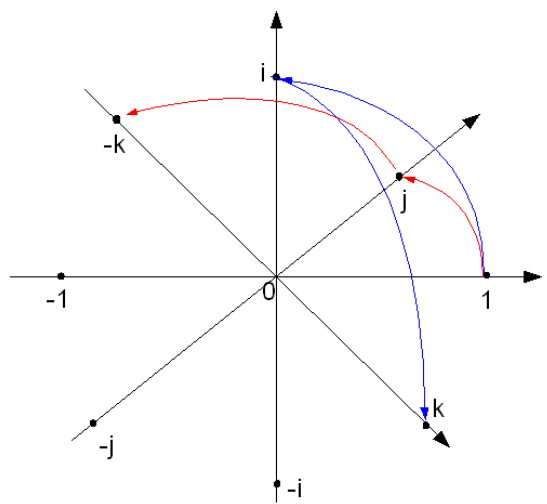
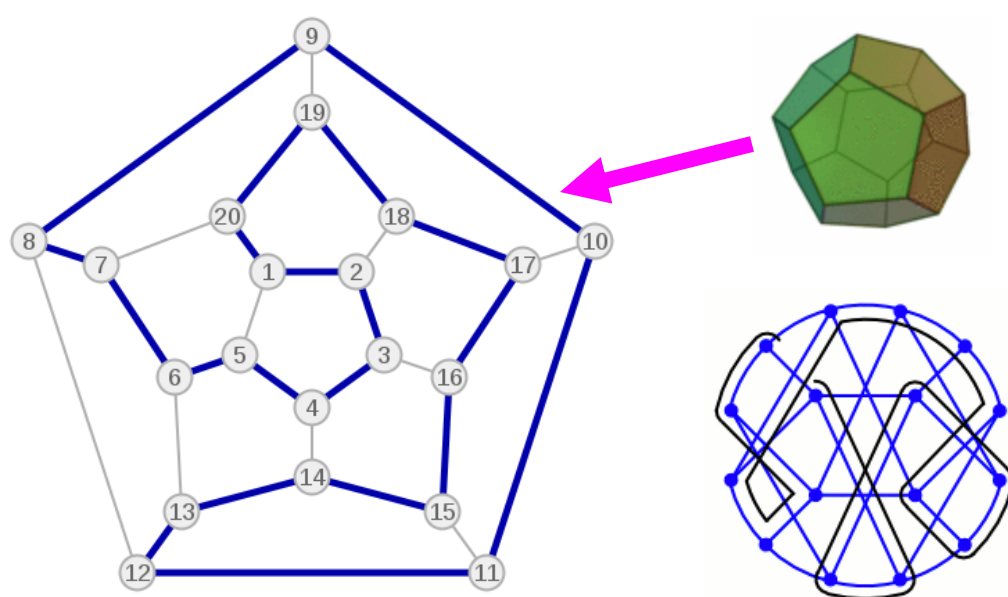
Historical Perspectives

William R. Hamilton (1805-1865)

- Mathematician, physicist, and astronomer
- Contributed to algebra, mechanics, optics
- Formulated **Hamiltonian mechanics**
- Discovered **quaternions**, conical refraction, Hamilton function, Hamilton principle, Hamiltonian group
- Invented “**Icosian Calculus**”, dot & cross products, **Hamiltonian paths**
- Influenced **computer graphics**, mechanics, electromagnetism, relativity, quantum theory, vector algebra



Here as he walked by
on the 16th of October 1843
Sir William Rowan Hamilton
in a flash of genius discovered
the fundamental formula for
quaternion multiplication
 $i^2 = j^2 = k^2 = ijk = -1$
& cut it on a stone of this bridge



Graphical representation of quaternion units product as 90°-rotation in 4D-space

Non-commutative: $ij=k$ $ji=-k$

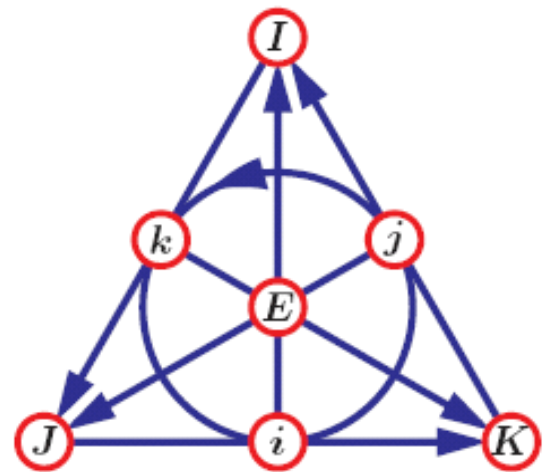
- $ij = k$
- $ji = -k$
- $ij = -ji$



Octonions: Generalization of Quaternions

- **Non-associative!** (e.g., $(ij)K = -E \neq E = i(jK)$)
- Discovered by John Graves (1843), friend of **Hamilton**
- Useful in general relativity, quantum logic, string theory

