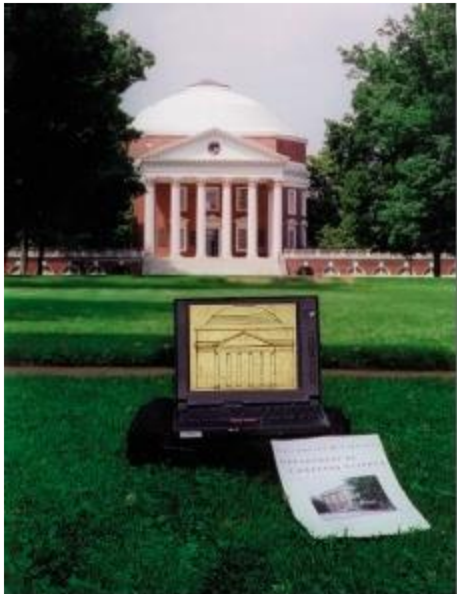


Theory of Computation CS6160

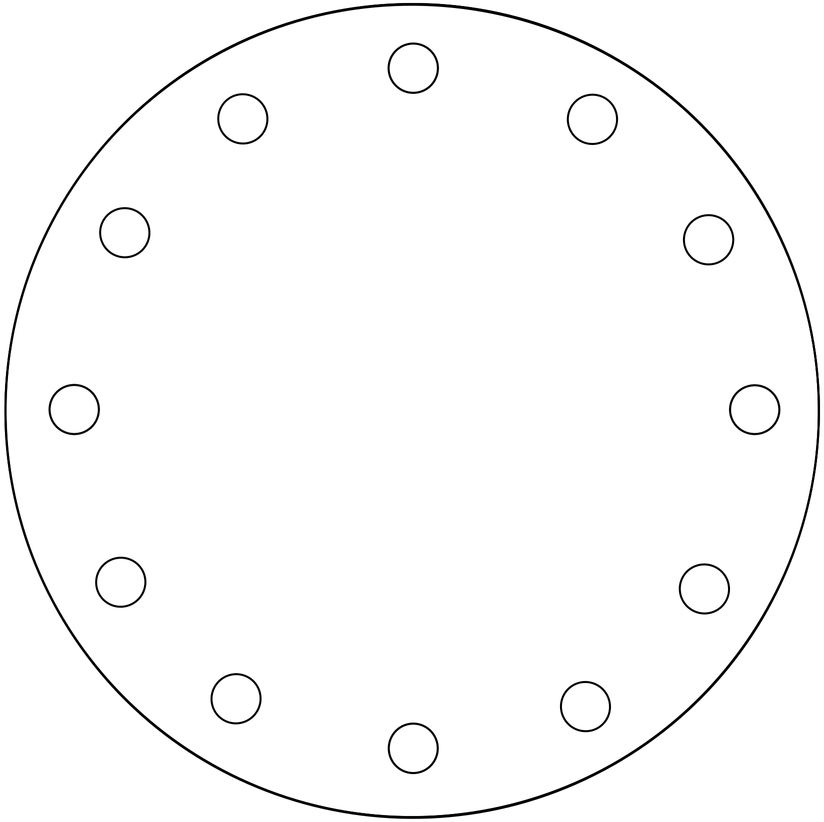
Gabriel Robins

Department of
Computer Science

University of Virginia

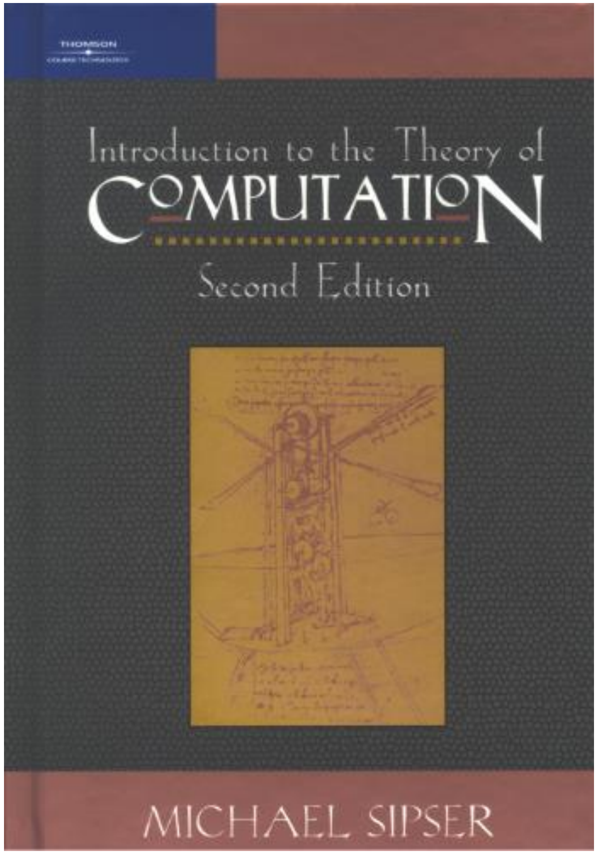
www.cs.virginia.edu/robins/theory_grad

Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?



- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

Theory of Computation (CS6160) - Textbook



Textbook:

Introduction to the Theory of Computation, by Michael Sipser (MIT), 2nd Edition, 2005

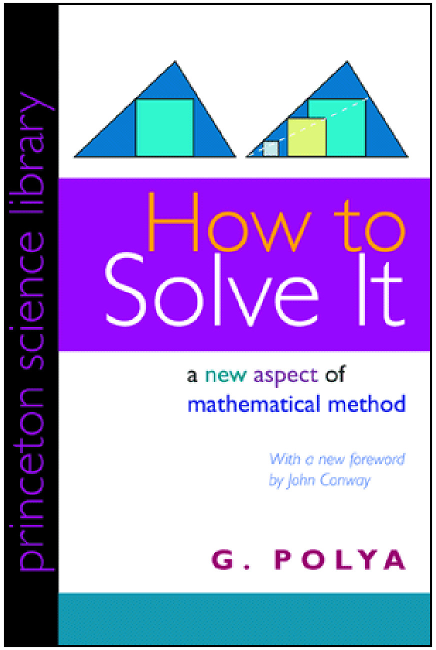
Good Articles / videos:

www.cs.virginia.edu/~robins/CS_readings.html

Theory of Computation (CS6160)

Required reading:

How to Solve It, by George Polya (MIT), Princeton University Press, 1945



- A classic on problem solving

George Polya (1887-1985)

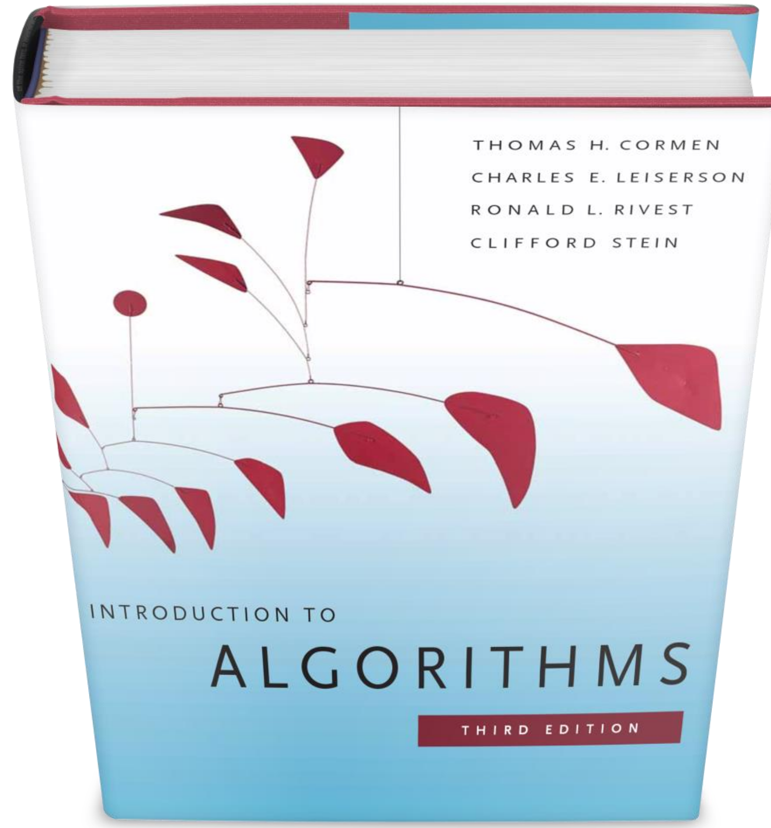
Theory of Computation (CS6160)

Good algorithms textbook:

Introduction to Algorithms

by Cormen et al (MIT)

Third Edition, 2009

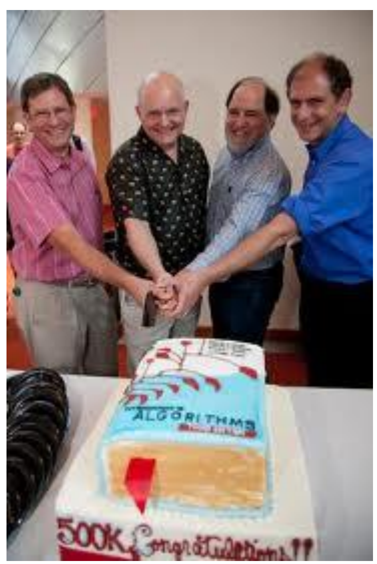


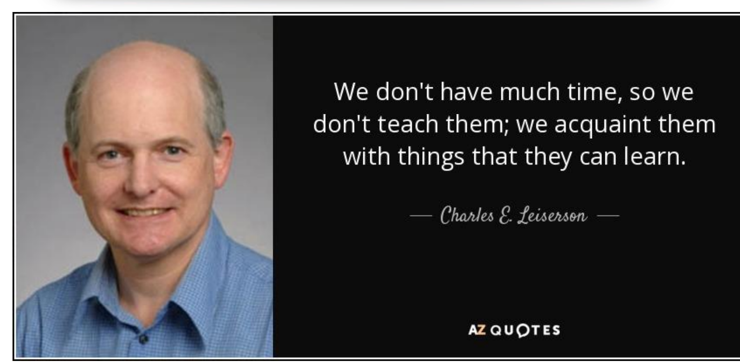
Thomas Cormen

Charles Leiserson

Ronald Rivest

Clifford Stein





computer science

Introduction to Algorithms

Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein

Third Edition

Some books on algorithms are rigorous but incomplete; others cover masses of material but lack rigor. *Introduction to Algorithms* uniquely combines rigor and comprehensiveness. The book covers a broad range of algorithms in depth, yet makes their design and analysis accessible to all levels of readers. Each chapter is relatively self-contained and can be used as a unit of study. The algorithms are described in English and in a pseudocode designed to be readable by anyone who has done a little programming. The explanations have been kept elementary without sacrificing depth of coverage or mathematical rigor.

The first edition became a widely used text in universities worldwide as well as the standard reference for professionals. The second edition featured new chapters on the role of algorithms, probabilistic analysis and randomized algorithms, and linear programming. The third edition has been revised and updated throughout. It includes two completely new chapters, on van Emde Boas trees and multithreaded algorithms, and substantial additions to the chapter on recurrences (now called "Divide-and-Conquer"). It features improved treatment of dynamic programming and greedy algorithms and a new notion of edge-based flow in the material on flow networks. Many new exercises and problems have been added for this edition.

As of the third edition, this textbook is published exclusively by the MIT Press.

Thomas H. Cormen is Professor of Computer Science and former Director of the Institute for Writing and Rhetoric at Dartmouth College. Charles E. Leiserson is Professor of Computer Science and Engineering at MIT. Ronald L. Rivest is Andrew and Erna Viterbi Professor of Electrical Engineering and Computer Science at MIT. Clifford Stein is Professor of Industrial Engineering and Operations Research at Columbia University.

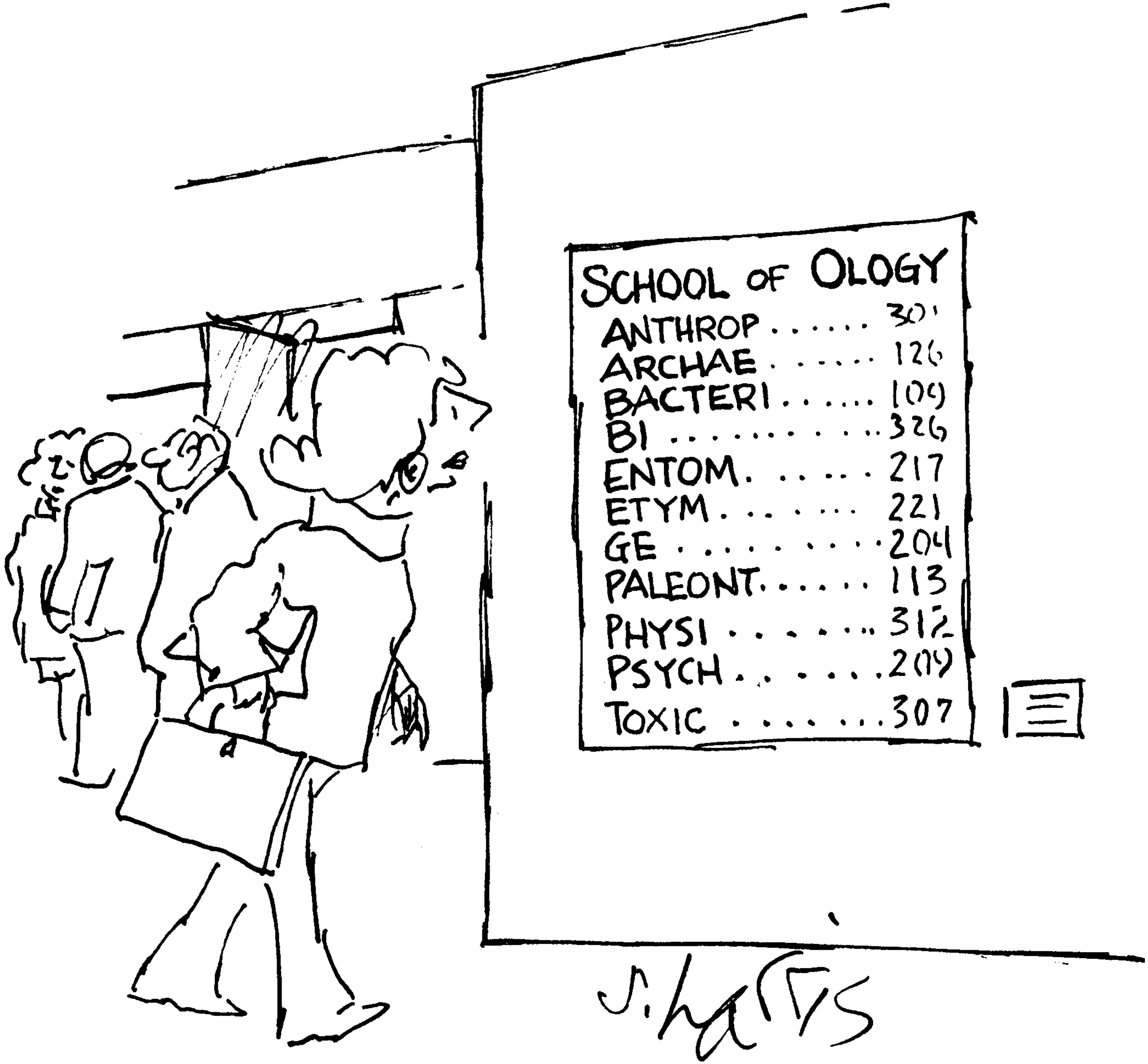
"In light of the explosive growth in the amount of data and the diversity of computing applications, efficient algorithms are needed now more than ever. This beautifully written, thoughtfully organized book is the definitive introductory book on the design and analysis of algorithms. The first half offers an effective method to teach and study algorithms; the second half then engages more advanced readers and curious students with compelling material on both the possibilities and the challenges in this fascinating field."
—Shang-Hua Teng, University of Southern California

"*Introduction to Algorithms*, the 'bible' of the field, is a comprehensive textbook covering the full spectrum of modern algorithms: from the fastest algorithms and data structures to polynomial-time algorithms for seemingly intractable problems, from classical algorithms in graph theory to special algorithms for string matching, computational geometry, and number theory. The revised third edition notably adds a chapter on van Emde Boas trees, one of the most useful data structures, and on multithreaded algorithms, a topic of increasing importance."
—Daniel Spielman, Department of Computer Science, Yale University

"As an educator and researcher in the field of algorithms for over two decades, I can unequivocally say that the Cormen book is the best textbook that I have ever seen on this subject. It offers an incisive, encyclopedic, and modern treatment of algorithms, and our department will continue to use it for teaching at both the graduate and undergraduate levels, as well as a reliable research reference."
—Gabriel Robins, Department of Computer Science, University of Virginia

978-0-262-03384-8

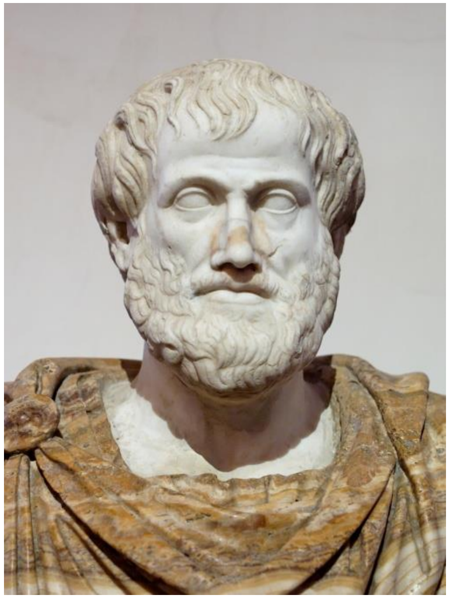
The MIT Press
Massachusetts Institute of Technology
Cambridge, Massachusetts 02142
http://mitpress.mit.edu

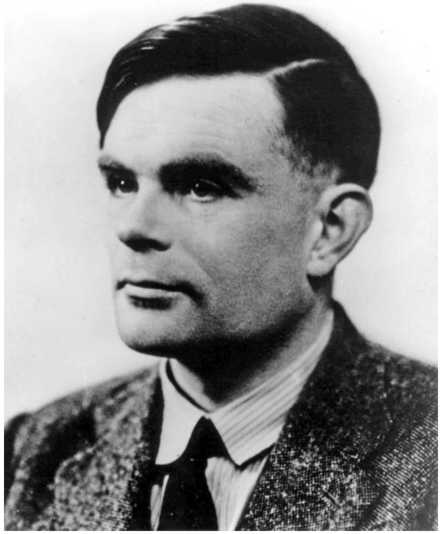

SCHOOL OF OLOGY
ANTHROP 301
ARCHAE 126
BACTERI 109
BI 326
ENTOM 217
ETYM 221
GE 204
PALEONT 113
PHYSI 312
PSYCH 209
TOXIC 307
J. Harris

Theory of Computation (CS6160) - Syllabus

A brief history of computing:

- Aristotle, Euclid, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, Euler, Gauss, Hamilton
- Boole, De Morgan, Babbage, Ada Agusta
- Venn, Carroll, Cantor, Hilbert, Russell
- Hardy, Ramanujan, Ramsey
- Godel, Church, Turing, von Neumann
- Shannon, Kleene, Chomsky, Hoare
- McCarthy, Erdos, Knuth, Backus, Dijkstra

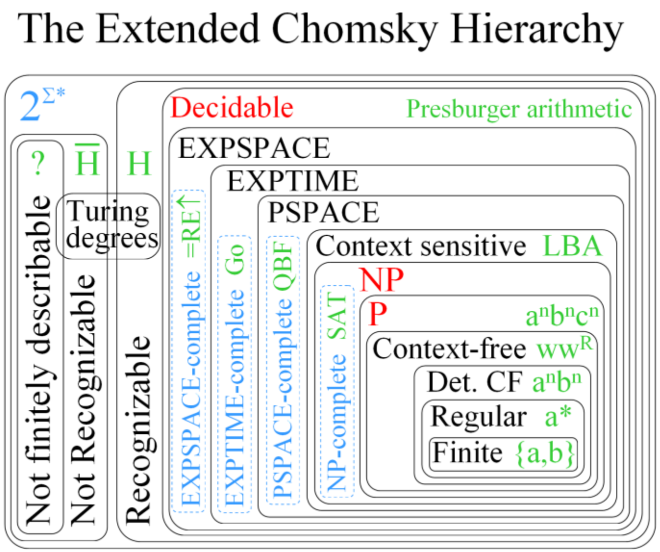


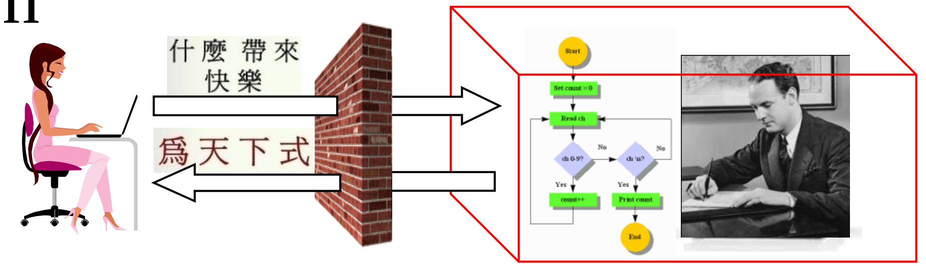


Theory of Computation Syllabus (continued)

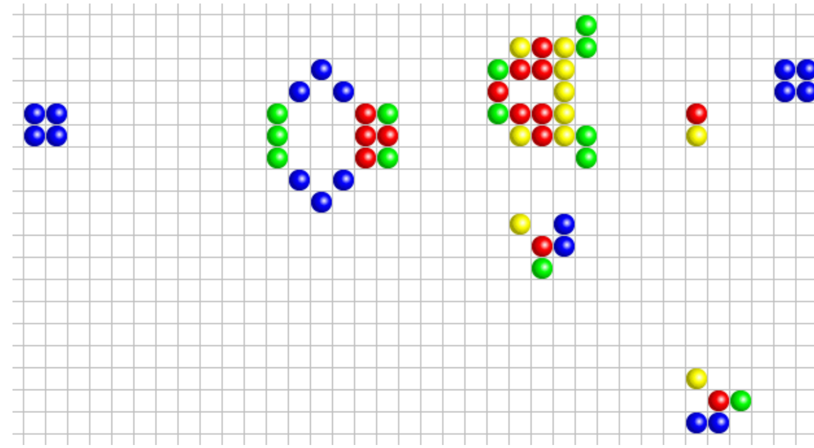
Beyond the Chomsky Hierarchy:

- Review of automata & languages
- Two-way and infinite automata
- Generalized finite automata
- State set minimization
- Deterministic context-free languages
- Counter automata and languages
- Ambiguity in grammars and languages
- Nondeterminism & alternation
- Context-sensitive grammars
- The Turing Test



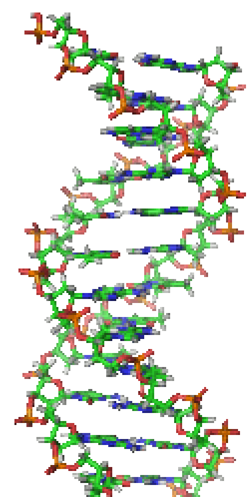


Theory of Computation Syllabus (continued)



Advanced Undecidability:

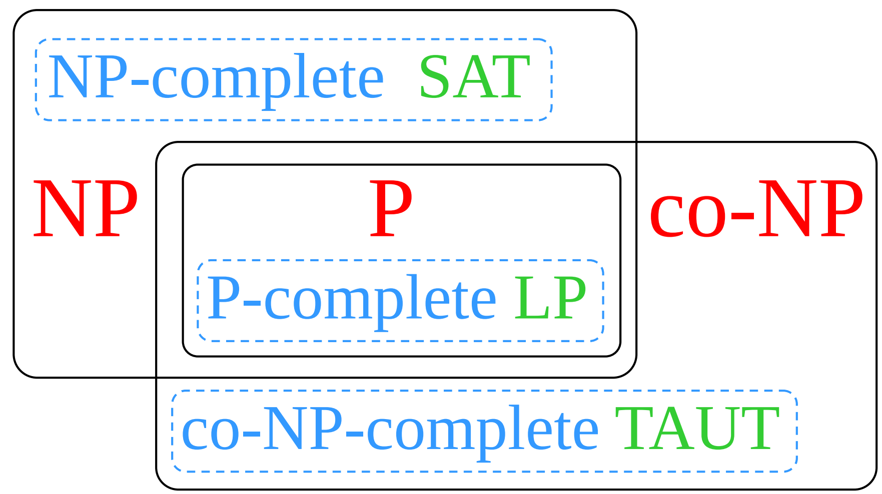
- Context-free intersections
- Post correspondence problem
- Linear-bounded automata
- Turing reducibilities
- Computational universality
- Conway's Game of Life
- Busy beaver problem
- The recursion theorem
- Oracles and relativizations
- Non-recognizability
- Turing degrees
- Randomness and entropy