\times	i	j	k	E	I	J	K
i	-1	k	$-j$	I	$-E$	$-K$	J
j	$-k$	-1	i	J	K	$-E$	$-I$
k	j	$-i$	-1	K	$-J$	I	$-E$
E	$-I$	$-J$	$-K$	-1	i	j	k
I	E	$-K$	J	$-i$	-1	$-k$	j
J	K	E	$-I$	$-j$	k	-1	$-i$
K	$-J$	I	E	$-k$	$-j$	i	-1



Mnemonic diagram for unit octonions products

“The **real numbers** are the dependable breadwinner of the family, the complete ordered field we all rely on. The **complex numbers** are a slightly flashier but still respectable younger brother: not ordered, but algebraically complete. The **quaternions**, being noncommutative, are the eccentric cousin who is shunned at important family gatherings. But the **octonions** are the crazy old uncle nobody lets out of the attic: they are *nonassociative*.”

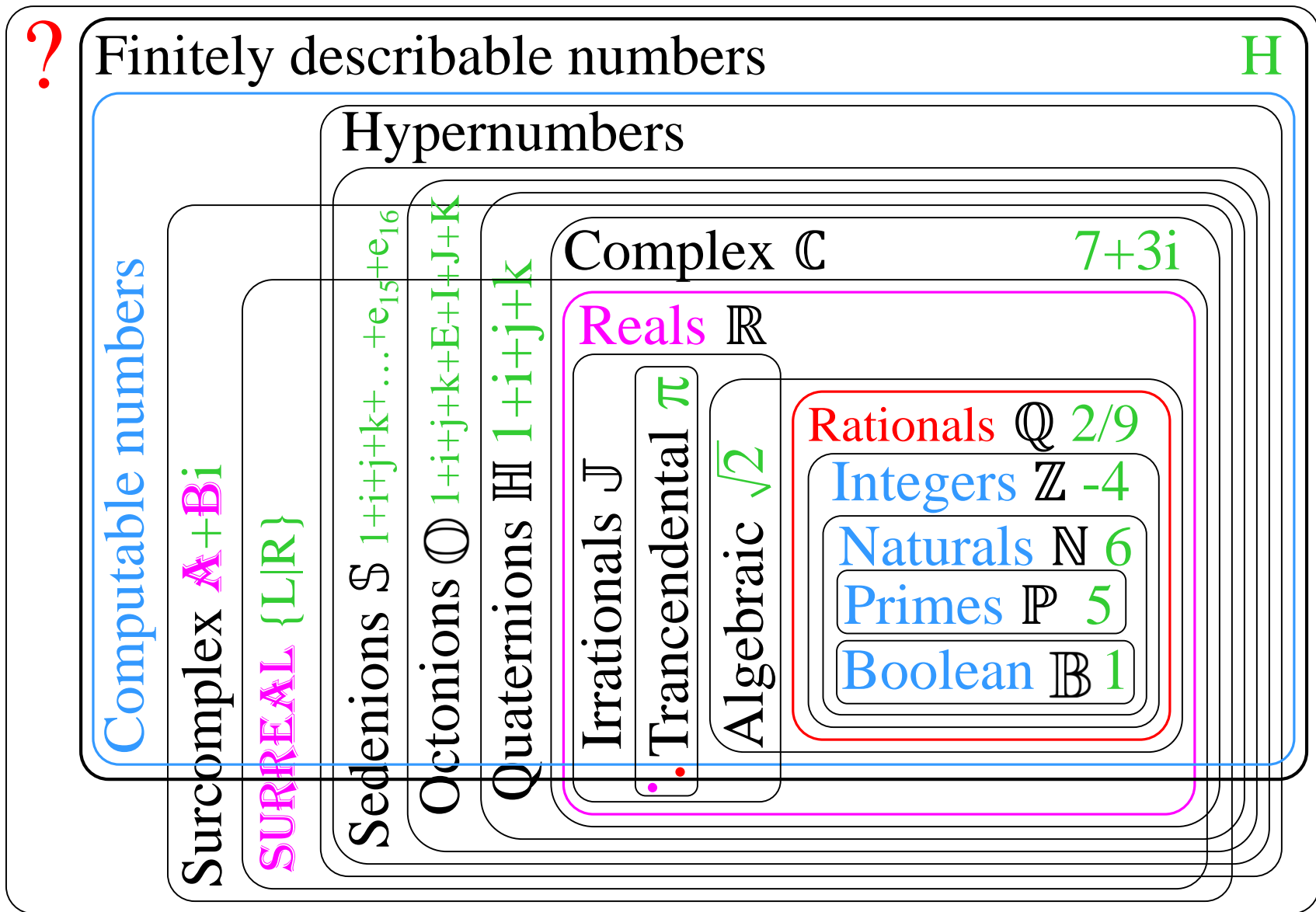
— John Baez (1961-), physicist
works on spin foams
and loop quantum gravity