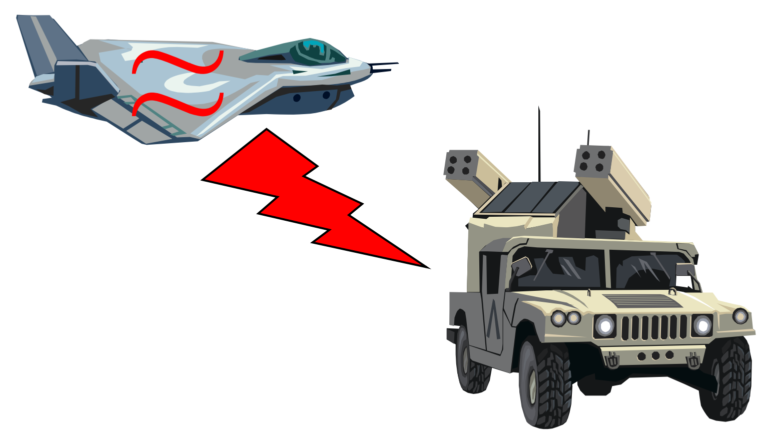


Theory of Computation Syllabus (continued)

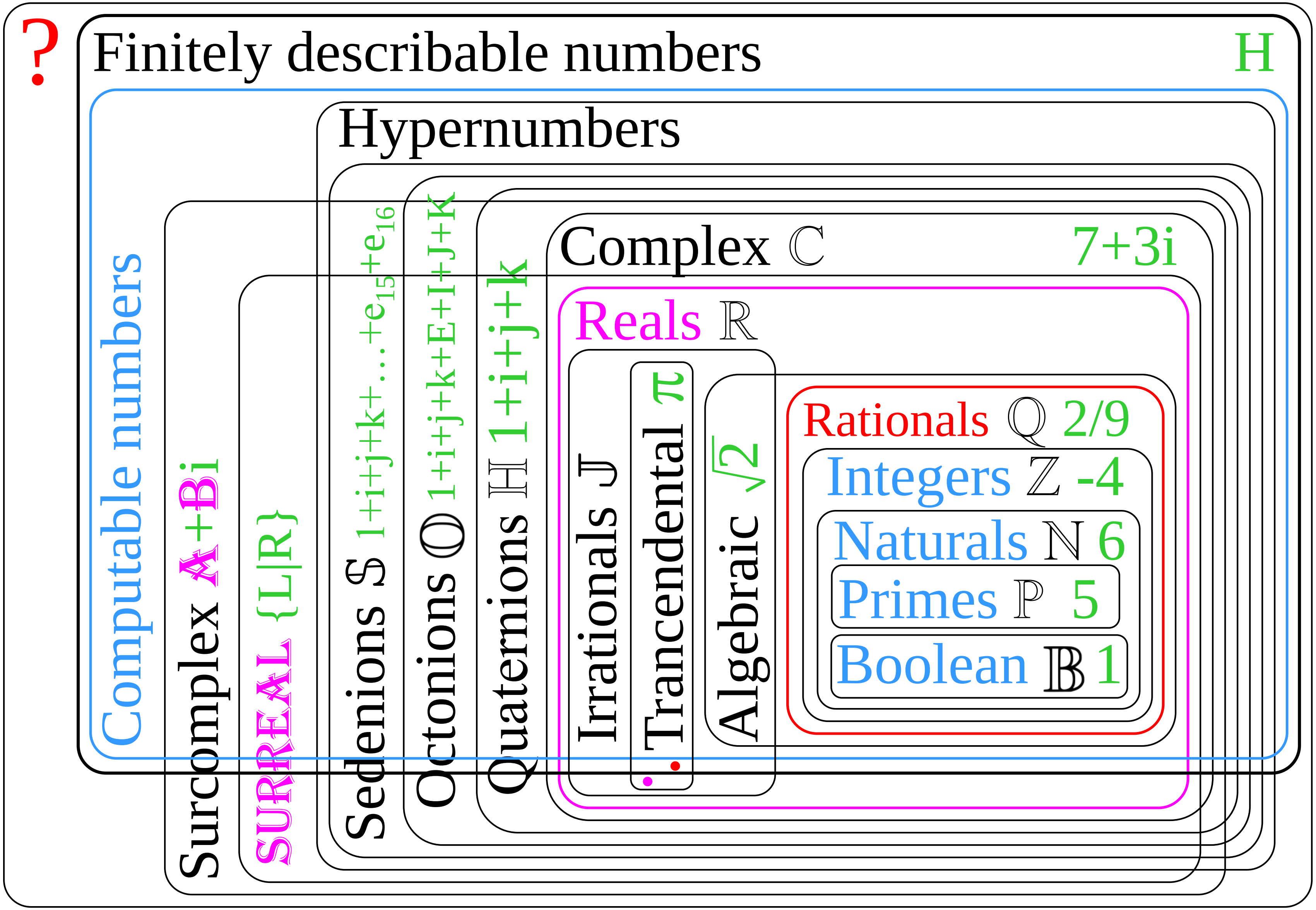
Advanced complexity theory:

- Time and space complexity classes
- Complexity hierarchies / separations
- Density and gap theorems
- NP-completeness reloaded
- Problem reductions
- Graph colorability
- Set cover problem
- Knapsacks and subset sums
- Savitch's Theorem
- PSPACE completeness
- NL completeness
- Approximation algorithms
- Zero-knowledge proofs





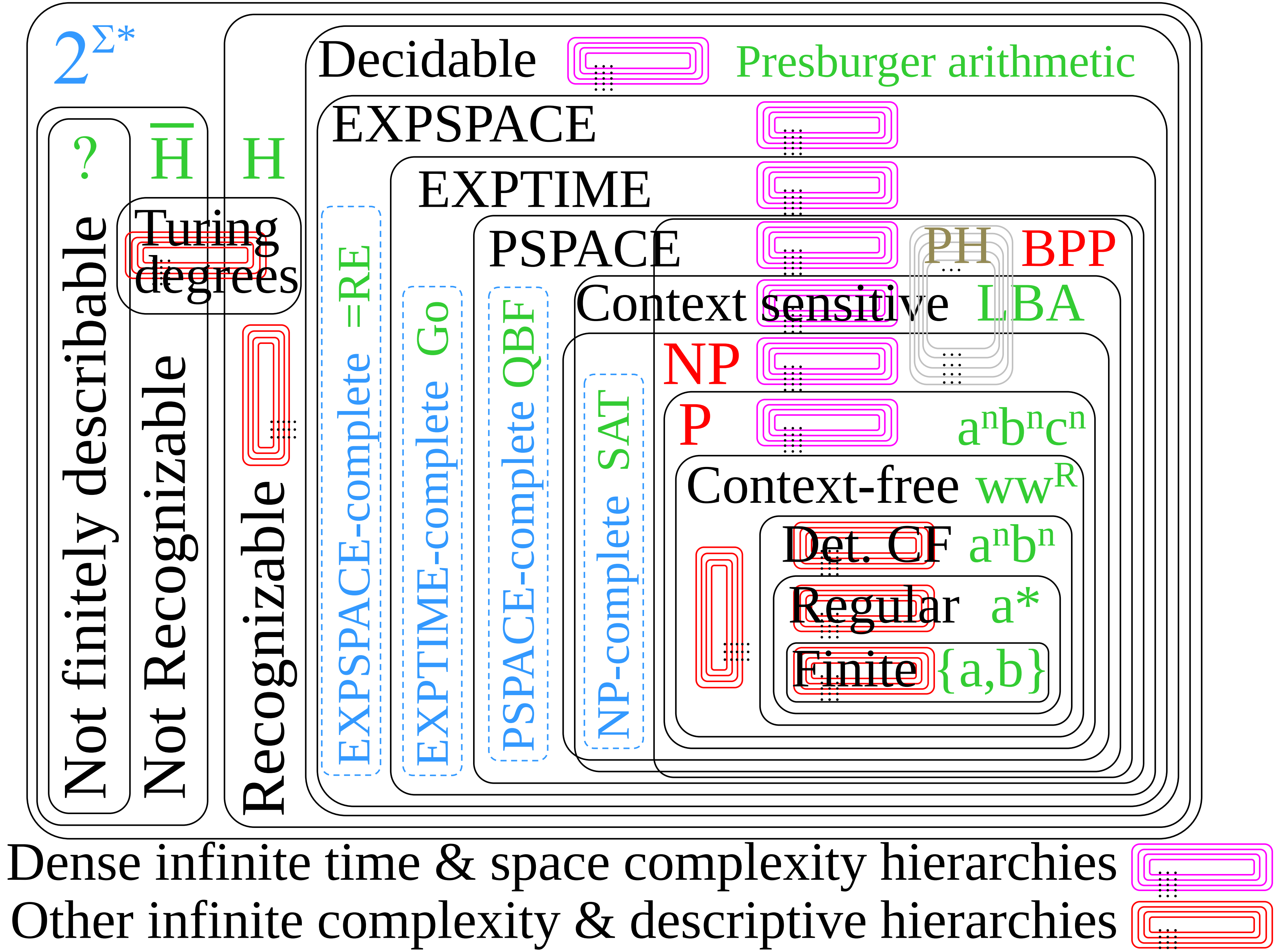
Generalized Numbers



Theorem: some real numbers are not finitely describable!

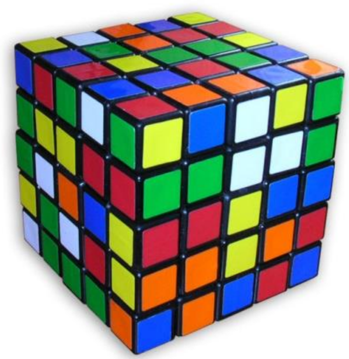
Theorem: some finitely describable real numbers are not computable!

The Extended Chomsky Hierarchy



Overarching Philosophy

- Focus on the "big picture" & "scientific method"
- Emphasis on problem solving & creativity
- Discuss applications & practice
- A primary objective: have fun!



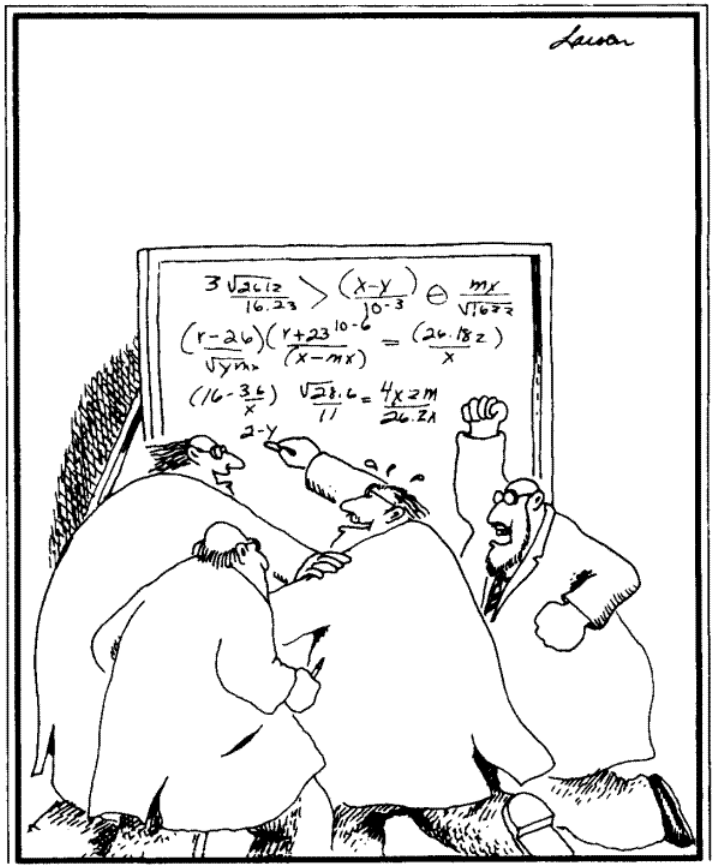
Prerequisites

- Some discrete math & algorithms knowledge
- Ideally, should have taken CS2102
- Course will "bootstrap" (albeit quickly) from first principles
- Critical: Tenacity, patience

Course Organization

- Exams: probably take home
  - Decide by vote
  - Flexible exam schedule
- Problem sets:
  - Lots of problem solving
  - Work in groups! (max size 6 people)
  - Not formally graded
  - Most exam questions will come from these sets!
- Homeworks:
  - Will come from problem sets
  - Formally graded
- Readings: papers / videos / books
- Extra credit problems
  - In class & take-home
  - Find mistakes in slides, handouts, etc.
- Course materials posted on Web site

  www.cs.virginia.edu/robins/theory_grad



"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"

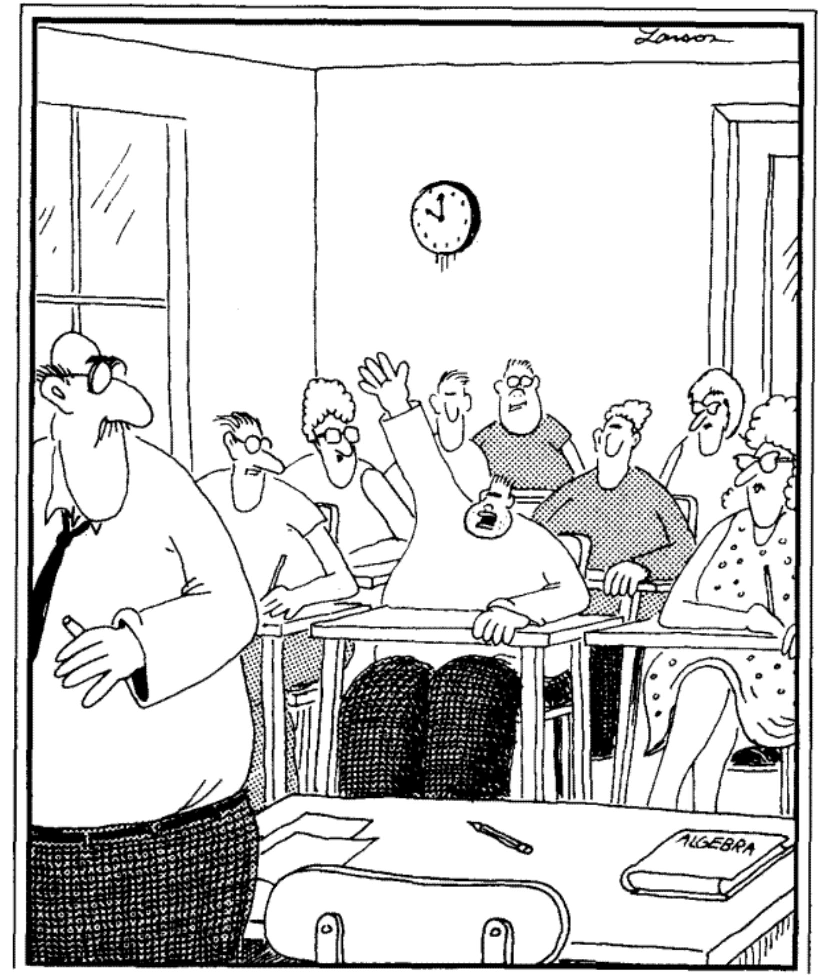
Grading Scheme

- Attendance 10%
- Homeworks 20%
- Readings 20%
- Midterm 25%
- Final 25%
- Extra credit 10%

Total: 110% +

Best strategy:

- Solve lots of problems!
- Do lots of readings / EC!
- “Ninety percent of success is just showing up.” – Woody Allen

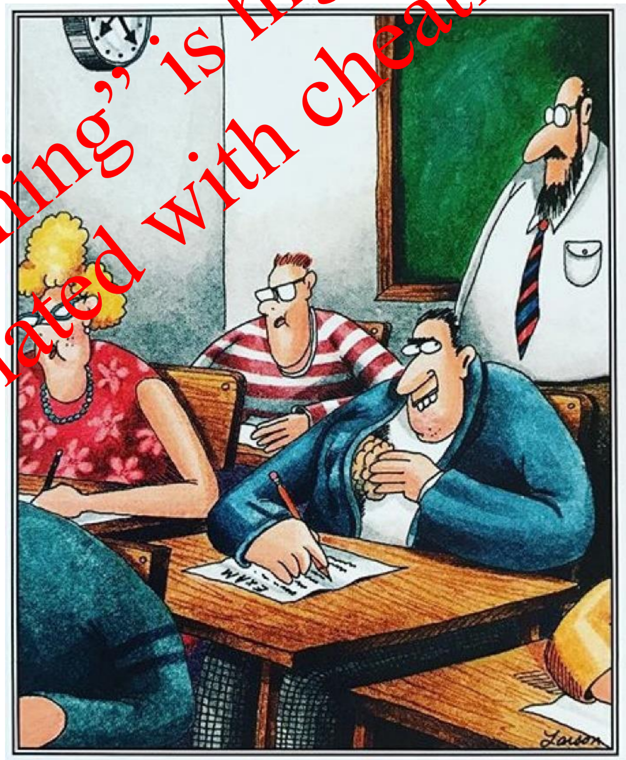


“Mr. Osborne, may I be excused? My brain is full.”

Cheating Policy

- Cheating / plagiarism is strictly prohibited
- Serious penalties for violators
- Please review the UVa Honor Code
- Examples of Cheating / plagiarism:
  - Copying of solutions from others / Web
  - Sharing of solutions with others / Web
  - Cutting-and-pasting from other people / Web
  - Copying article/book/movie reviews from people / Web
  - Other people / Web solving entire problems for you
  - Providing other people / Web with verbatim solutions
  - Submitting answers that you don't understand!
  - This list is not exhaustive!
- We have automated cheating / plagiarism detection tools!
- We encourage collaborations / brainstorming
- Lets keep it positive (and lets not play "gotcha")

"Cramming" is highly correlated with cheating!



Midway through the exam, Allen pulls out a bigger brain.

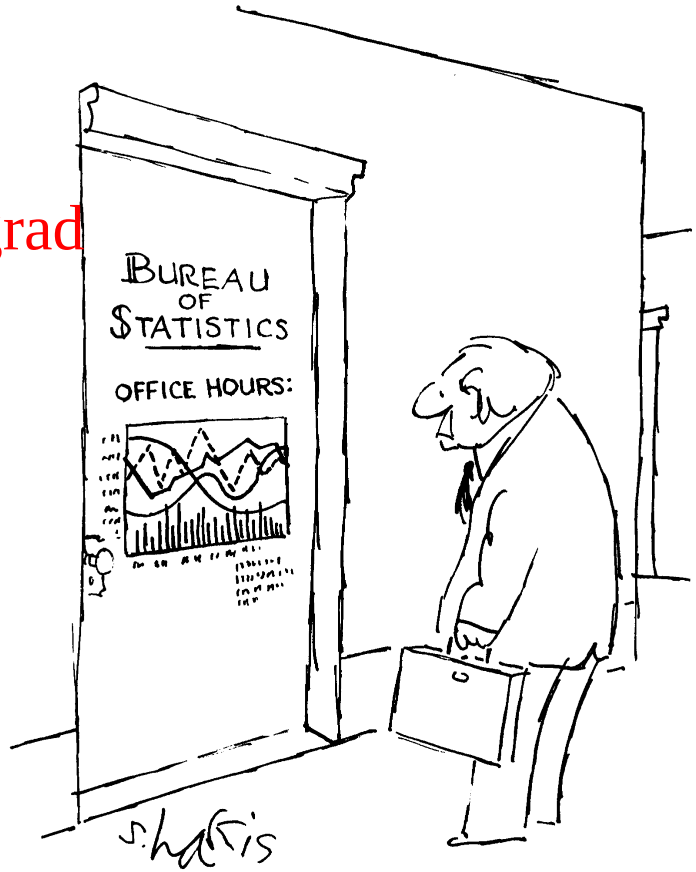
Contact Information

Professor	Gabriel Robins
Office:	406 Rice Hall
Phone:	(434) 982-2207
Email:	robins@cs.virginia.edu
Web:	www.cs.virginia.edu/robins

www.cs.virginia.edu/robins/theory_grad

Office hours: right after class

- Any other time
- By email (preferred)
- By appointment
- Q&A blog posted on class Web site



Course Readings

www.cs.virginia.edu/robins/CS_readings.html

Goal: broad exposure to lots of cool ideas & technologies!

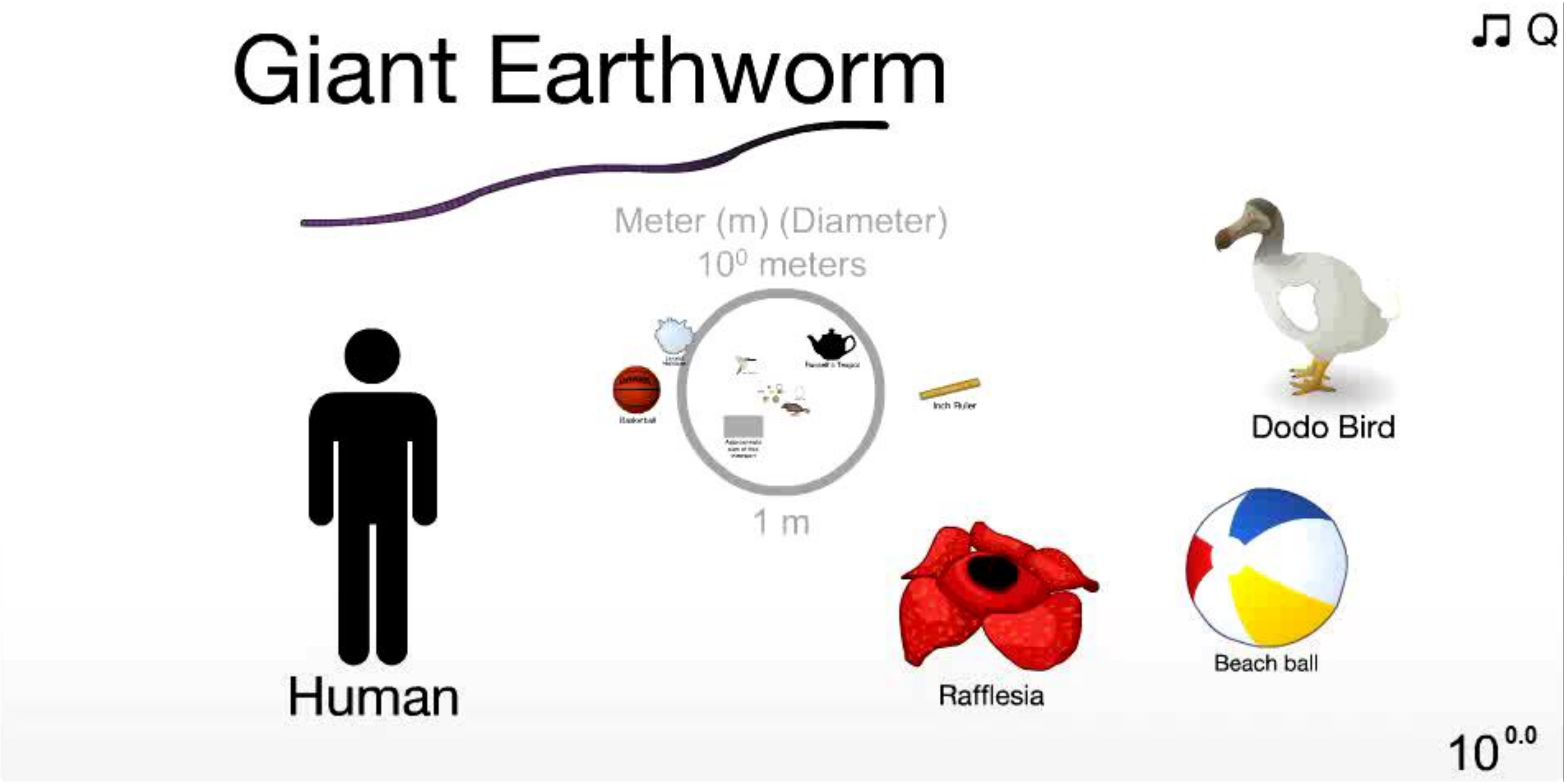
- Required: total of at least 36 items over the semester
- Diversity: minimums in each of 3 categories:
  1. Minimum of 15 videos
  2. Minimum of 15 papers / Web sites
  3. Minimum of 6 books
- More than 36 total is even better! (extra credit)
- Some required items in each category
  - Remaining "elective" items should be a diverse mix
- Submissions form: www.cs.virginia.edu/robins/theory_homework_grad

Required Readings
www.cs.virginia.edu/robins/CS_readings.html

- Required videos:
  - Last Lecture, Randy Pausch, 2007
  - Time Management, Randy Pausch, 2007
  - Powers of Ten, Charles and Ray Eames, 1977

Required Reading

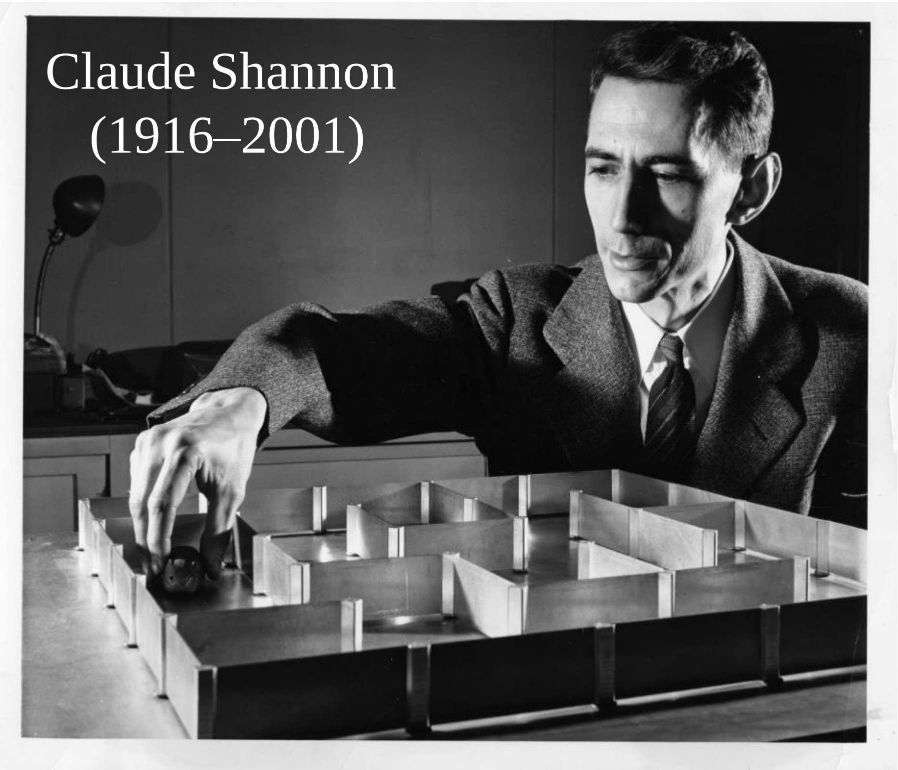
- "Scale of the Universe", Cary and Michael Huang, 2012



- 10^{-24} to 10^{26} meters ⇒ 50 orders of magnitude!