Sedenions: Generalization of Octonions

- **Non-alternative!** (i.e., $x(xy)=(xx)y$ doesn't hold)

x	1	e ₁	e ₂	e ₃	e ₄	e ₅	e ₆	e ₇	e ₈	e ₉	e ₁₀	e ₁₁	e ₁₂	e ₁₃	e ₁₄	e ₁₅
1	1	e ₁	e ₂	e ₃	e ₄	e ₅	e ₆	e ₇	e ₈	e ₉	e ₁₀	e ₁₁	e ₁₂	e ₁₃	e ₁₄	e ₁₅
e ₁	e ₁	-1	e ₃	-e ₂	e ₅	-e ₄	-e ₇	e ₆	e ₉	-e ₈	-e ₁₁	e ₁₀	-e ₁₃	e ₁₂	e ₁₅	-e ₁₄
e ₂	e ₂	-e ₃	-1	e ₁	e ₆	e ₇	-e ₄	-e ₅	e ₁₀	e ₁₁	-e ₈	-e ₉	-e ₁₄	-e ₁₅	e ₁₂	e ₁₃
e ₃	e ₃	e ₂	-e ₁	-1	e ₇	-e ₆	e ₅	-e ₄	e ₁₁	-e ₁₀	e ₉	-e ₈	-e ₁₅	e ₁₄	-e ₁₃	e ₁₂
e ₄	e ₄	-e ₅	-e ₆	-e ₇	-1	e ₁	e ₂	e ₃	e ₁₂	e ₁₃	e ₁₄	e ₁₅	-e ₈	-e ₉	-e ₁₀	-e ₁₁
e ₅	e ₅	e ₄	-e ₇	e ₆	-e ₁	-1	-e ₃	e ₂	e ₁₃	-e ₁₂	e ₁₅	-e ₁₄	e ₉	-e ₈	e ₁₁	-e ₁₀
e ₆	e ₆	e ₇	e ₄	-e ₅	-e ₂	e ₃	-1	-e ₁	e ₁₄	-e ₁₅	-e ₁₂	e ₁₃	e ₁₀	-e ₁₁	-e ₈	e ₉
e ₇	e ₇	-e ₆	e ₅	e ₄	-e ₃	-e ₂	e ₁	-1	e ₁₅	e ₁₄	-e ₁₃	-e ₁₂	e ₁₁	e ₁₀	-e ₉	-e ₈
e ₈	e ₈	-e ₉	-e ₁₀	-e ₁₁	-e ₁₂	-e ₁₃	-e ₁₄	-e ₁₅	-1	e ₁	e ₂	e ₃	e ₄	e ₅	e ₆	e ₇
e ₉	e ₉	e ₈	-e ₁₁	e ₁₀	-e ₁₃	e ₁₂	e ₁₅	-e ₁₄	-e ₁	-1	-e ₃	e ₂	-e ₅	e ₄	e ₇	-e ₆
e ₁₀	e ₁₀	e ₁₁	e ₈	-e ₉	-e ₁₄	-e ₁₅	e ₁₂	e ₁₃	-e ₂	e ₃	-1	-e ₁	-e ₆	-e ₇	e ₄	e ₅
e ₁₁	e ₁₁	-e ₁₀	e ₉	e ₈	-e ₁₅	e ₁₄	-e ₁₃	e ₁₂	-e ₃	-e ₂	e ₁	-1	-e ₇	e ₆	-e ₅	e ₄
e ₁₂	e ₁₂	e ₁₃	e ₁₄	e ₁₅	e ₈	-e ₉	-e ₁₀	-e ₁₁	-e ₄	e ₅	e ₆	e ₇	-1	-e ₁	-e ₂	-e ₃
e ₁₃	e ₁₃	-e ₁₂	e ₁₅	-e ₁₄	e ₉	e ₈	e ₁₁	-e ₁₀	-e ₅	-e ₄	e ₇	-e ₆	e ₁	-1	e ₃	-e ₂
e ₁₄	e ₁₄	-e ₁₅	-e ₁₂	e ₁₃	e ₁₀	-e ₁₁	e ₈	e ₉	-e ₆	-e ₇	-e ₄	e ₅	e ₂	-e ₃	-1	e ₁
e ₁₅	e ₁₅	e ₁₄	-e ₁₃	-e ₁₂	e ₁₁	e ₁₀	-e ₉	e ₈	-e ₇	e ₆	-e ₅	-e ₄	e ₃	e ₂	-e ₁	-1

Generalized Numbers



Theorem: some real numbers are not finitely describable!

Theorem: some finitely describable real numbers are not computable!

Problem: Are the complex numbers closed under exponentiation ? E.g., what is the value of i^i ?

$$i^i = \frac{1}{\sqrt{e^\pi}} = 0.207879\dots$$

$$i^i = \frac{1}{\sqrt{e^{\pi+2k\pi}}}$$

$$e^{ix} = \cos(x) + i \sin(x)$$

i^i is multi-valued!