Required Readings
www.cs.virginia.edu/robins/CS_readings.html

- More required videos:
  - Claude Shannon - Father of the Information Age, UCTV
  - The Pattern Behind Self-Deception, Michael Shermer, 2010

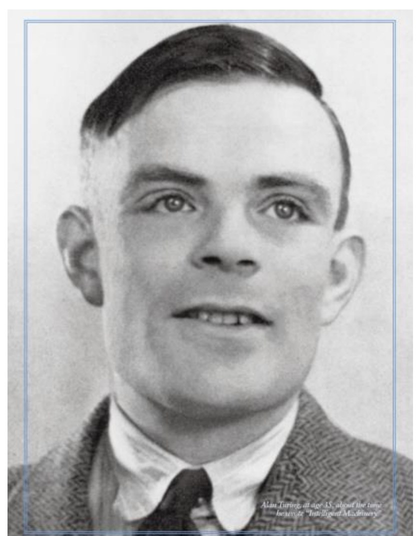


Required Readings

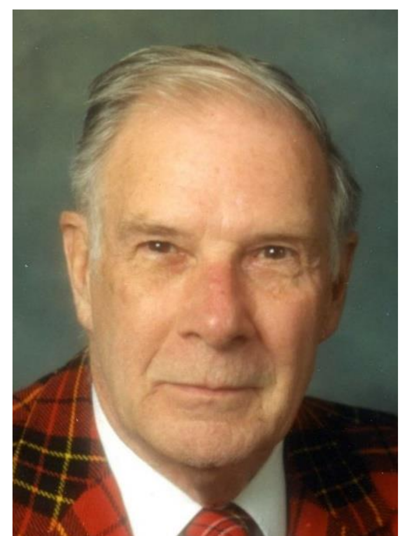
www.cs.virginia.edu/robins/CS_readings.html

- Required articles:
  - Decoding an Ancient Computer, Freeth, 2009
  - Alan Turing's Forgotten Ideas, Copeland and Proudfoot, 1999
  - You and Your Research, Richard Hamming, 1986
  - Who Can Name the Bigger Number, Scott Aaronson, 1999

Antikythera computer, 200BC



Alan Turing



Richard Hamming

Scott Aaronson

Extra credit!

Basic Concepts and Notation

Gabriel Robins

Required reading

"When I use a word," Humpty Dumpty said, in a rather scornful tone, "it means just what I choose it to mean -- neither more nor less."

A **set** is formally an undefined term, but intuitively it is a (possibly empty) collection of arbitrary objects. A set is usually denoted by curly braces and some (optional) restrictions. Examples of sets are {1,2,3}, {hi, there}, and {k | k is a perfect square}. The symbol ∈ denotes set **membership**, while the symbol ∉ denotes set **non-membership**; for example, 7∈{p | p prime} states that 7 is a prime number, while q∉{0,2,4,6,...} states that q is not an even number. Some **common sets** are denoted by special notation:

The **natural numbers**: $\mathbb{N} = \{1,2,3,...\}$

The **integers**: $\mathbb{Z} = \{...,-3,-2,-1,0,1,2,3,...\}$

The **rational numbers**: $\mathbb{Q} = \{ \frac{a}{b} \mid a,b \in \mathbb{Z}, b \neq 0\}$

The **real numbers**: $\mathbb{R} = \{x \mid x \text{ is a real number}\}$

The **empty set**: $\varnothing = \{\}$

http://www.cs.virginia.edu/robins/cs6160/basics.pdf

JOHANNES KEPLER'S UPHILL BATTLE

Discrete Math Review Slides

Required reading

Symbolic Logic

Def: *proposition* - statement either true (T) or false (F)

Ex: 1 + 1 = 2

2 + 2 = 3

3 < 7

x + 4 = 5

"today is Monday"

Boolean Functions

- "and" $\wedge$
- "or" $\vee$
- "not" $\neg$
- "xor" $\oplus$
- "nand"
- "nor"
- "implication" $\Rightarrow$
- "equivalence" $\Leftrightarrow$

Logical Implication

- "implies" $\Rightarrow$

Truth table:

p	q	p⇒q
T	T	T
T	F	F
F	T	T
F	F	T

Ex: $(x<0) \wedge (x>0) \Rightarrow (x=0)$

$1 < x < y \Rightarrow x^3 < y^3$

"today is Sunday" $\Rightarrow$ 1+1=3

Logical Equivalence

- "biconditional" $\Leftrightarrow$

or "if and only if" ("iff")

or "necessary and sufficient"

or "logically equivalent" $\equiv$

Truth table:

p	q	p⇔q
T	T	T
T	F	F
F	T	F
F	F	T

Ex: $p \Leftrightarrow p$

$[(x=0) \vee (y=0)] \Leftrightarrow (xy=0)$

$\min(x,y)=\max(x,y) \Leftrightarrow x=y$

Predicates

Def: *predicate* - a function or formula involving some variables

Ex: let P(x) = "x > 3"

x is the variable

"x>3" is the predicate

P(5)

P(1)

Ex: Q(x,y,z) = " $x^2+y^2=z^2$ "

Q(2,3,4)

Q(3,4,5)

Quantifiers

- Universal: "for all" $\forall$

$\forall x\ P(x)$

$\Leftrightarrow P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \ldots$

Ex: $\forall x \quad x < x + 1$

$\forall x \quad x < x^3$

- Existential: "there exists" $\exists$

$\exists x\ P(x)$

$\Leftrightarrow P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots$

Ex: $\exists x \quad x = x^2$

$\exists x \quad x < x - 1$

Combinations:

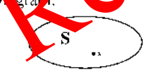
$\forall x \exists y \quad y>x$

Sets

Def: *set* - an unordered collection of elements

Ex: {1, 2, 3} or {hi, there}

Venn Diagram:



Def: two sets are *equal* iff they contain the same elements

Ex: {1, 2, 3} = {2, 3, 1}

{0} ≠ {1}

{3, 5} = {3, 5, 3, 3, 5}

Common Sets

Naturals: N = {1, 2, 3, 4, ...}

Integers: Z = {...,-2, -1, 0, 1, 2,...}

Rationals: Q = { $\frac{a}{b}$ | a,b∈Z, b≠0}

Reals: ℜ = {x | x a real #}

Empty set: Ø = {}

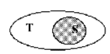
Z^+ = non-negative integers

$ℜ^-$ = non-positive reals, etc.

Subsets

- Subset notation: ⊆

S ⊆ T ⇔ (x∈S ⇒ x∈T)



- Proper subset: ⊂

S ⊂ T ⇔ ((S ⊆ T) ^ (S≠T))

S=T ⇔ ((T ⊆ S) ^ (S ⊆ T))

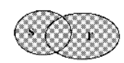
∀S Ø ⊆ S

∀S S ⊆ S

Union / Intersection

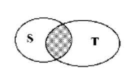
- Union: ∪

S∪T={x | x∈S ∨ x∈T}



- Intersection: ∩

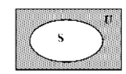
S∩T={x | x∈S ∧ x∈T}



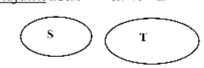
Complement

- Universal set: U (everything)
- Set complement: S' or $\bar{S}$

S' = {x | x∉S} = U - S



- Disjoint sets: S∩T=Ø

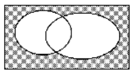


S - T= S ∩ T'

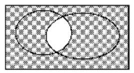
S - S = Ø

DeMorgan's Laws

(S∪T)' = S'∩T'



(S∩T)' = S'∪T'



Boolean logic version:

(X∧Y)'=X'∨Y'

(X∨Y)'=X'∧Y'

Function Types

- One-to-one function: "1-1"

a,b∈S ^ a≠b ⇒ f(a)≠f(b)

Ex: f: ℜ→ℜ, f(x)=2x is 1-1

g(x)= x^2 is not 1-1

- Onto function:

∀ t ∈T ∃ s∈S ∋ f(s)=t

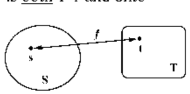
Ex: f: Z→Z, f(x)=13-x is onto

g(x)= x^2 is not onto

1-to-1 Correspondence

- 1-to-1 correspondence: f:S↔T

f is both 1-1 and onto



Ex: f: ℜ↔ℜ ∋ f(x)=x (identity)

h: N↔Z ∋ h(x)= $\frac{x-1}{2}$, x odd,

$\frac{-x}{2}$, x even.

Generalized Cardinality

- S is at least as large as T:

|S ≥ T| ⇒ ∃ f:S→T, f onto

i.e., "S covers T"

Ex: r:ℜ→Z, r(x)=round(x)

⇒ |ℜ|≥|Z|

- S and T have same cardinality:

|S =|T| ⇒ |S|≥|T| ^ |T|≥|S

or

∃ 1-1 correspondence S↔T

- Generalizes finite cardinality:

{1, 2, 3, 4, 5} ≥ {a, b, c}

Infinite Sets

- Infinite set: |S > k ∀k∈Z

or

∃ 1-1 corres. f:S↔T, S⊂T

Ex: {p | p prime}, ℜ

- Countable set: |S ≤ N|

Ex: Ø, {p | p prime}, N, Z

- S is strictly smaller than T:

|S < |T| ⇒ |S|≤|T| ^ |S|≠|T|

- Uncountable set: |N| < |S|

Ex: |N| < ℜ

|N| < [0,1] = {x | x∈ℜ, 0≤x≤1}

Countability of Q

Thm: ∃ 1-1 correspondence Q↔N

Pf (dove-tailing):

6	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$ ...
5	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$	$\frac{6}{5}$ ...
4	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$	$\frac{6}{4}$ ...
3	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{6}{3}$ ...
2	$\frac{1}{2}$	$\frac{2}{2}$	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	$\frac{6}{2}$ ...
1	$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{1}$	$\frac{4}{1}$	$\frac{5}{1}$	$\frac{6}{1}$ ...
	1	2	3	4	5	6

Uncountability of ℜ

Thm: |ℜ|> N|

Pf (diagonalization):

Assume ∃ 1-1 corres. f: ℜ↔N

Construct X ∈ ℜ:

f(1)=2.718281828... →
f(2)=1.414213562... →
f(3)=1.618033989... →

X = 0. ... ≠f(K) ∀K∈N

⇒ f not a 1-1 correspondence

→ contradiction

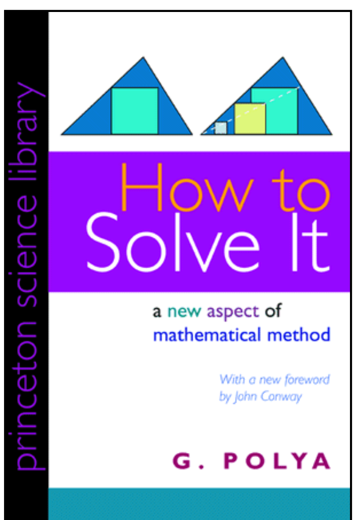
⇒ ℜ is uncountable

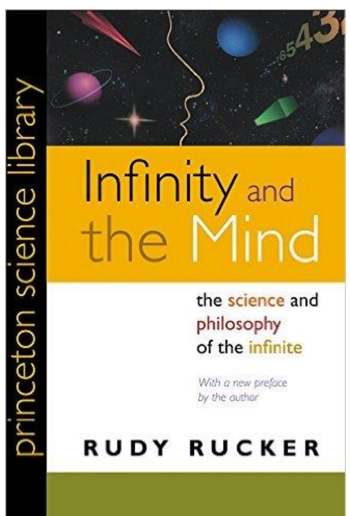
http://www.cs.virginia.edu/robins/cs6160/discrete_math_review_slides.pdf

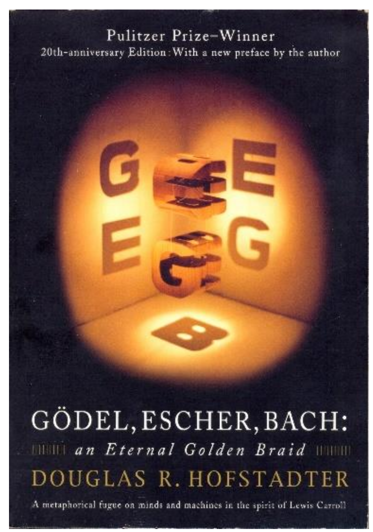
Required Readings

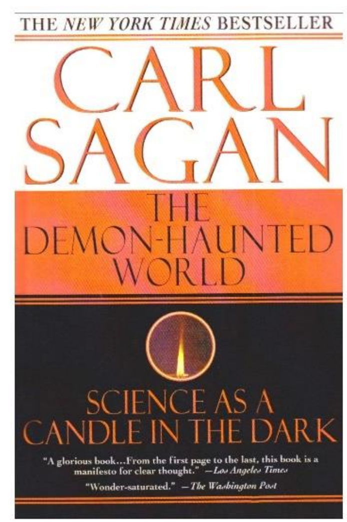
www.cs.virginia.edu/robins/CS_readings.html

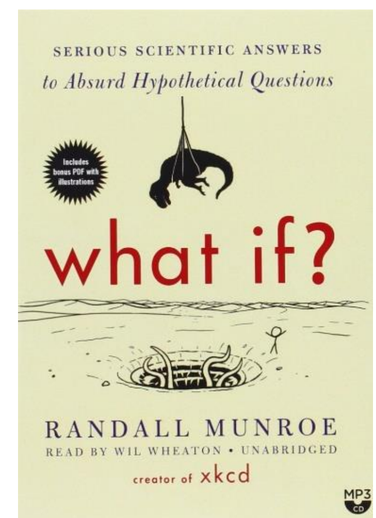
- Required books:
  - “How to Solve It”, Polya, 1957
  - “Infinity and the Mind”, Rucker, 1995
  - “Godel, Escher, Bach”, Hofstadter, 1979
  - “The Demon-Haunted World”, Sagan, 2009
  - “What If”, Munroe, 2014











Required Readings

www.cs.virginia.edu/robins/CS_readings.html

- Remaining videos / articles / books are "electives"
- At least 2 submissions per week (due 11:59pm Mon)
- At most 2 submissions per day
- This policy is intended to help you avoid "cramming"
- "Cramming" is highly correlated with cheating!
- Length: 1-2 paragraphs per article / video

  1-2 pages per book
- Books are worth more credit than articles / videos
- Additional readings beyond 36 are welcome! (extra credit)
- Submissions form: www.cs.virginia.edu/robins/theory_homework_grad

Other “Elective” Readings
www.cs.virginia.edu/robins/CS_readings.html

- Theory and Algorithms:
  - Who Can Name the Bigger Number, Scott Aaronson, 1999
  - The Limits of Reason, Gregory Chaitin, Scientific American, March 2006, pp. 74-81.
  - Breaking Intractability, Joseph Traub and Henryk Wozniakowski, Scientific American, January 1994, pp. 102-107.
  - Confronting Science's Logical Limits, John Casti, Scientific American, October 1996, pp. 102-105.
  - Go Forth and Replicate, Moshe Sipper and James Reggia, Scientific American, August 2001, pp. 34-43.
  - The Science Behind Sudoku, Jean-Paul Delahaye, Scientific American, June 2006, pp. 80-87.
  - The Traveler's Dilemma, Kaushik Basu, Scientific American, June 2007, pp. 90-95.

Other "Elective" Readings
www.cs.virginia.edu/robins/CS_readings.html

- Biological Computing:
  - Computing with DNA, Leonard Adleman, Scientific American, August 1998, pp. 54-61.
  - Bringing DNA Computing to Life, Ehud Shapiro and Yaakov Benenson, Scientific American, May 2006, pp. 44-51.
  - Engineering Life: Building a FAB for Biology, David Baker et al., Scientific American, June 2006, pp. 44-51.
  - Big Lab on a Tiny Chip, Charles Choi, Scientific American, October 2007, pp. 100-103.
  - DNA Computers for Work and Play, Macdonald et al, Scientific American, November 2007, pp. 84-91.

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

- Quantum Computing:
  - Quantum Mechanical Computers, Seth Lloyd, Scientific American, 1997, pp. 98-104.
  - Quantum Computing with Molecules, Gershenfeld and Chuang, Scientific American, June 1998, pp. 66-71.
  - Black Hole Computers, Seth Lloyd and Jack Ng, Scientific American, November 2004, pp. 52-61.
  - Computing with Quantum Knots, Graham Collins, Scientific American, April 2006, pp. 56-63.
  - The Limits of Quantum Computers, Scott Aaronson, Scientific American, March 2008, pp. 62-69.
  - Quantum Computing with Ions, Monroe and Wineland, Scientific American, August 2008, pp. 64-71.

Other "Elective" Readings
www.cs.virginia.edu/robins/CS_readings.html

- History of Computing:
  - The Origins of Computing, Campbell-Kelly, Scientific American, September 2009, pp. 62-69.
  - Ada and the First Computer, Eugene Kim and Betty Toole, Scientific American, April 1999, pp. 76-81.
- Security and Privacy:
  - Malware Goes Mobile, Mikko Hypponen, Scientific American, November 2006, pp. 70-77.
  - RFID Powder, Tim Hornyak, Scientific American, February 2008, pp. 68-71.
  - Can Phishing be Foiled, Lorrie Cranor, Scientific American, December 2008, pp. 104-110.

Other “Elective” Readings
www.cs.virginia.edu/robins/CS_readings.html

- Future of Computing:
  - Microprocessors in 2020, David Patterson, Scientific American, September 1995, pp. 62-67.
  - Computing Without Clocks, Ivan Sutherland and Jo Ebergen, Scientific American, August 2002, pp. 62-69.
  - Making Silicon Lase, Bahram Jalali, Scientific American, February 2007, pp. 58-65.
  - A Robot in Every Home, Bill Gates, Scientific Am, January 2007, pp. 58-65.
  - Ballbots, Ralph Hollis, Scientific American, October 2006, pp. 72-77.
  - Dependable Software by Design, Daniel Jackson, Scientific American, June 2006, pp. 68-75.
  - Not Tonight Dear - I Have to Reboot, Charles Choi, Scientific American, March 2008, pp. 94-97.
  - Self-Powered Nanotech, Zhong Lin Wang, Scientific American, January 2008, pp. 82-87.

Other "Elective" Readings

www.cs.virginia.edu/robins/CS_readings.html

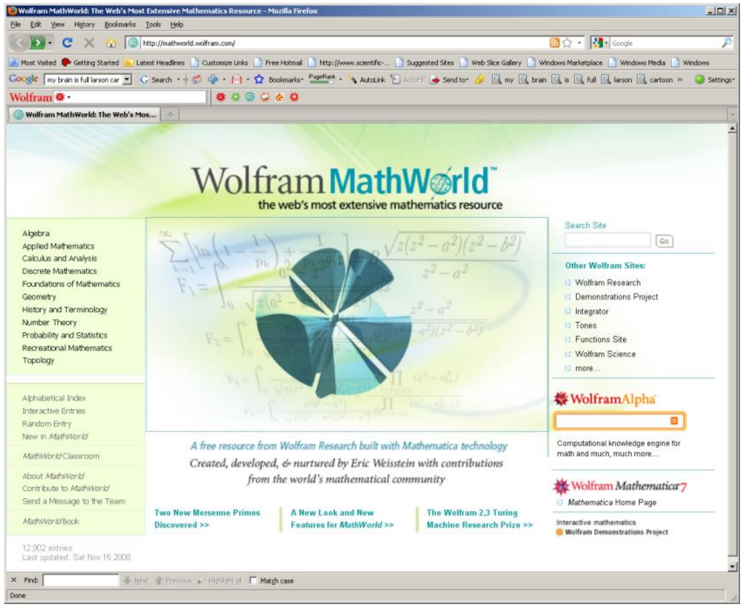
- The Web:
  - The Semantic Web in Action, Lee Feigenbaum et al., Scientific American, December 2007, pp. 90-97.
  - Web Science Emerges, Nigel Shadbolt and Tim Berners-Lee, Scientific American, October 2008, pp. 76-81.
- The Wikipedia Computer Science Portal:
  - Theory of computation and Automata theory
  - Formal languages and grammars
  - Chomsky hierarchy and the Complexity Zoo
  - Regular, context-free &Turing-decidable languages
  - Finite & pushdown automata; Turing machines
  - Computational complexity
  - List of data structures and algorithms

Submissions form: www.cs.virginia.edu/robins/theory_homework_grad

Other “Elective” Readings

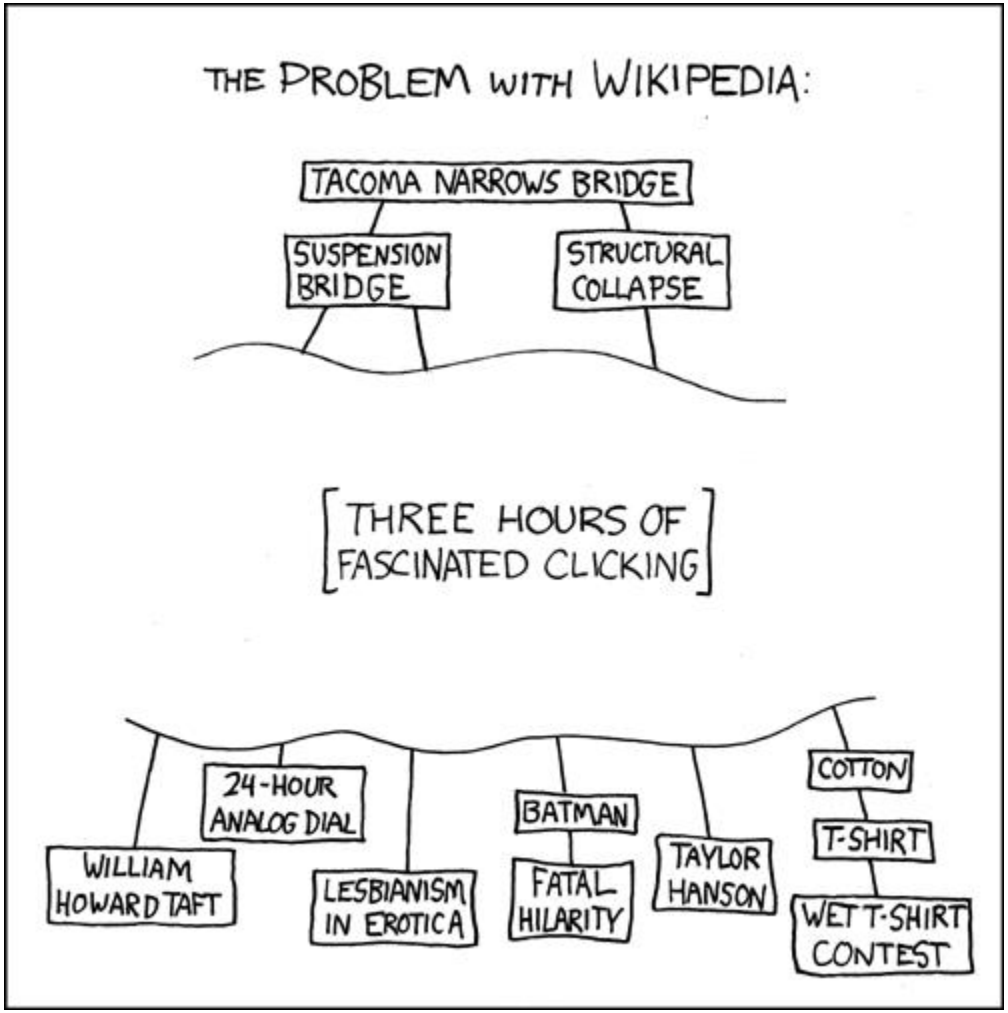
www.cs.virginia.edu/robins/CS_readings.html

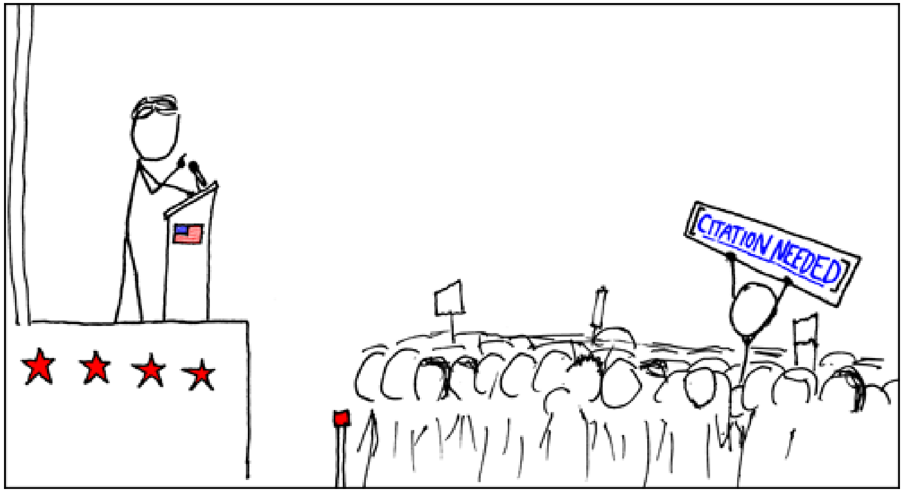
- The Wikipedia Math Portal:
  - Problem solving
  - List of Mathematical lists
  - Sets and Infinity
  - Discrete mathematics
  - Proof techniques and list of proofs
  - Information theory & randomness
  - Game theory
- Mathematica's “Math World”

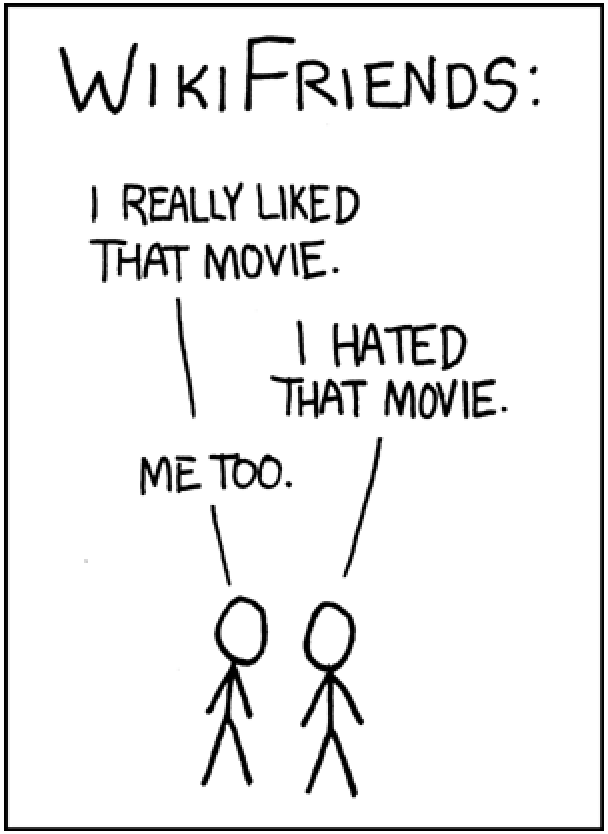


Submissions Web form:

www.cs.virginia.edu/robins/theory_homework_grad

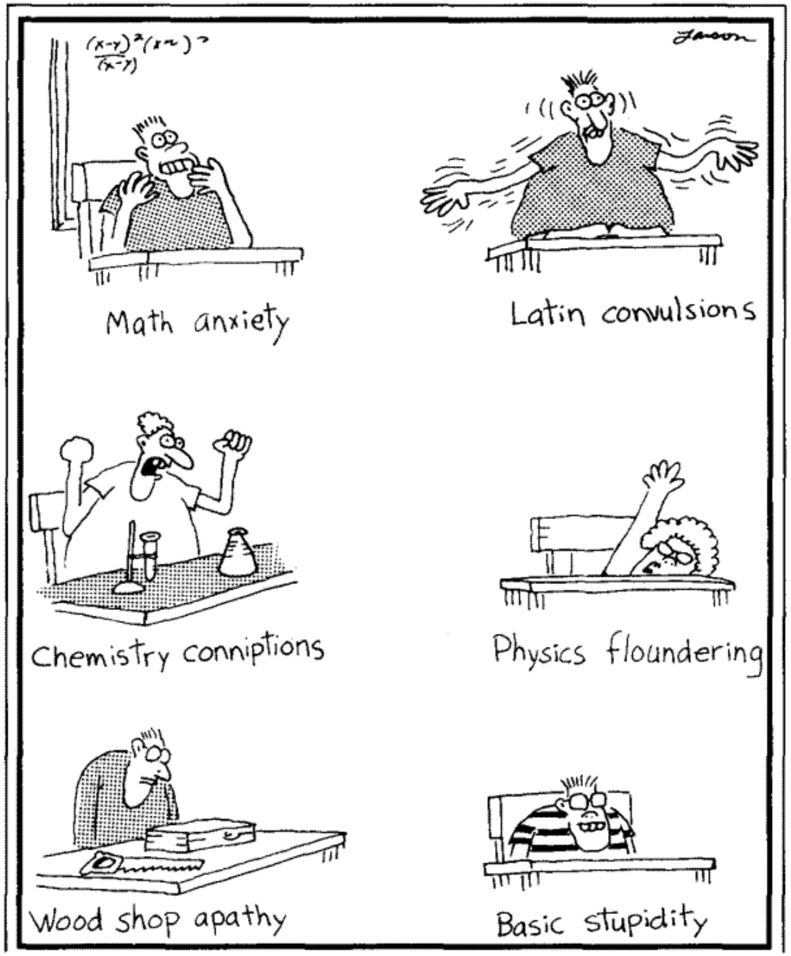

THE PROBLEM WITH WIKIPEDIA:
TACOMA NARROWS BRIDGE
SUSPENSION BRIDGE
STRUCTURAL COLLAPSE
THREE HOURS OF FASCINATED CLICKING
24-HOUR ANALOG DIAL
WILLIAM HOWARD TAFT
LESBIANISM IN EROTICA
BATMAN
FATAL HILARITY
TAYLOR HANSON
COTTON
T-SHIRT
WET T-SHIRT CONTEST


CITATION NEEDED


WIKIFRIENDS:
I REALLY LIKED THAT MOVIE.
I HATED THAT MOVIE.
ME TOO.

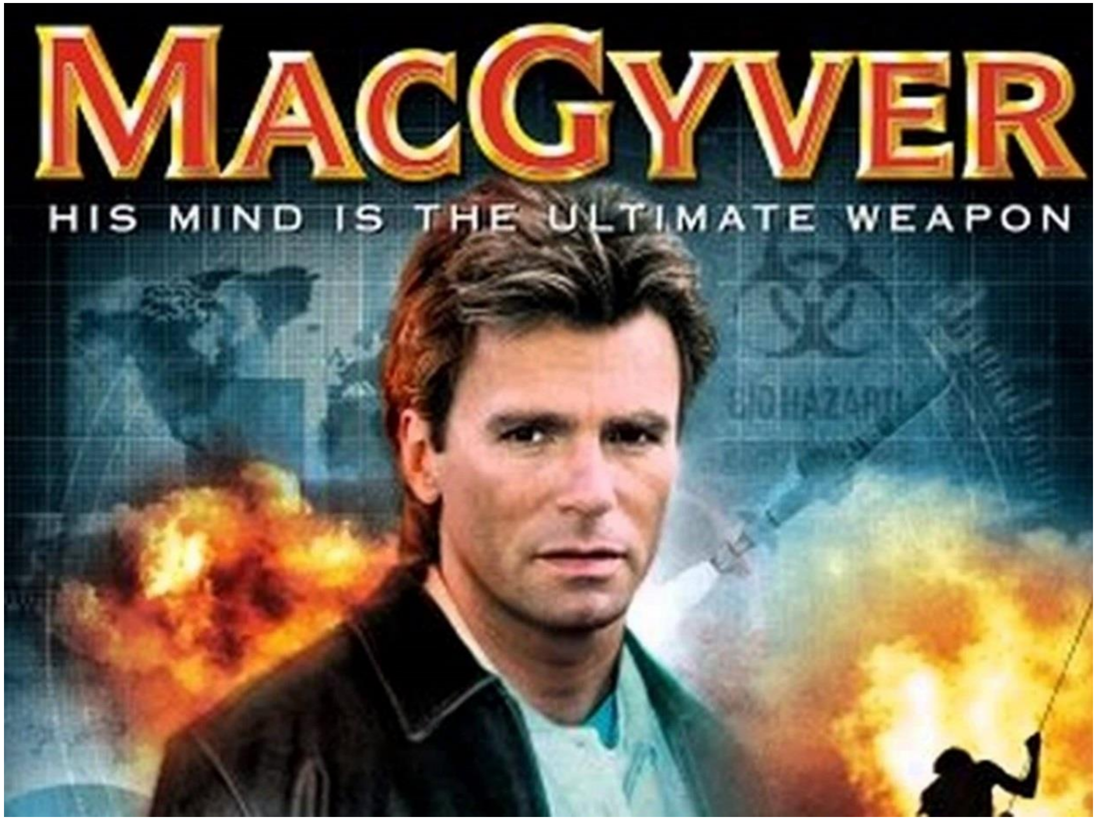
Good Advice

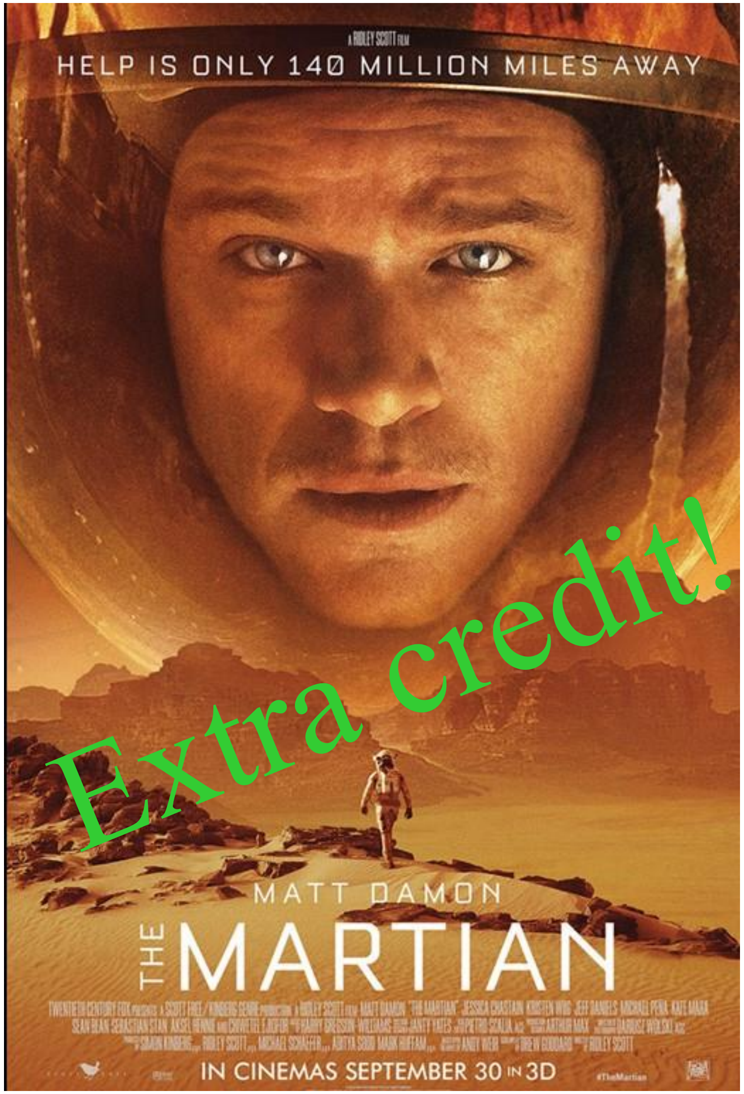
- Ask questions ASAP
- Solve problems ASAP
- Work in study groups
- Do not fall behind
- "Cramming" won't work
- Do lots of extra credit
- Attend every lecture
- Visit class Website often
- Solve lots of problems



Classroom afflictions

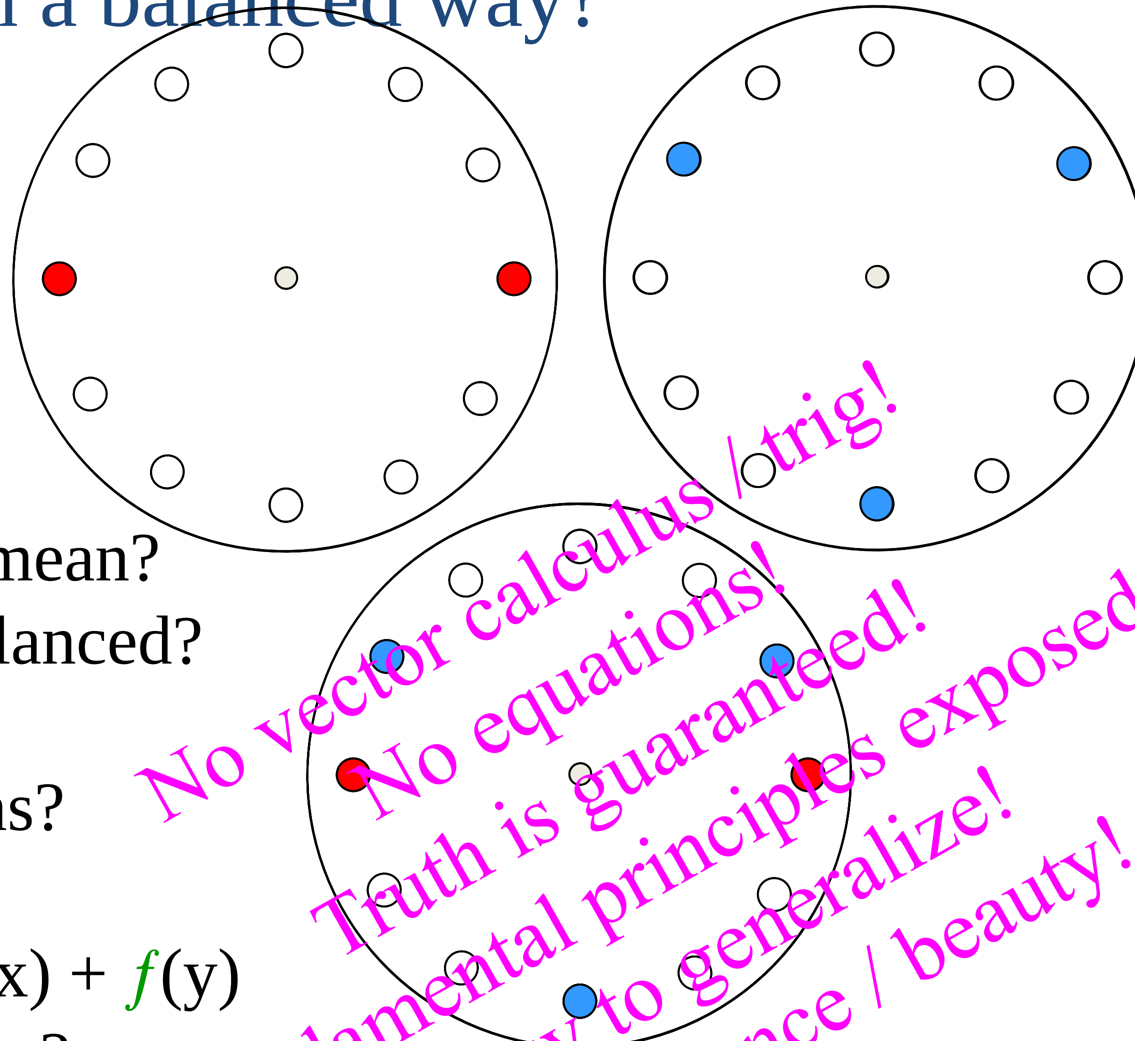
Goal: Become a more effective problem solver!





Submissions Web form: www.cs.virginia.edu/robins/theory_homework_grad

Problem: Can 5 test tubes be spun simultaneously in a 12-hole centrifuge in a balanced way?



- What does "balanced" mean?
- Why are 3 test tubes balanced?
- Symmetry!
- Can you merge solutions?
- Superposition!
- Linearity! $f(x + y) = f(x) + f(y)$
- Can you spin 7 test tubes?
- Complementarity!
- Empirical testing…

No vector calculus / trig!
No equations!
Truth is guaranteed!
Fundamental principles exposed!
Easy to generalize!
High elegance / beauty!

Problem: 1 + 2 + 3 + 4 + …+ 100 = ?

= (100*101)/2

= 5050

Proof: Induction…

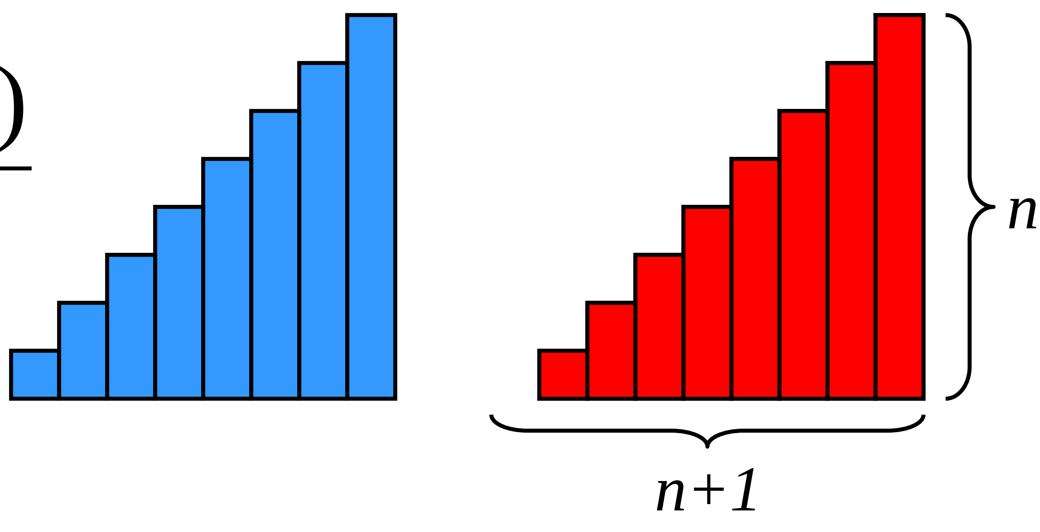
1 + 2 + 3 + … + 99 + 100

100 + 99 + 98 + … + 2 + 1

---

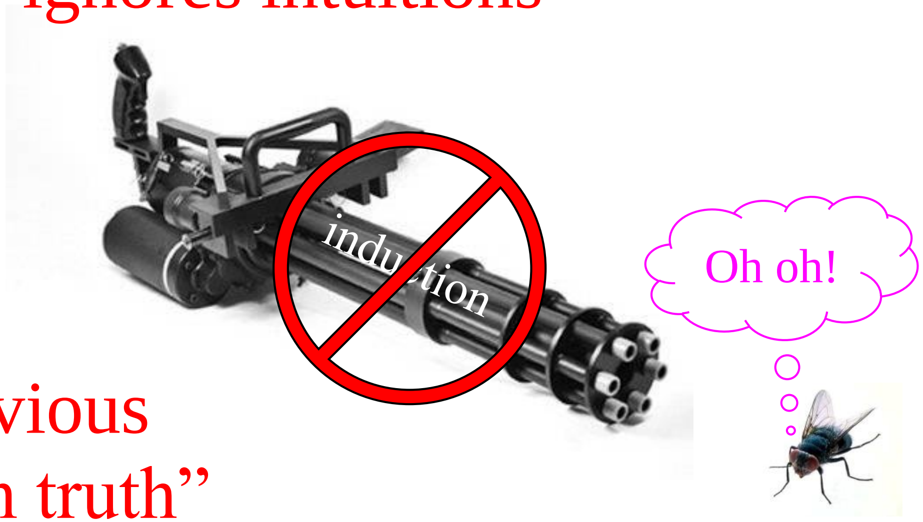
101 + 101 + 101 + … + 101 + 101 = 100*101

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$



Drawbacks of Induction

- You must a priori know the formula / result
- Easy to make mistakes in inductive proof
- Mostly "mechanical" – ignores intuitions
- Tedious to construct
- Difficult to check
- Hard to understand
- Not very convincing
- Generalizations not obvious
- Does not "shed light on truth"
- Obfuscates connections



Conclusion: only use induction as a last resort! (i.e., rarely)

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?$

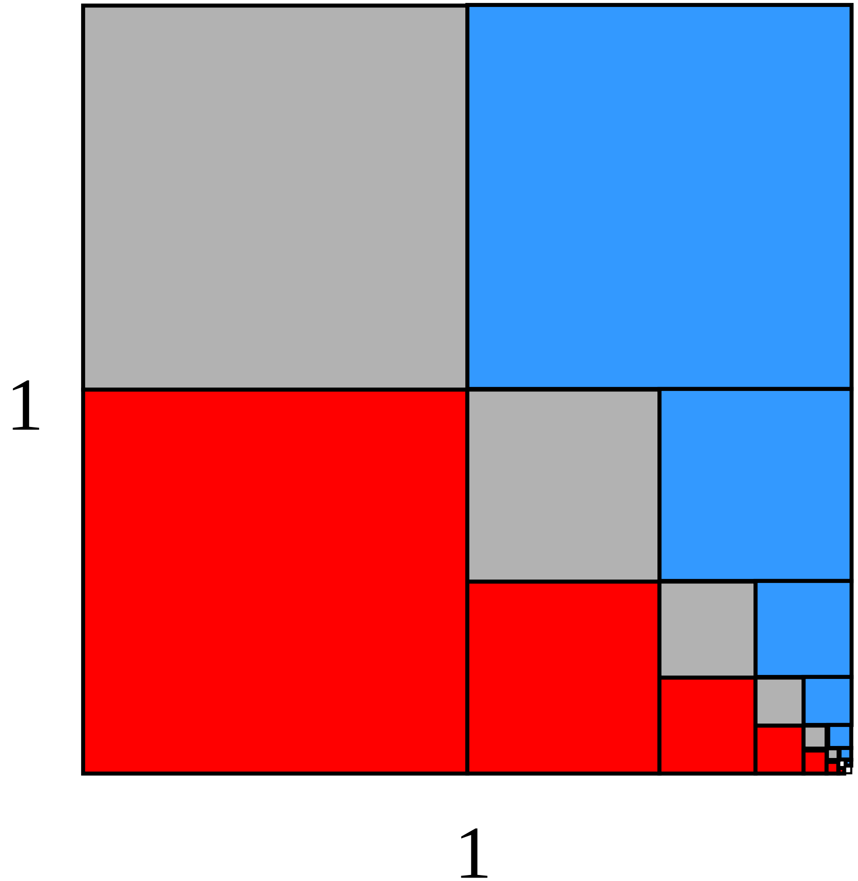
$$\sum_{i=1}^{\infty} \frac{1}{4^i} = ?$$

Extra Credit:

Find a short, geometric, induction-free proof.

Problem: $(1/4) + (1/4)^2 + (1/4)^3 + (1/4)^4 + \ldots = ?$

Find a short, geometric, induction-free proof.



$$\sum_{i=1}^{\infty} \frac{1}{4^i} = \frac{1}{3}$$

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ?$

$$\sum_{i=1}^{\infty} \frac{1}{8^i} = ?$$

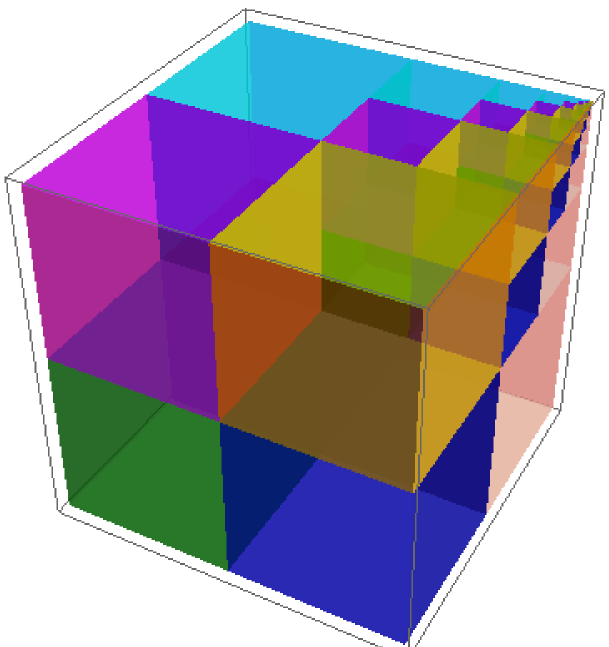
Extra Credit:

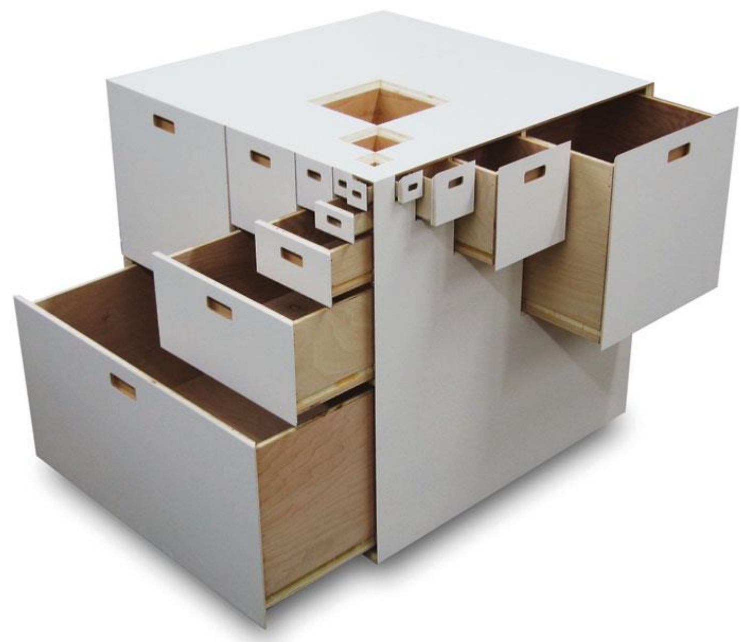
Find a short, geometric, induction-free proof.

Problem: $(1/8) + (1/8)^2 + (1/8)^3 + (1/8)^4 + \ldots = ?$

Find a short, geometric, induction-free proof.

$$\sum_{i=1}^{\infty} \frac{1}{8^i} = \frac{1}{7}$$



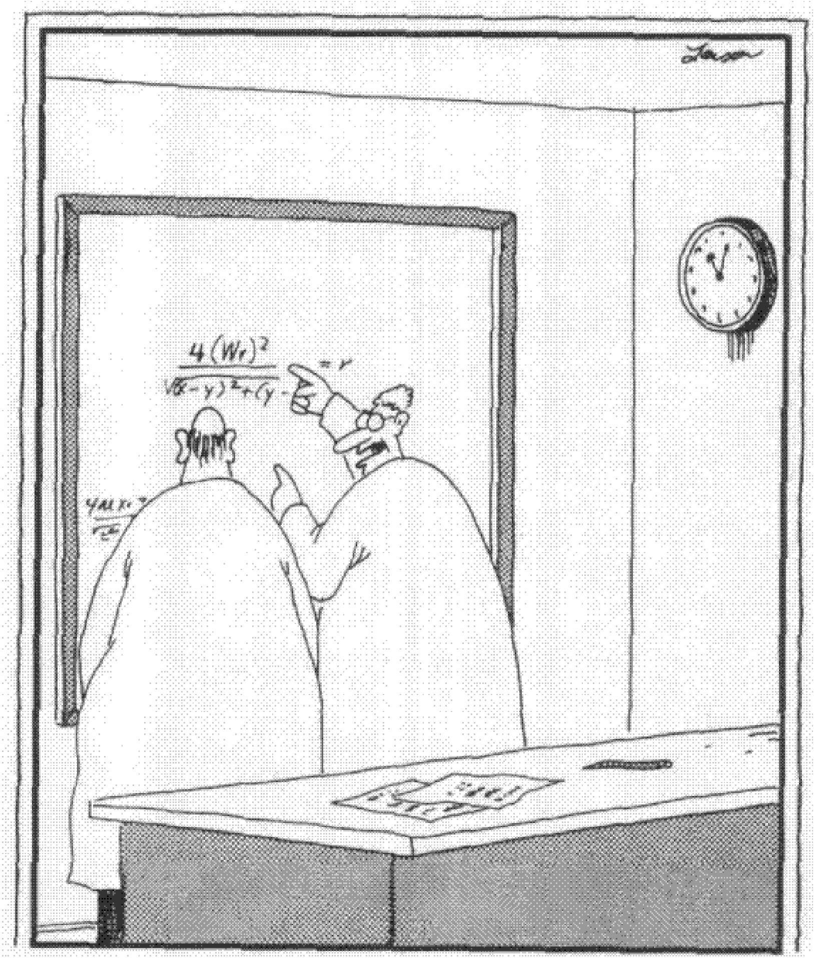


Problem: $1^3 + 2^3 + 3^3 + 4^3 + \ldots + n^3 = ?$

$$\sum_{i=1}^{n} i^3 = ?$$

Extra Credit:

find a short, geometric, induction-free proof.



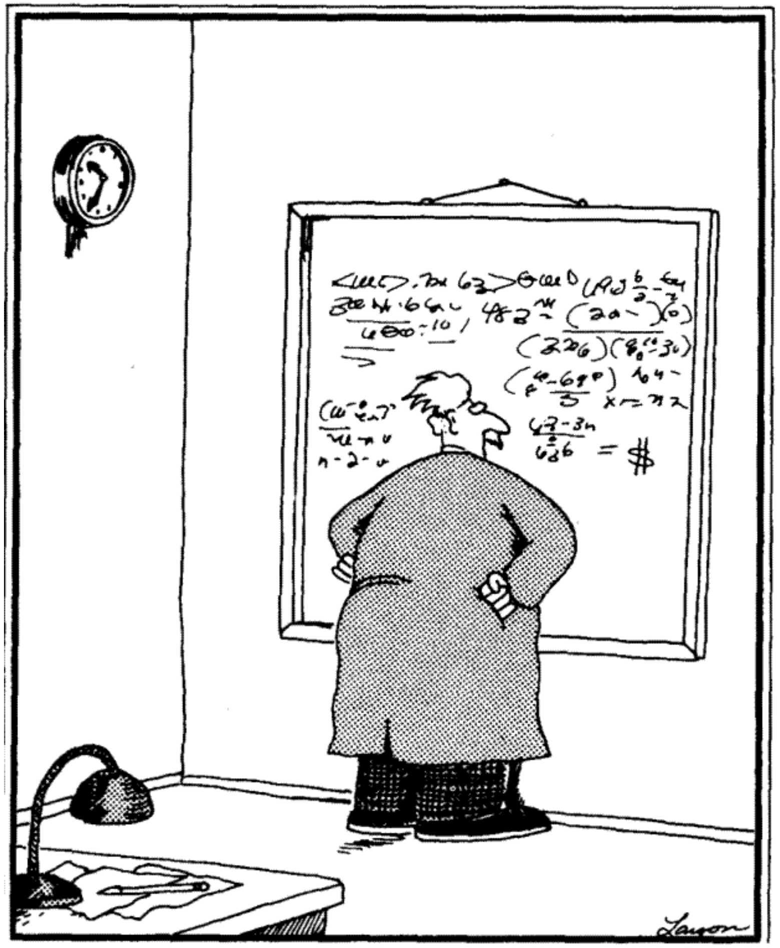
"Yes, yes, I *know* that, Sidney ... *everybody* knows *that*! ... But look: *Four wrongs squared*, minus two wrongs to the fourth power, divided by this formula, *do* make a right."

Submissions Web form: www.cs.virginia.edu/robins/theory_homework_grad

Problem: Prove that $\sqrt{2}$ is irrational.

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Einstein discovers that time is actually money.

Problem: Prove that there are an infinity of primes.

Extra Credit: Find a short, induction-free proof.

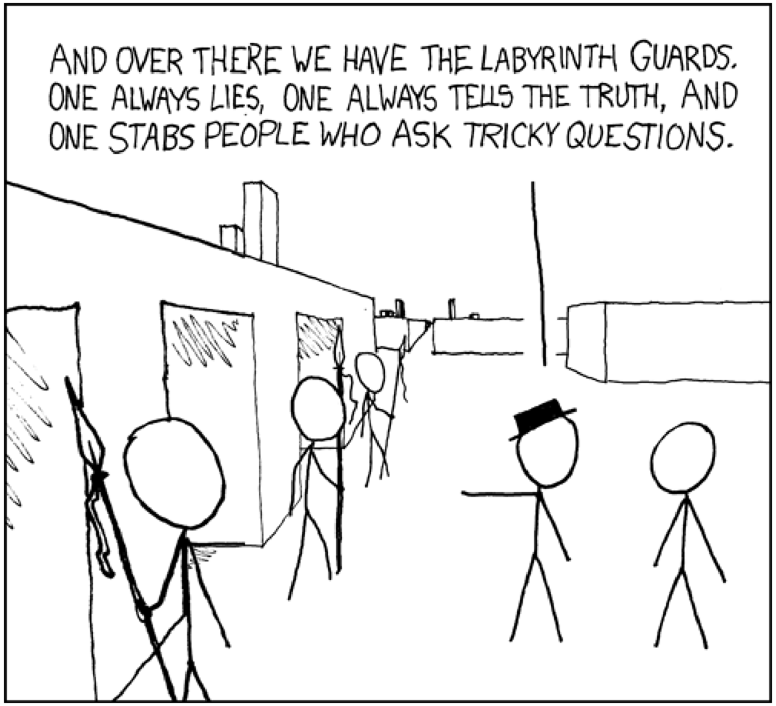
- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

Submissions Web form: www.cs.virginia.edu/robins/theory_homework_grad

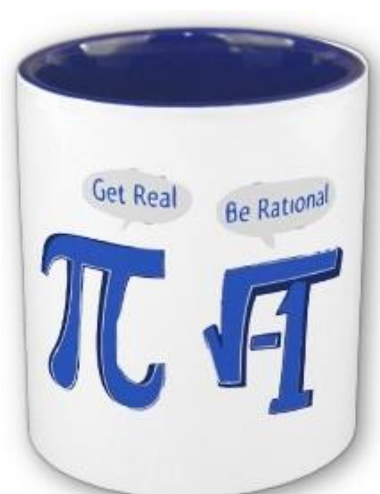
Problem: True or false: there arbitrary long blocks of consecutive composite integers.

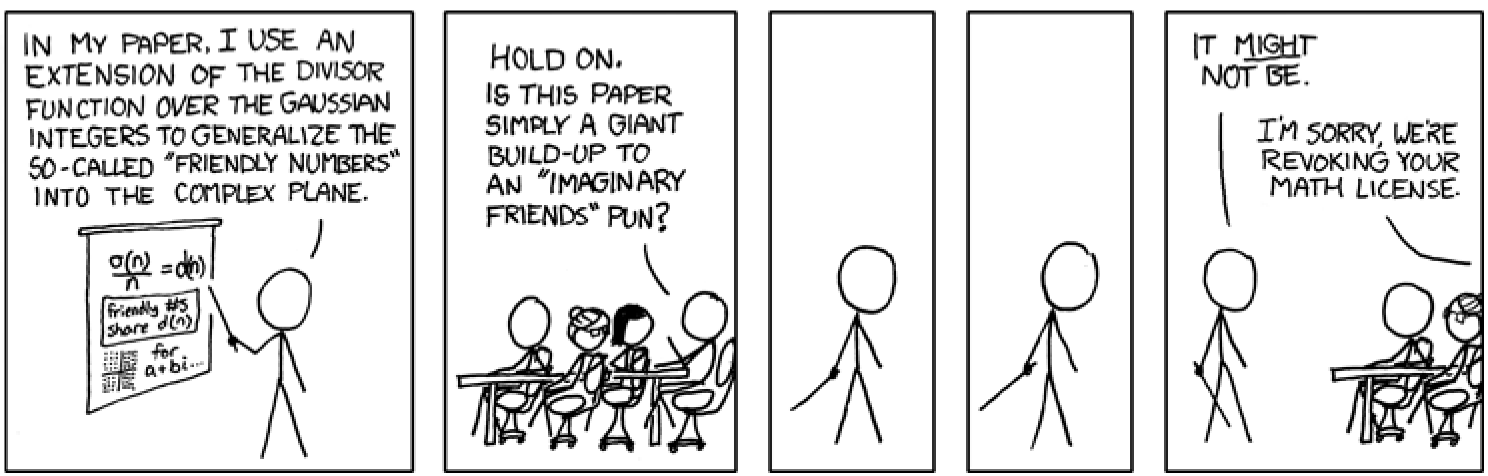
Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



Problem: Are the complex numbers closed under exponentiation ? E.g., what is the value of i^i?

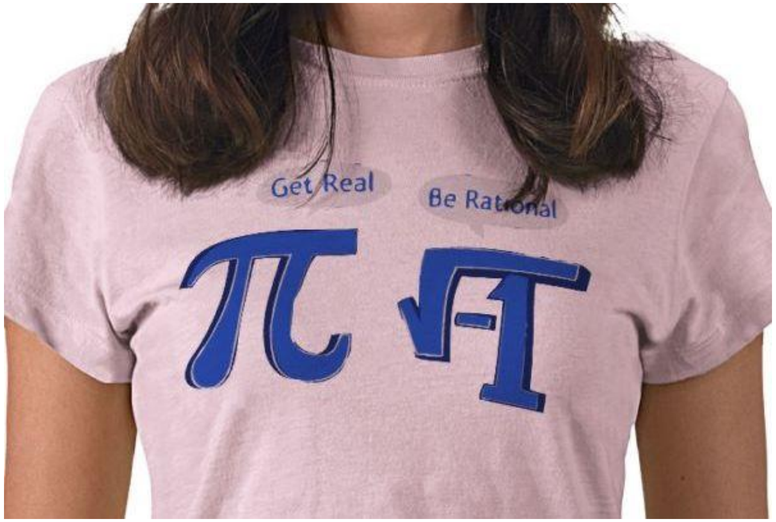




Problem: Does exponentiation preserve irrationality?
i.e., are there two irrational numbers x and y such that x^y is rational?

Extra Credit: find a short, induction-free proof.

- What approaches fail?
- What techniques work and why?
- Lessons and generalizations

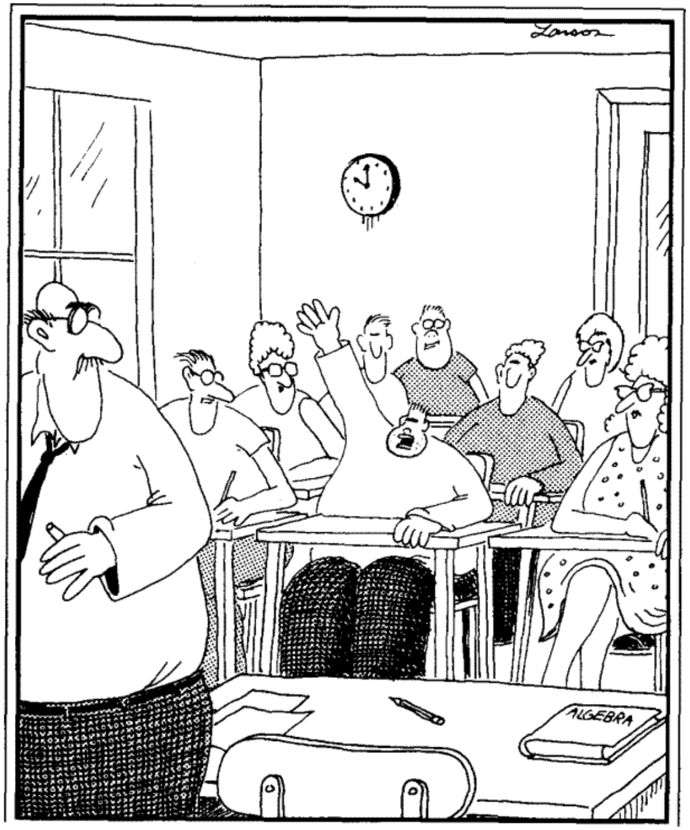


Problem: Solve the following equation for X:

$$X^{X^{X^{X^{\cdot^{\cdot^{\cdot}}}}}} = 2$$

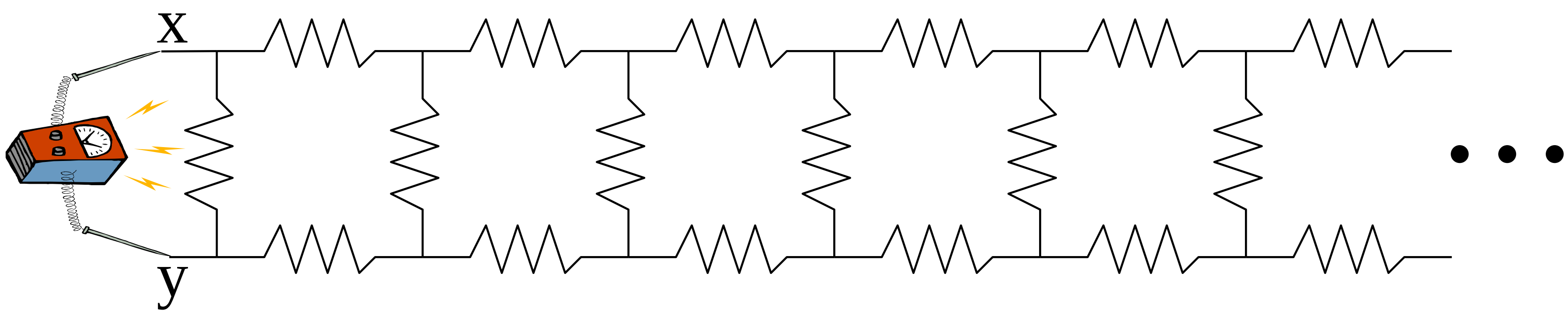
where the stack of exponentiated x's extends forever.

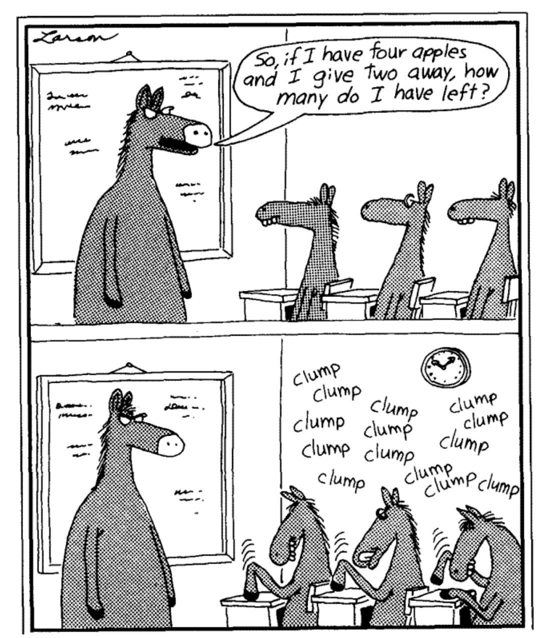
- What approaches fail?
- What techniques work and why?
- Lessons and generalizations



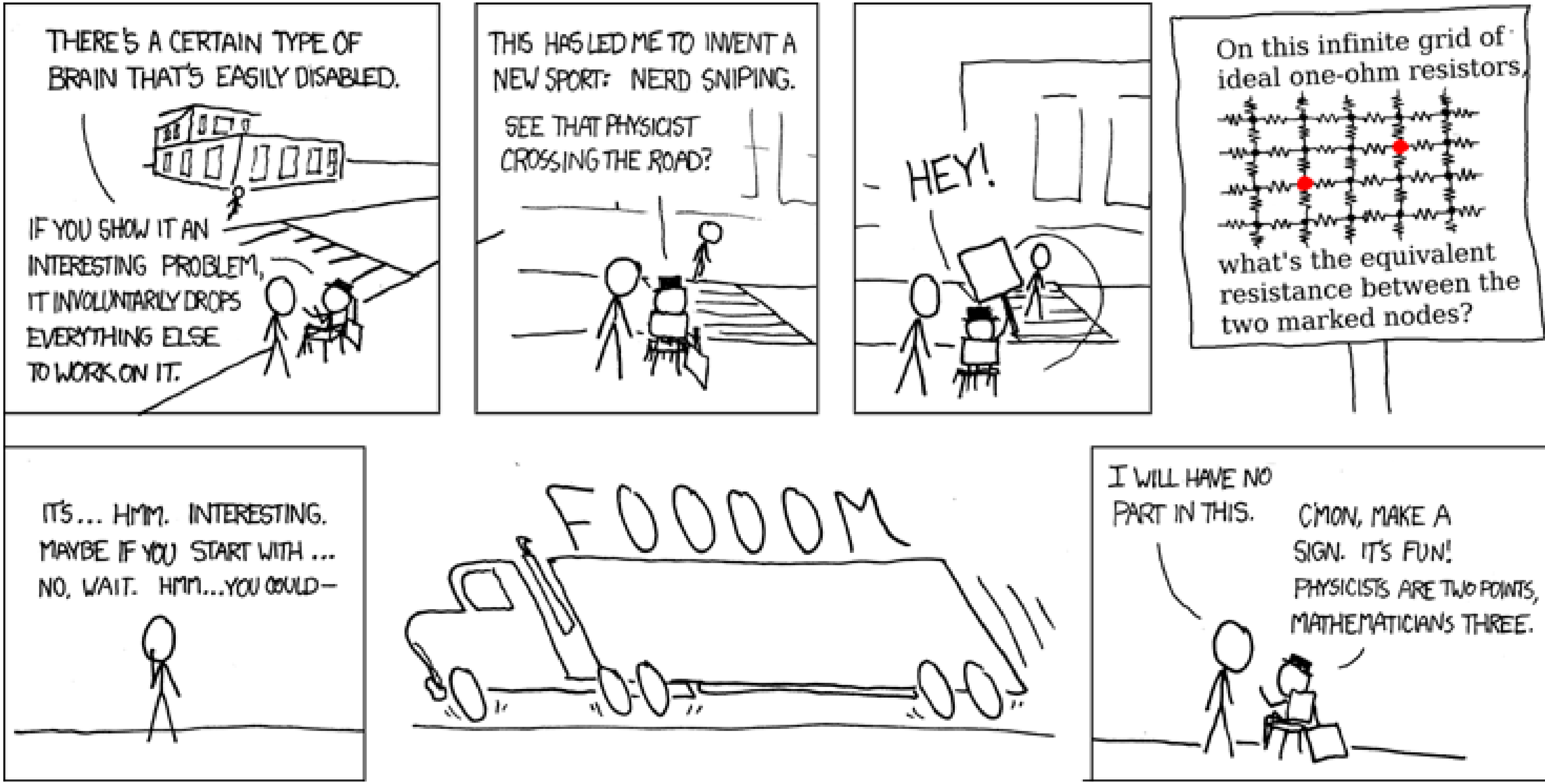
"Mr. Osborne, may I be excused? My brain is full."

Problem: For the given infinite ladder of resistors of resistance R each, what is the resistance measured between points x and y?





- What approaches fail?
- What techniques work and why?
- Lessons and generalizations


THERE'S A CERTAIN TYPE OF BRAIN THAT'S EASILY DISABLED.
IF YOU SHOW IT AN INTERESTING PROBLEM, IT INVOLUNTARILY DROPS EVERYTHING ELSE TO WORK ON IT.
THIS HAS LED ME TO INVENT A NEW SPORT: NERD SNIPING.
SEE THAT PHYSICIST CROSSING THE ROAD?
HEY!
On this infinite grid of ideal one-ohm resistors, what's the equivalent resistance between the two marked nodes?
IT'S... HMM. INTERESTING. MAYBE IF YOU START WITH... NO, WAIT. HMM...YOU COULD—
FOOOOM
I WILL HAVE NO PART IN THIS.
C'MON, MAKE A SIGN. IT'S FUN! PHYSICISTS ARE TWO POINTS, MATHEMATICIANS THREE.

Historical Perspectives

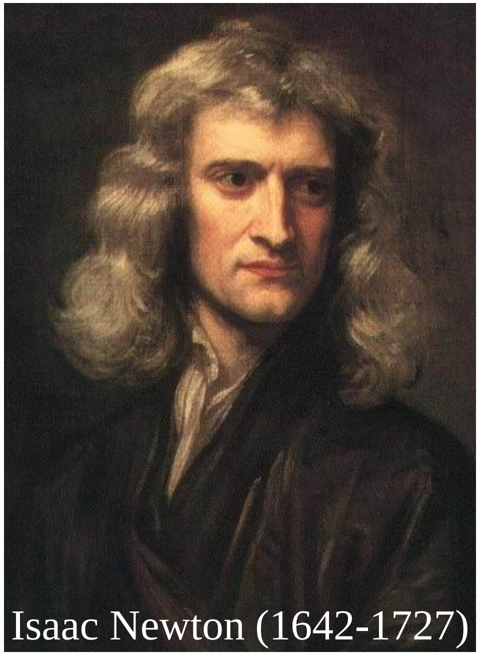
Historical Perspectives

- Knowing the "big picture" is empowering
- Science and mathematics builds heavily on past
- Often the simplest ideas are the most subtle
- Most fundamental progress was done by a few
- We learn much by observing the best minds
- Research benefits from seeing connections
- The field of computer science has many "parents"
- We get inspired and motivated by excellence
- The giants can show us what is possible to achieve
- It is fun to know these things!

"Standing on the Shoulders of Giants"

- Aristotle, Euclid, Archimedes, Eratosthenes
- Abu Ali al-Hasan ibn al-Haytham
- Fibonacci, Descartes, Fermat, Pascal
- Newton, Euler, Gauss, Hamilton
- Boole, De Morgan
- Babbage, Ada Lovelace
- Venn, Carroll


Ada Lovelace (1815-1852)

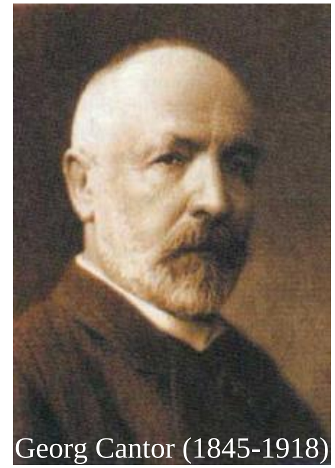

Isaac Newton (1642-1727)


Euclid (300 BC)

“Standing on the Shoulders of Giants”

- Cantor, Hilbert, Russell
- Hardy, Ramanujan, Ramsey
- Gödel, Church, Turing
- von Neumann, Shannon
- Kleene, Chomsky
- Hoare, McCarthy, Erdos
- Knuth, Backus, Dijkstra

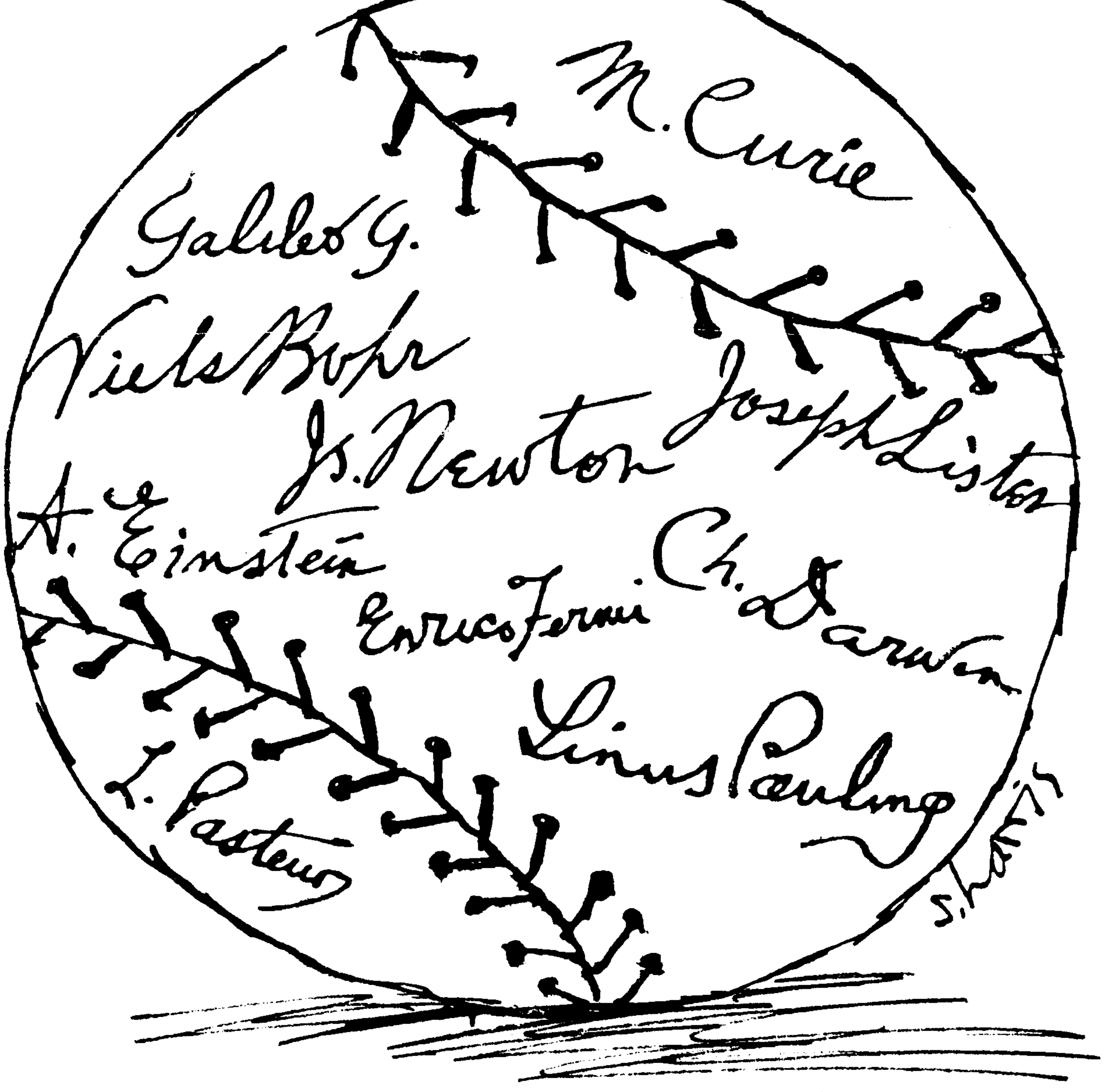
Many others…


Georg Cantor (1845-1918)


Bertrand Russell (1872-1970)


David Hilbert (1862-1943)


Kurt Gödel (1906-1978)

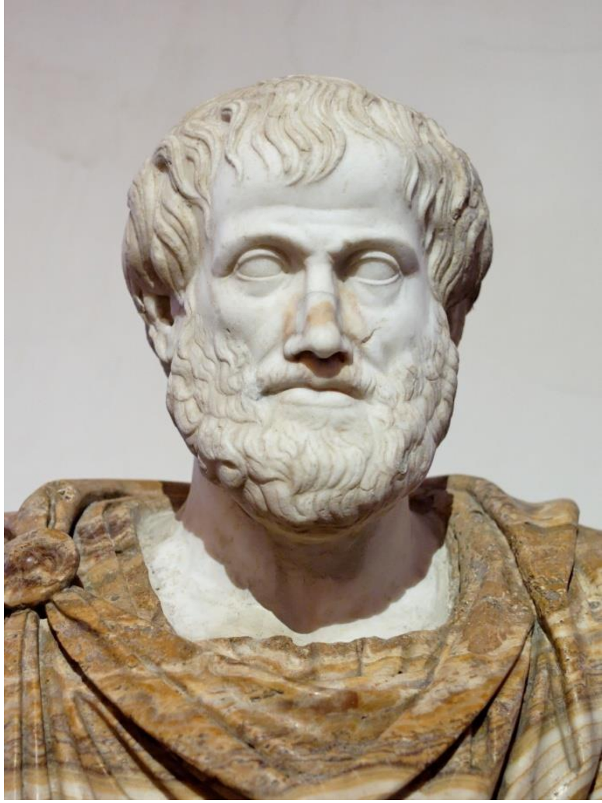


M. Curie
Galileo G.
Niels Bohr
Is. Newton
Joseph Lister
A. Einstein
Ch. Darwin
Enrico Fermi
Linus Pauling
L. Pasteur
S. Harris

Gauss
Newton
Archimedes
Euler
Cauchy
Poincare
Riemann
Cantor
Cayley
Hamilton
Eisenstein
Pascal
Abel
Hilbert
Klein
Leibniz
Descartes
Galois
Mobius
Jacob
Johann Bernoulli
Daniel Bernoulli
Dirichlet
Fermat
Pythagoras
Laplace
Lagrange
Kronecker
Jacobi
Bolyai
Lobatchewsky
Noether
Germain
Euclid
Legendre

MAKING PHILOSOPHY ACCESSIBLE: POP-UP PLATO

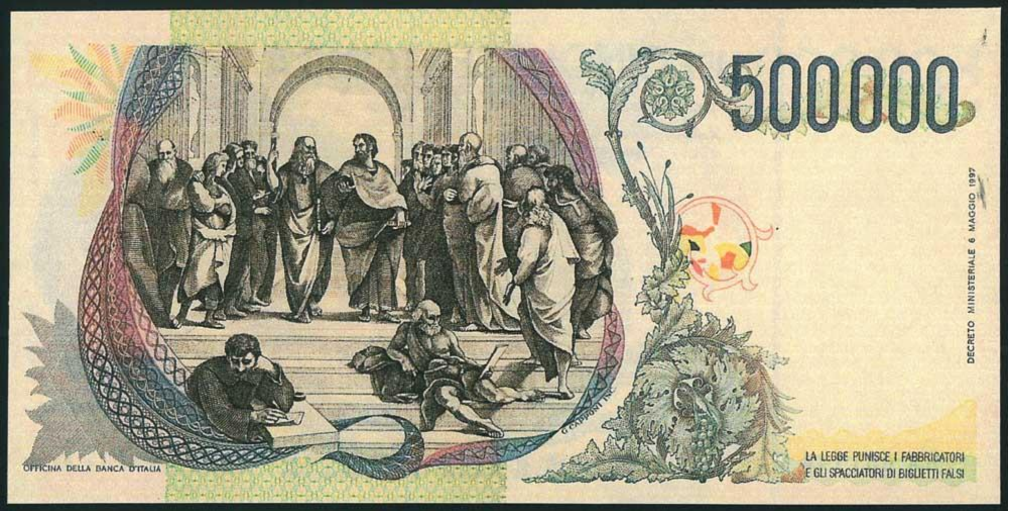
Historical Perspectives

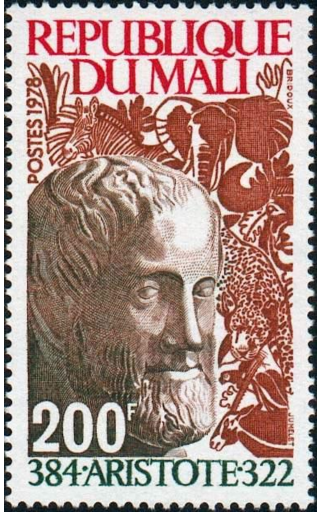


Aristotle (384BC-322BC)

- Founded Western philosophy
- Student of Plato
- Taught Alexander the Great
- "Aristotelianism"
- Developed the "scientific method"
- One of the most influential people ever
- Wrote on physics, theatre, poetry, music, logic, rhetoric, politics, government, ethics, biology, zoology, morality, optics, science, aesthetics, psychology, metaphysics, …
- Last person to know everything known in his own time!

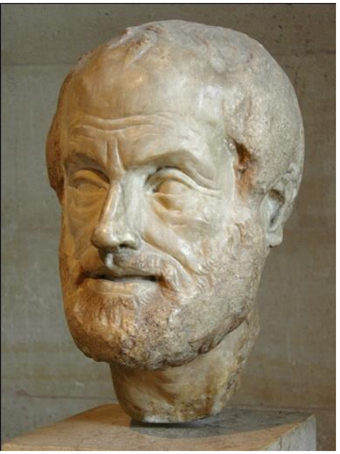
"Almost every serious intellectual advance has had to begin with an attack on some Aristotelian doctrine." – Bertrand Russell

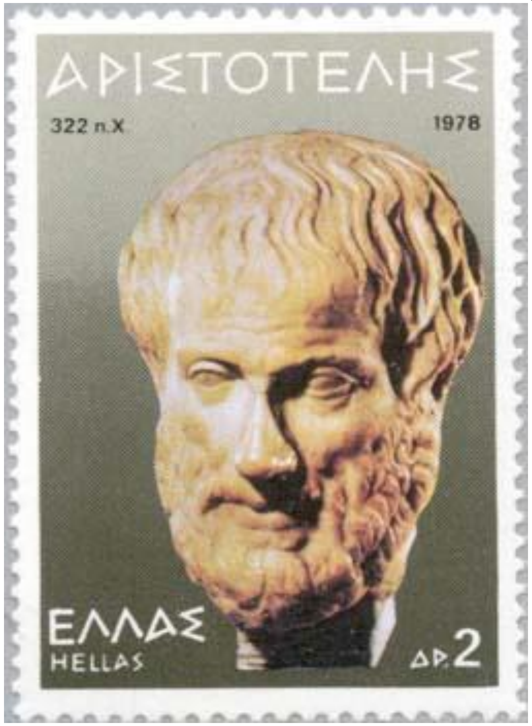


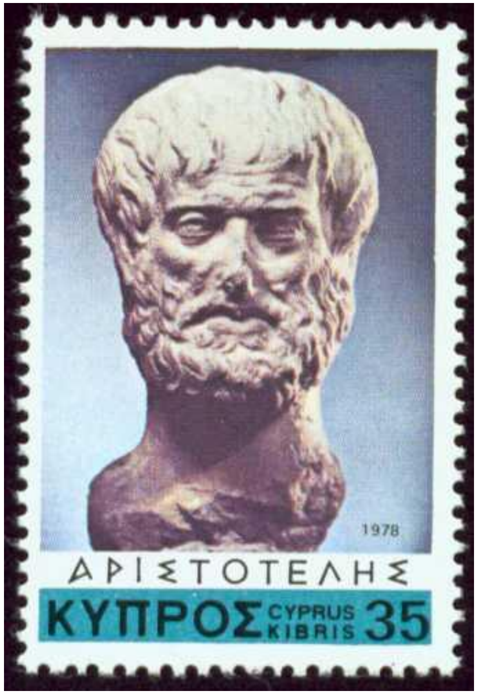


“Wit is educated insolence.”

- Aristotle (384-322 B.C.)







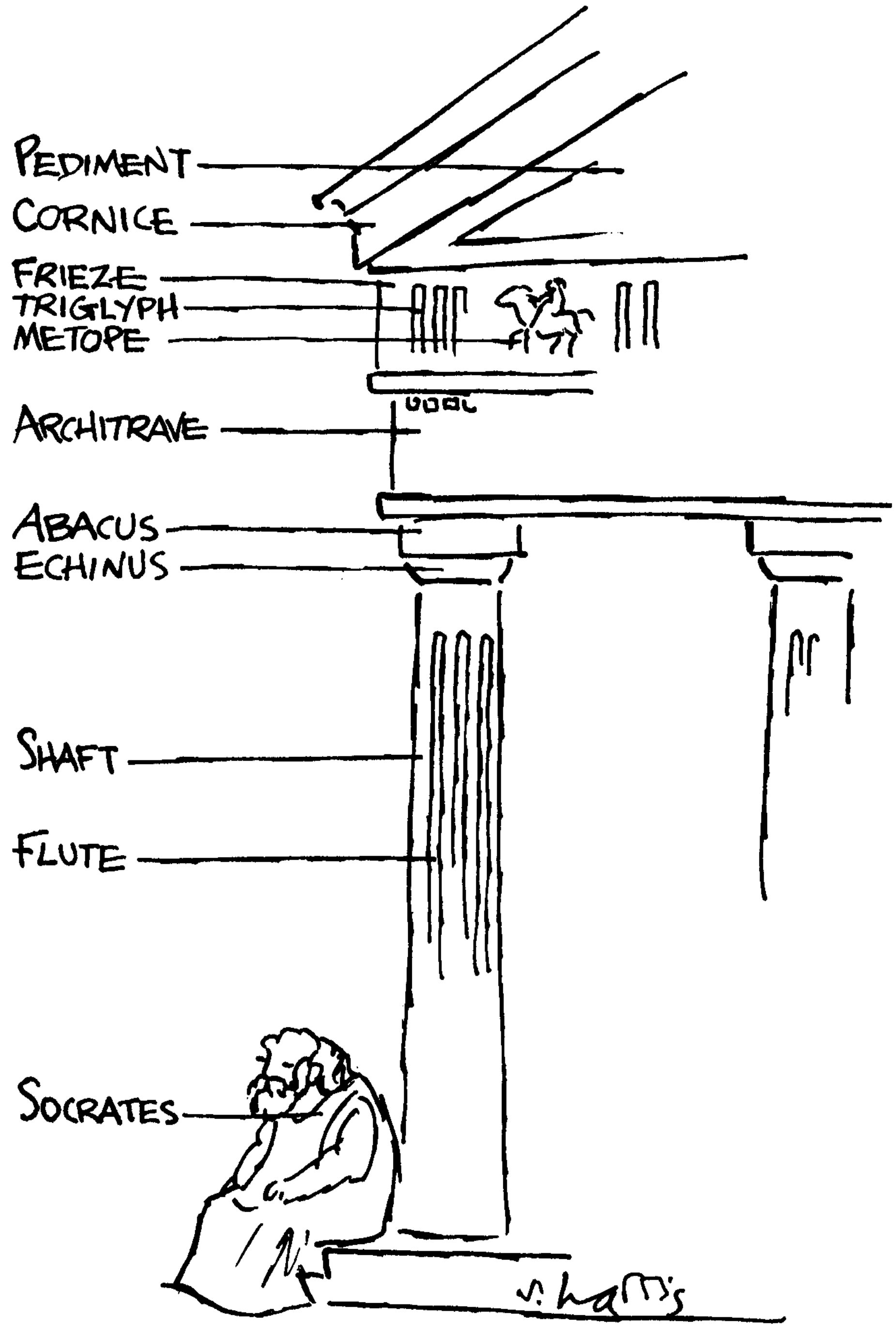
“The School of Athens” (by Raphael, 1483-1520)


ARISTOTELES
87 km
97 / 10 / 09
D=254mm f/D=10
Moon "LIGHT"
© António J. Cidadão
8
B/W QuickCam
a.cidadao@mail.telepac.pt


THE UNKNOWN PHILOSOPHER
WHO FIRST REALIZED LIFE IS NO PICNIC

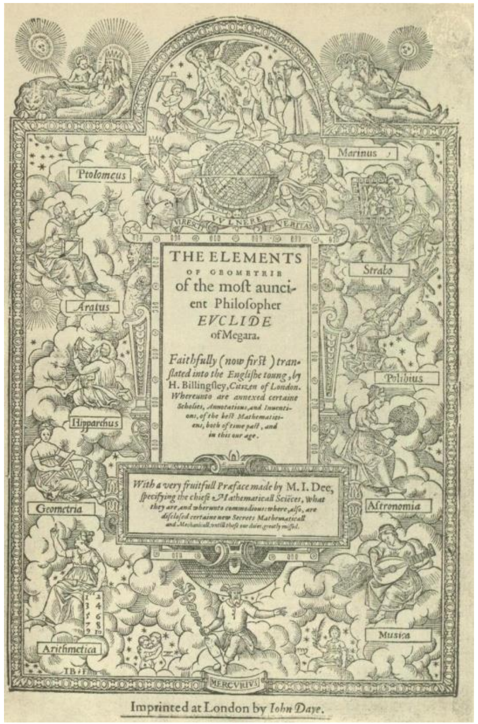

Birds fly because they're lighter than air. Some trees have different fruits each year. At night, clouds rest on the ground.
Are you sure he's Aristotle?
S. Harris

"What I especially like about being a philosopher-scientist is that I don't have to get my hands dirty."

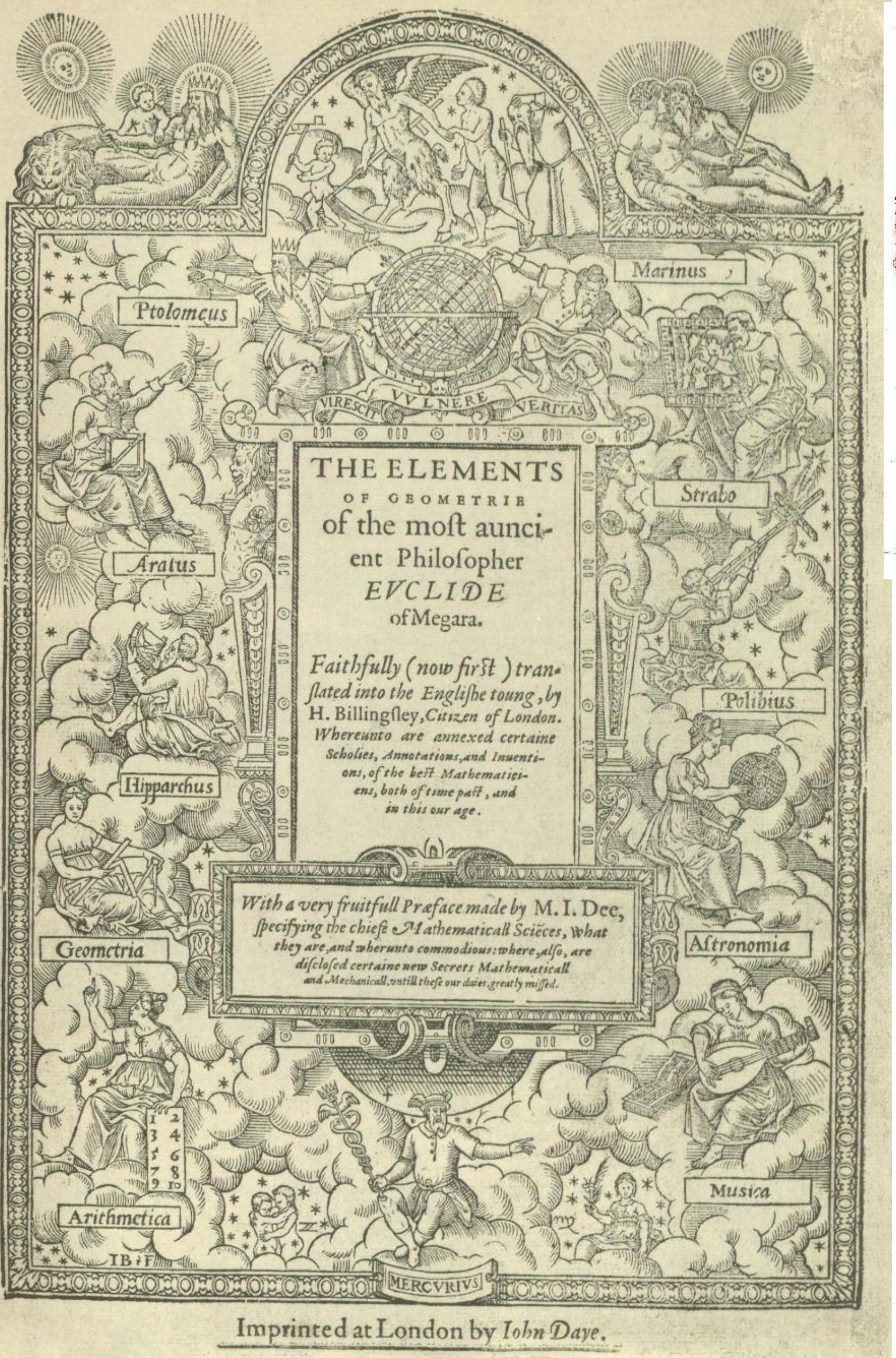

PEDIMENT
CORNICE
FRIEZE
TRIGLYPH
METOPE
ARCHITRAVE
ABACUS
ECHINUS
SHAFT
FLUTE
SOCRATES
S. Harris

Historical Perspectives

Euclid (325BC-265BC)



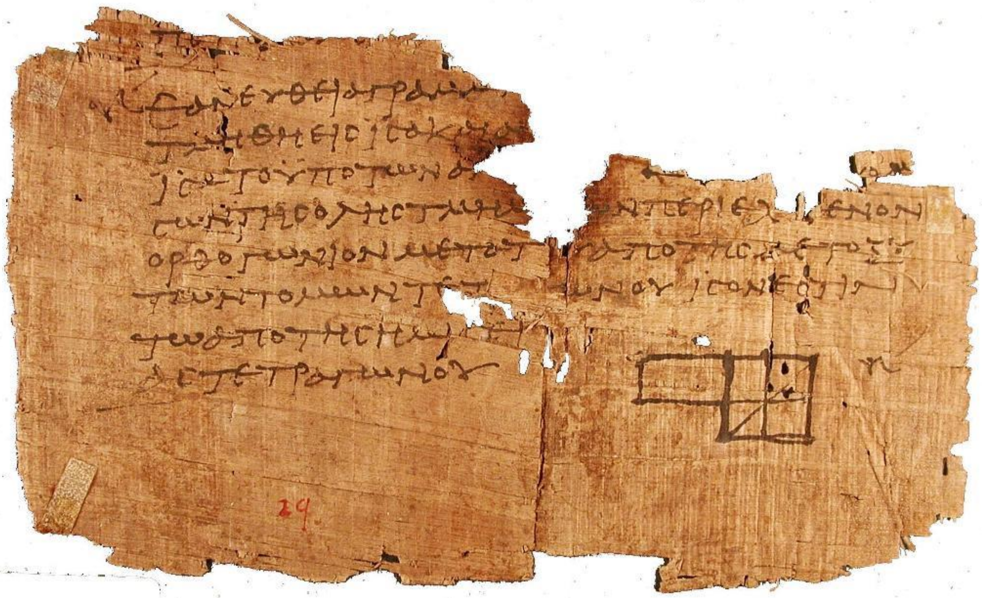
- Founder of geometry & the axiomatic method
- "Elements" – oldest and most impactful textbook
- Unified logic & math
- Introduced rigor and "Euclidean" geometry
- Influenced all other fields of science: Copernicus, Kepler, Galileo, Newton, Russell, Lincoln, Einstein & many others


THE ELEMENTS
OF GEOMETRIE
of the most aunci-
ent Philosopher
EVCLIDE
of Megara.

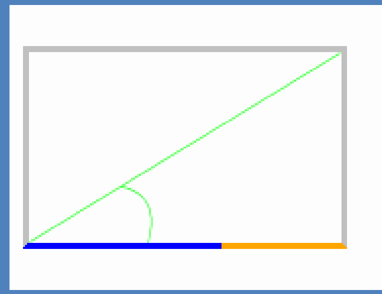
Faithfully (now first) translated into the Englishe toung, by H. Billingsley, Citizen of London. Whereunto are annexed certaine Scholies, Annotations, and Inventions, of the best Mathematiciens, both of time past, and in this our age.

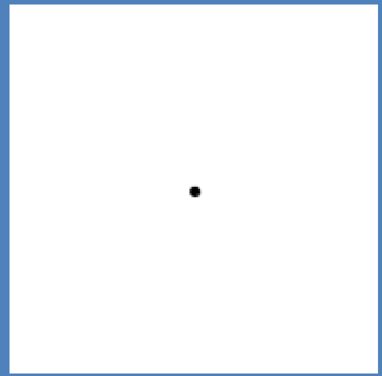
With a very fruitfull Præface made by M. I. Dee, specifying the chiefe Mathematicall Sciēces, what they are, and wherunto commodious: where, also, are disclosed certaine new Secrets Mathematicall and Mechanicall, vntill these our daies, greatly missed.

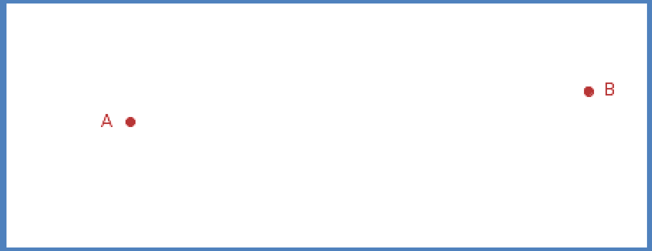
Imprinted at London by Iohn Daye.

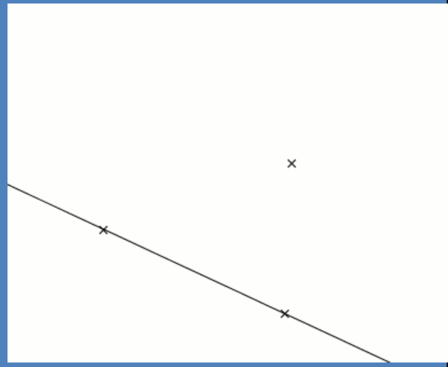


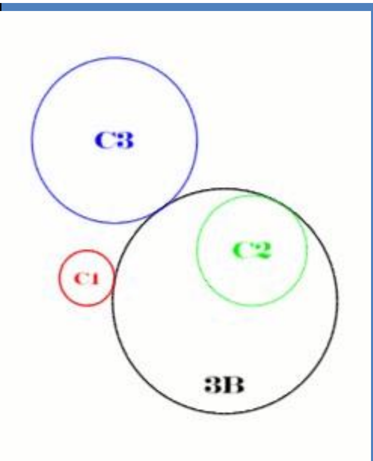
Euclid's Straight-Edge and Compass Geometric Constructions

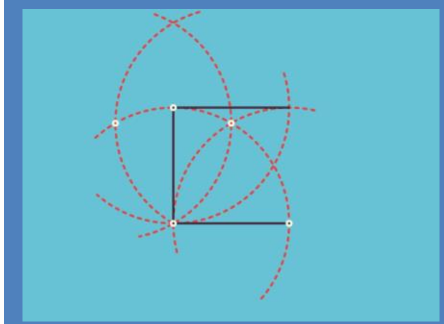




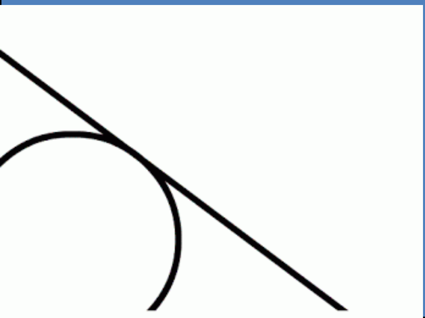


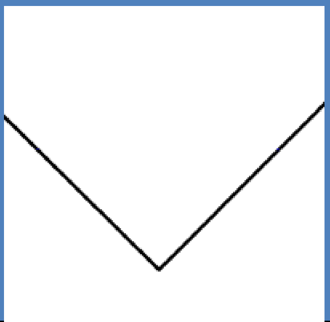


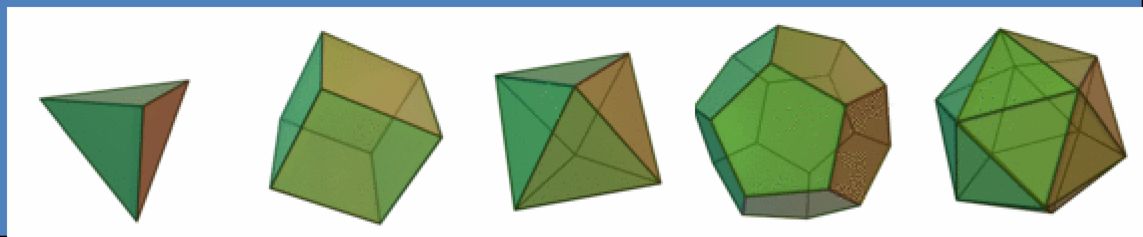




Extra credit!



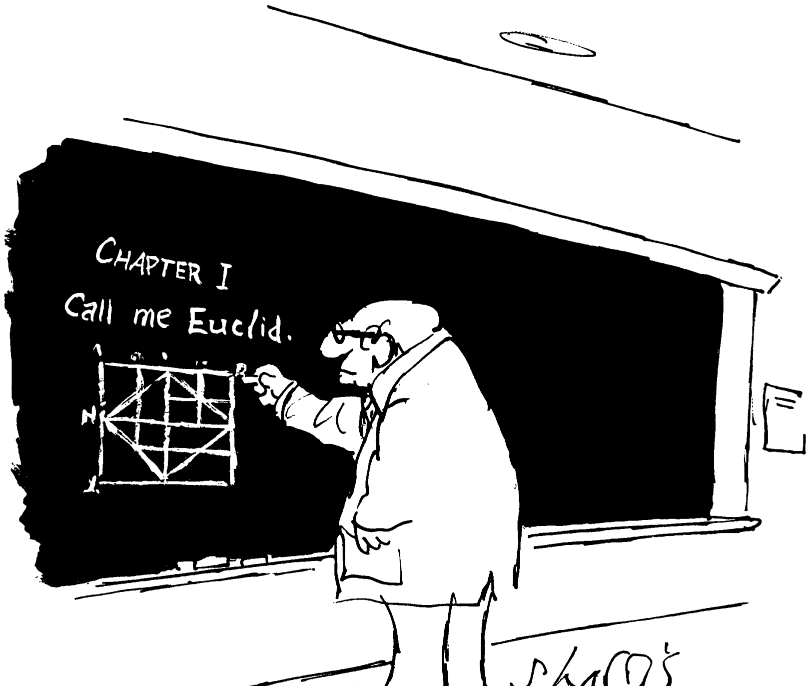


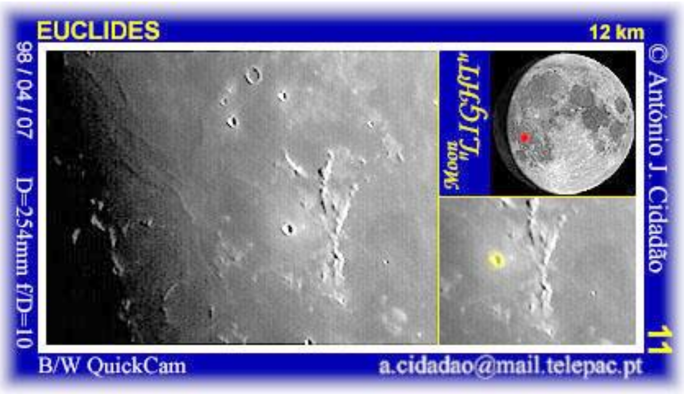



SCHOOL OF ATHENS
50c
SIERRA LEONE
500th ANNIVERSARY
BIRTH OF RAPHAEL
(detail) Euclid and his pupils
1983
SCALA NEW YORK/FLORENCE


MALDIVES
PRINCIPLES OF GEOMETRY
EUCLID
Rf3


POSTE VATICANE
2500
PONTIFICIA
ACADEMIA
SCIENTIARVM
I.P.Z.S.-ROMA-1986
P.N. ARGHITTU INC.


CHAPTER I
Call me Euclid.
S. Harris


EUCLIDES
12 km
98 / 04 / 07
D=254mm f/D=10
© António J. Cidadão
Moon "LIGHT"
11
B/W QuickCam
a.cidadao@mail.telepac.pt

Euclid's Axioms

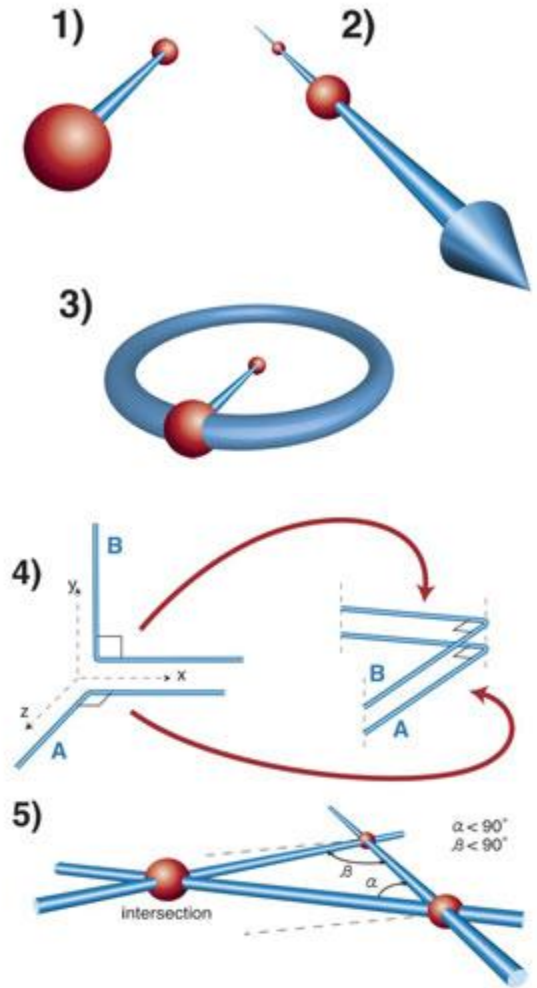
1: Any two points can be connected by exactly one straight line.

2: Any segment can be extended indefinitely into a straight line.

3: A circle exists for any given center and radius.

4: All right angles are equal to each other.

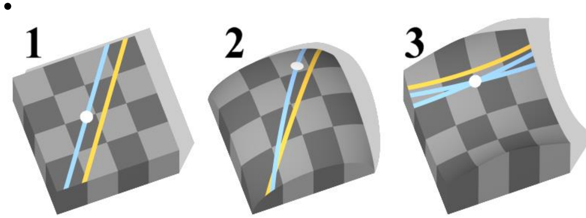
5: The parallel postulate: Given a line and a point off that line, there is exactly one line passing through the point, which does not intersect the first line.

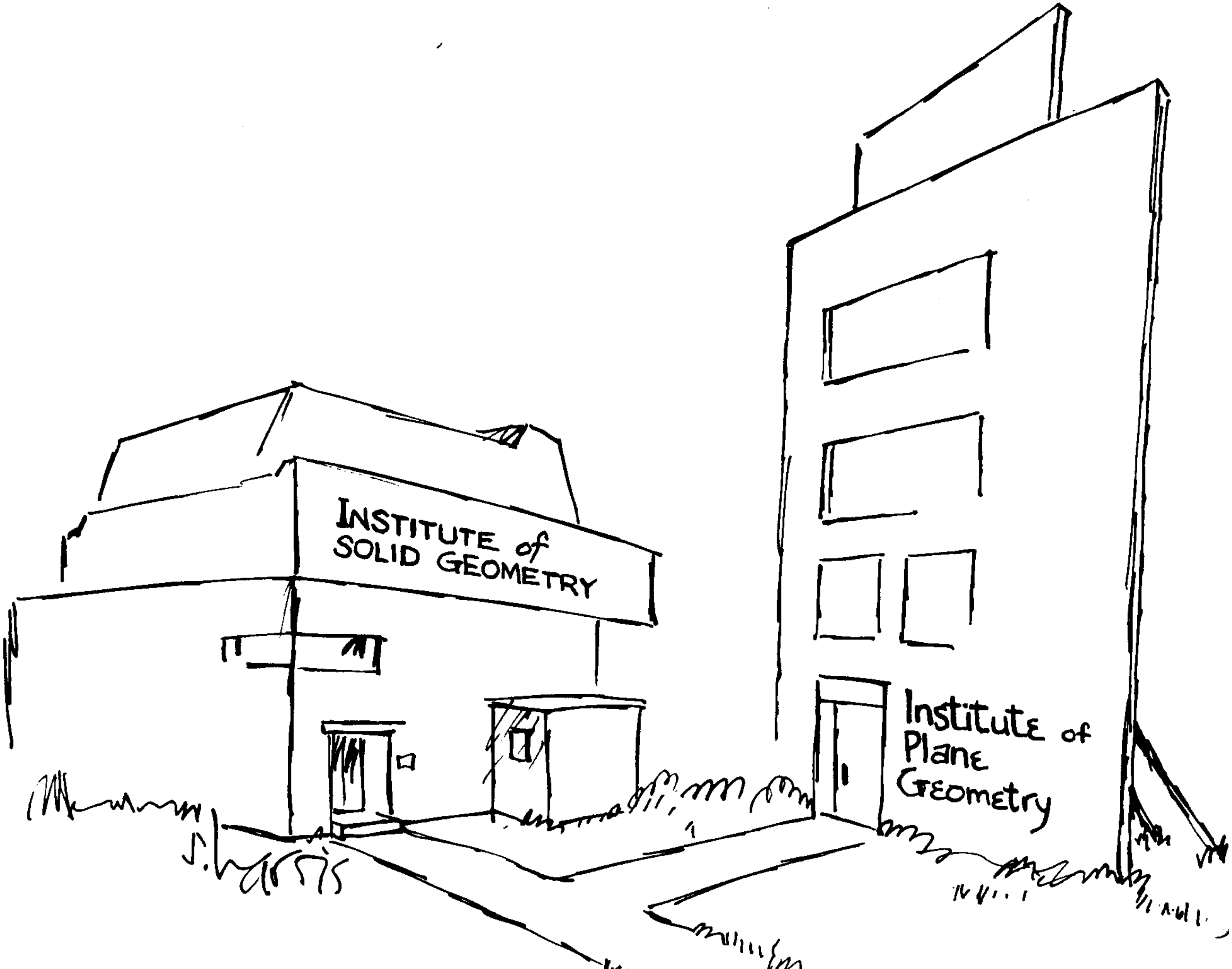


The first 28 propositions of Euclid's Elements were proven without using the parallel postulate!

Theorem [Beltrami, 1868]: The parallel postulate is independent of the other axioms of Euclidean geometry.

The parallel postulate can be modified to yield non-Euclidean geometries!

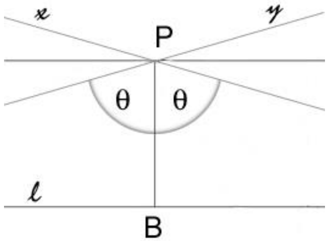




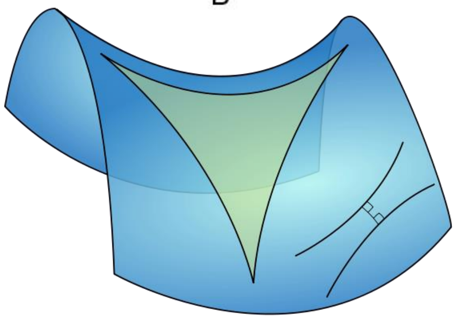
INSTITUTE of
SOLID GEOMETRY
Institute of
Plane
Geometry

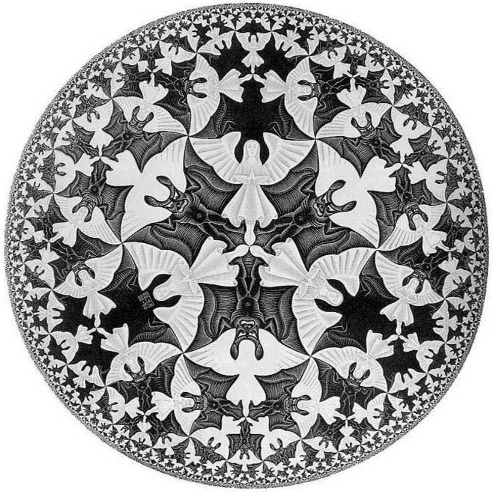
Non-Euclidean Geometries

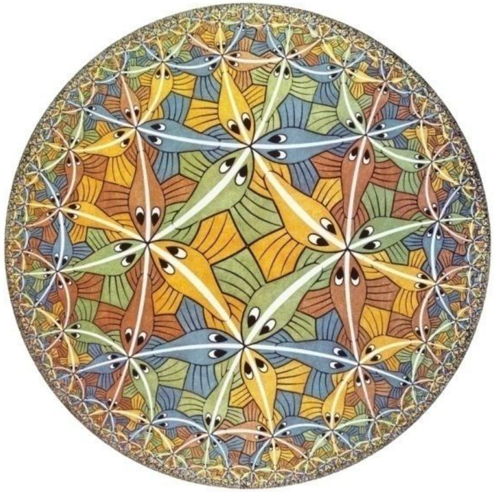
Hyperbolic geometry: Given a line and a point off that line, there are an infinity of lines passing through that point that do not intersect the first line.

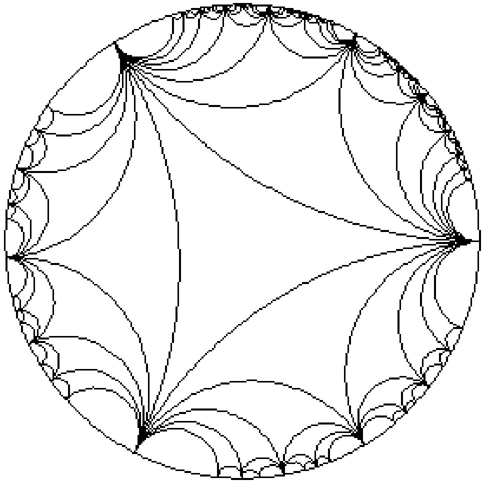


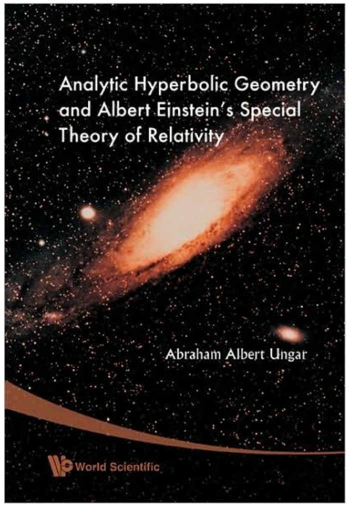
- Sum of triangle angles is less than 180°
- Different triangles have different angle sum
- Triangles with same angles have same area
- There are no similar triangles
- Used in relativity theory





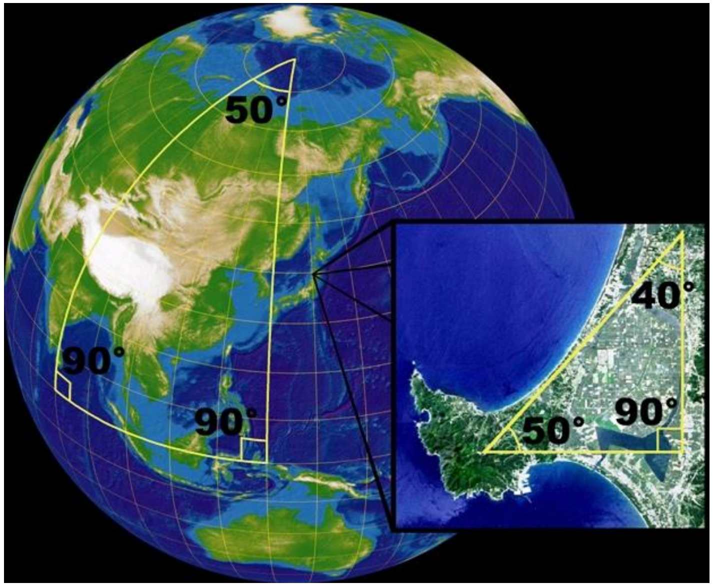




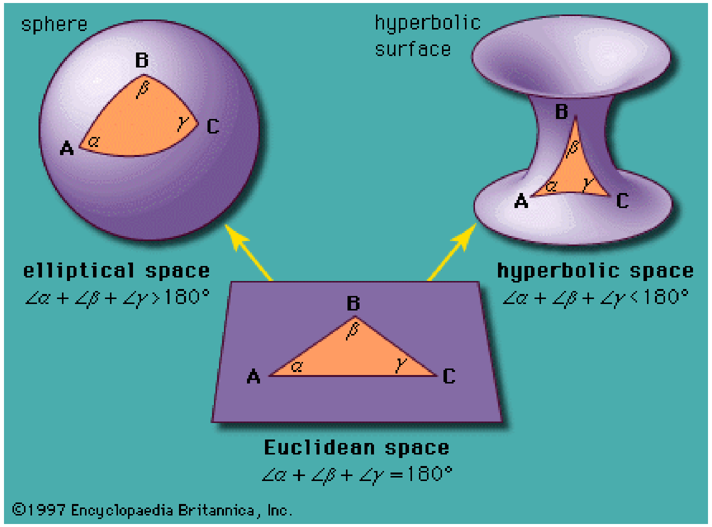


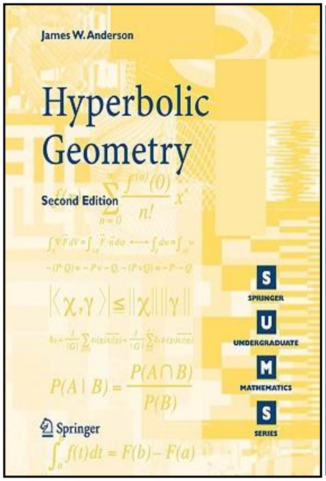
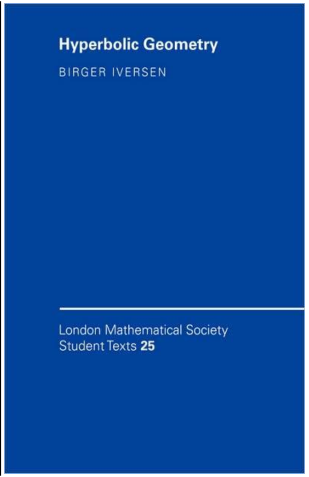
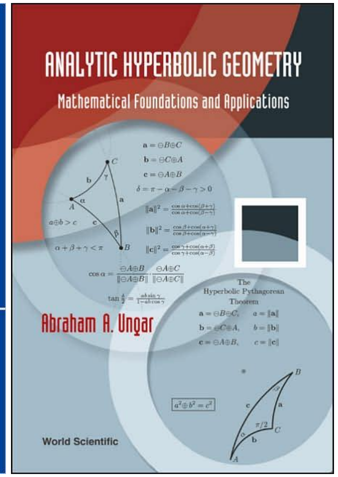
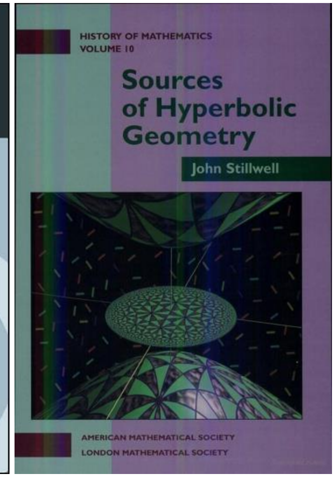
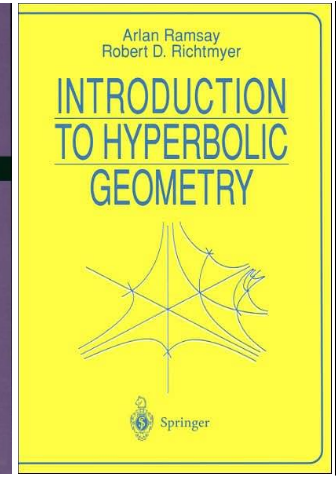
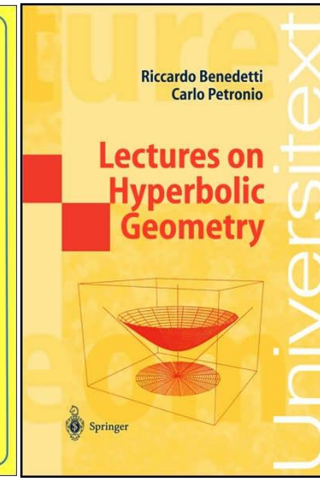
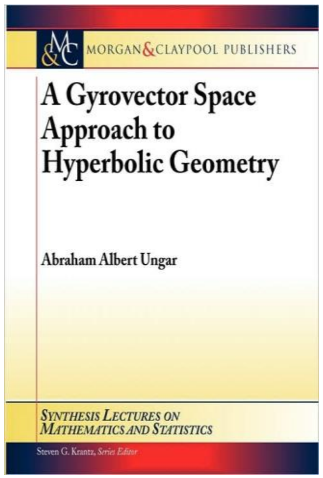
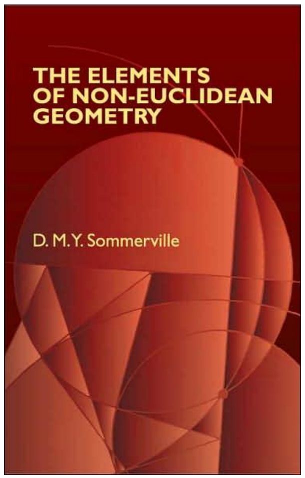
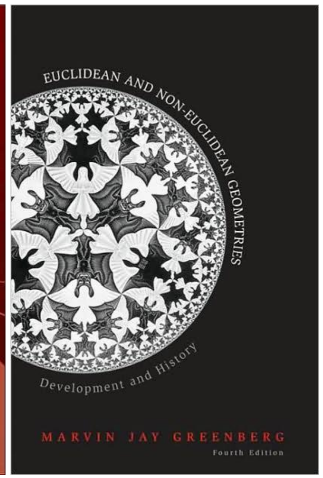
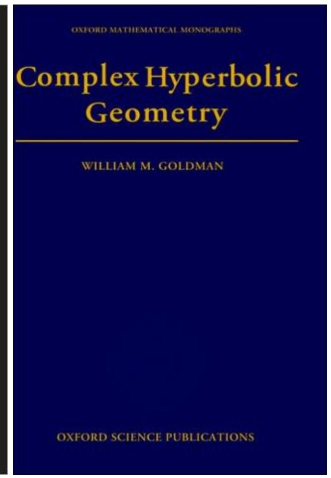
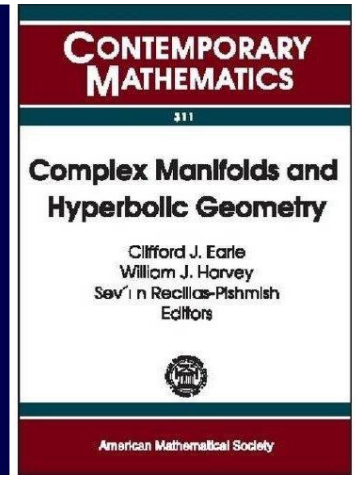
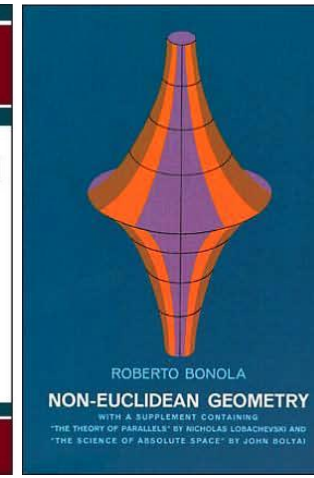
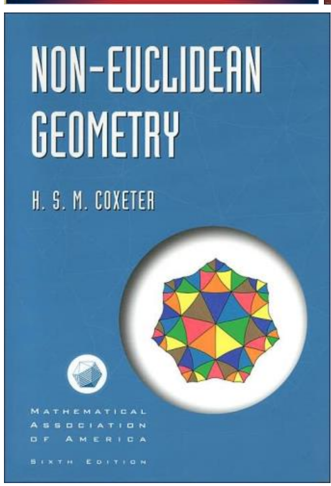
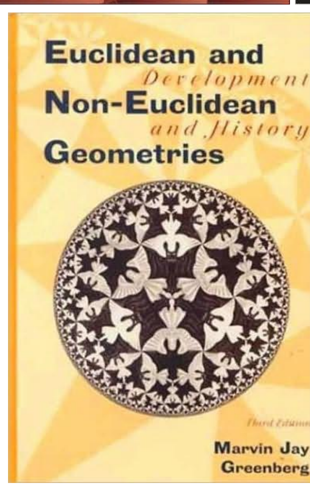
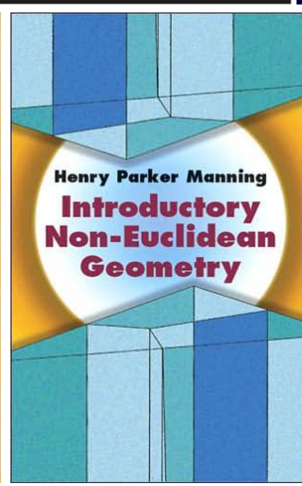
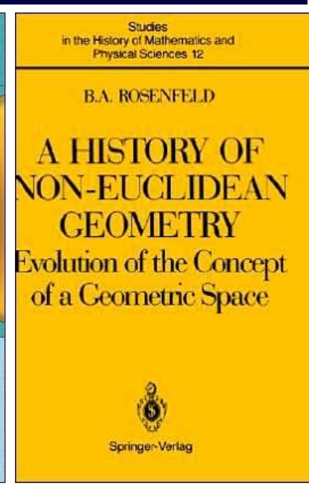
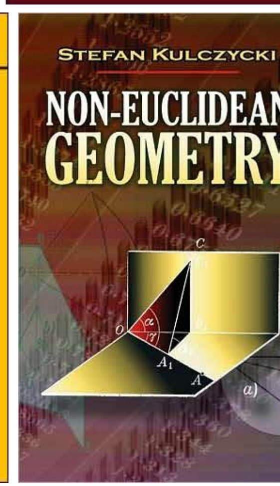
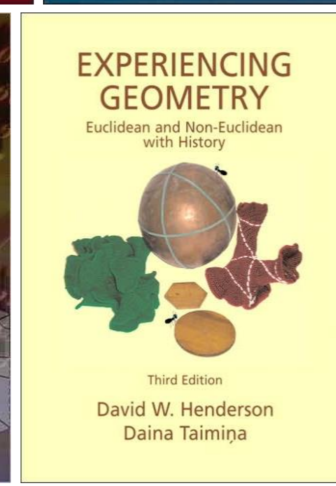
Non-Euclidean Geometries

Spherical / Elliptic geometry: Given a line and a point off that line, there are no lines passing through that point that do not intersect the first line.



- Lines are geodesics - "great circles"
- Sum of triangle angles is > 180°
- Not all triangles have same angle sum
- Figures can not scale up indefinitely
- Area does not scale as the square
- Volume does not scale as the cube
- The Pythagorean theorem fails
- Self-consistent, and complete

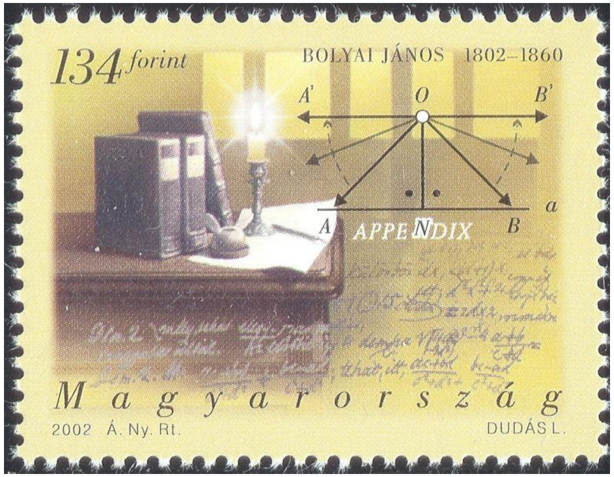


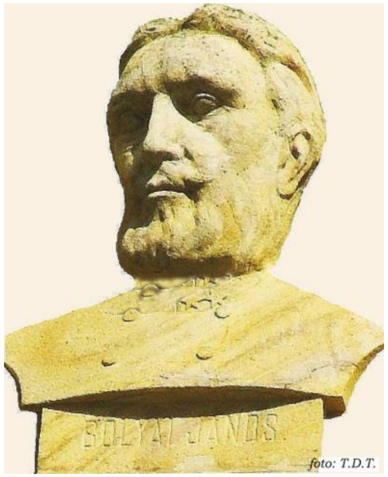

James W. Anderson
Hyperbolic Geometry
Second Edition
Springer
SPRINGER UNDERGRADUATE MATHEMATICS SERIES

Hyperbolic Geometry
BIRGER IVERSEN
London Mathematical Society Student Texts 25

ANALYTIC HYPERBOLIC GEOMETRY
Mathematical Foundations and Applications
Abraham A. Ungar
World Scientific

HISTORY OF MATHEMATICS
VOLUME 10
Sources of Hyperbolic Geometry
John Stillwell
AMERICAN MATHEMATICAL SOCIETY
LONDON MATHEMATICAL SOCIETY

Arlan Ramsay
Robert D. Richtmyer
INTRODUCTION TO HYPERBOLIC GEOMETRY
Springer

Riccardo Benedetti
Carlo Petronio
Lectures on Hyperbolic Geometry
Springer
Universitext

MORGAN&CLAYPOOL PUBLISHERS
A Gyrovector Space Approach to Hyperbolic Geometry
Abraham Albert Ungar
SYNTHESIS LECTURES ON MATHEMATICS AND STATISTICS

THE ELEMENTS OF NON-EUCLIDEAN GEOMETRY
D. M. Y. Sommerville

EUCLIDEAN AND NON-EUCLIDEAN GEOMETRIES
Development and History
MARVIN JAY GREENBERG
Fourth Edition

OXFORD MATHEMATICAL MONOGRAPHS
Complex Hyperbolic Geometry
WILLIAM M. GOLDMAN
OXFORD SCIENCE PUBLICATIONS

CONTEMPORARY MATHEMATICS
311
Complex Manifolds and Hyperbolic Geometry
Clifford J. Earle
William J. Harvey
Editors
American Mathematical Society

ROBERTO BONOLA
NON-EUCLIDEAN GEOMETRY
WITH A SUPPLEMENT CONTAINING
"THE THEORY OF PARALLELS" BY NICHOLAS LOBACHEVSKI AND
"THE SCIENCE OF ABSOLUTE SPACE" BY JOHN BOLYAI

NON-EUCLIDEAN GEOMETRY
H. S. M. COXETER
MATHEMATICAL ASSOCIATION OF AMERICA
SIXTH EDITION

Euclidean and Non-Euclidean Geometries
Development and History
Third Edition
Marvin Jay Greenberg

Henry Parker Manning
Introductory Non-Euclidean Geometry

Studies in the History of Mathematics and Physical Sciences 12
B.A. ROSENFELD
A HISTORY OF NON-EUCLIDEAN GEOMETRY
Evolution of the Concept of a Geometric Space
Springer-Verlag

STEFAN KULCZYCKI
NON-EUCLIDEAN GEOMETRY

EXPERIENCING GEOMETRY
Euclidean and Non-Euclidean with History
Third Edition
David W. Henderson
Daina Taimiņa


Non-Euclidean Plane Geometry And Trigonometry
H. S. CARSLAW
Euclidean and Non-Euclidean Geometries
M. Helena Noronha
EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY
AN ANALYTIC APPROACH
Patrick J. Ryan
COSIMO CLASSICS
[SCIENCE]
NON-EUCLIDEAN GEOMETRY
ROBERTO BONOLA
János Bolyai
Non-Euclidean Geometry
and the Nature of Space
JEREMY J. GRAY
Mathematics and Its Applications
András Prékopa
Emil Molnár
Non-Euclidean Geometries
Springer
The Elements of Non-Euclidean Geometry
Julian Lowell Coolidge
Non-Euclidean Geometry: A Critical And Historical Study Of Its Development (1912)
Roberto Bonola
H. S. Carslaw
Introduction To Non-Euclidean Geometry
Harold E. Wolfe
THE ELEMENTS OF NON-EUCLIDEAN GEOMETRY
D. M. Y. SOMMERVILLE
Undergraduate Texts in Mathematics
George E. Martin
The Foundations of Geometry and the Non-Euclidean Plane
Springer
PRINCETON LANDMARKS IN MATHEMATICS
Luther Pfahler Eisenhart
Riemannian Geometry
Universitext
Sylvestre Gallot
Dominique Hulin
Jacques Lafontaine
Riemannian Geometry
Third Edition
Springer
Cambridge studies in advanced mathematics 98
Riemannian Geometry
A Modern Introduction
Second Edition
ISAAC CHAVEL
Graduate Texts in Mathematics
Peter Petersen
Riemannian Geometry
Second Edition
Springer
MATHEMATICS: THEORY & APPLICATIONS
RIEMANNIAN GEOMETRY
MANFREDO PERDIGÃO DO CARMO
BIRKHÄUSER
Marcel Berger
A Panoramic View of Riemannian Geometry
Springer
OXFORD SCIENCE PUBLICATIONS
RIEMANNIAN GEOMETRY
T. J. WILLMORE

Founders of Non-Euclidean Geometry

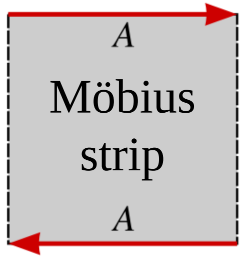
János Bolyai (1802-1860)

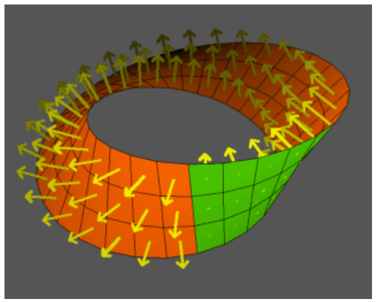


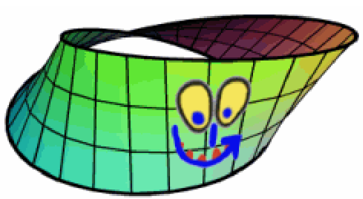


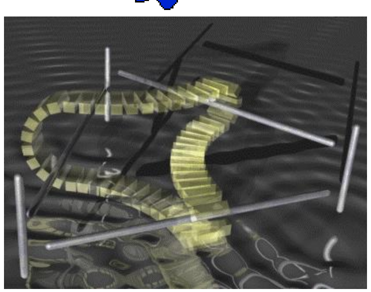
Nikolai Ivanovich Lobachevsky (1792-1856)

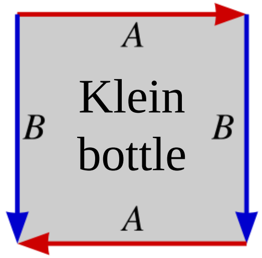
Non-Euclidean Non-Orientable Surfaces

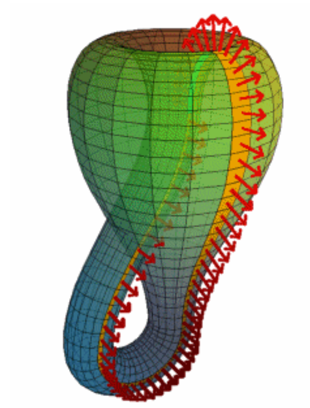


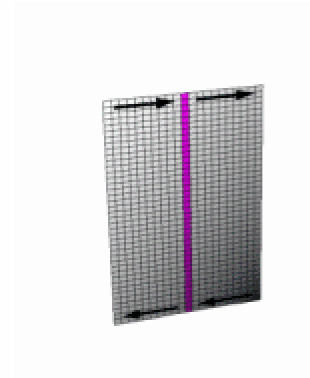


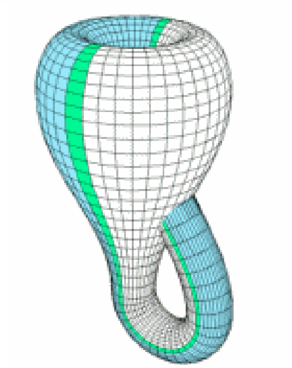


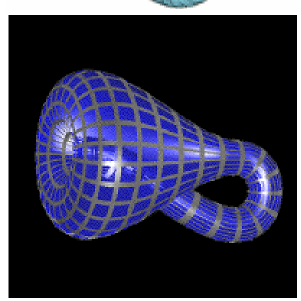




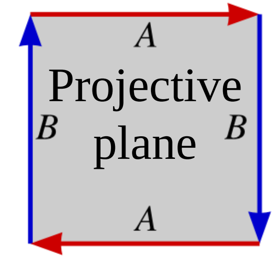


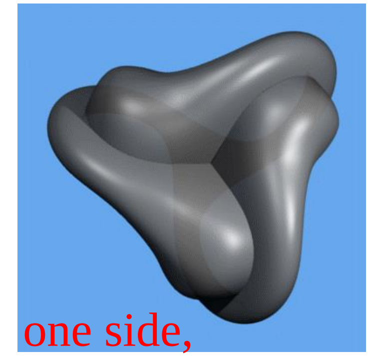




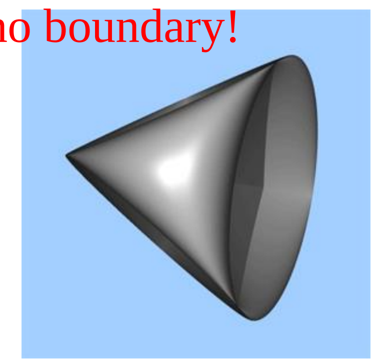


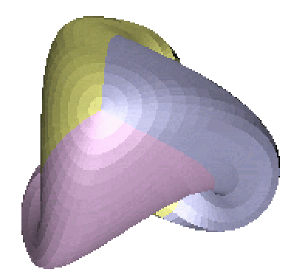
one side,
no boundary!



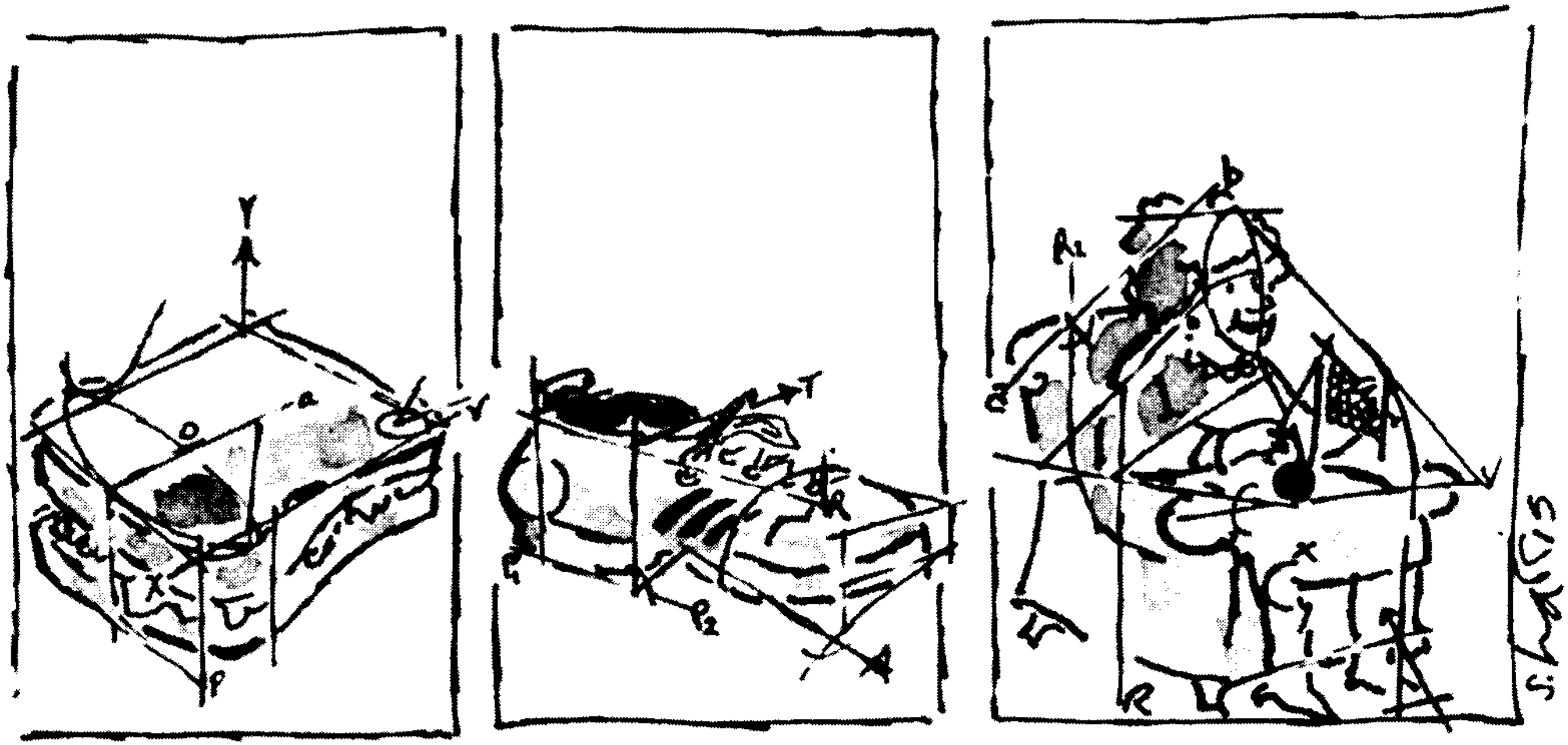


one side,
no boundary!



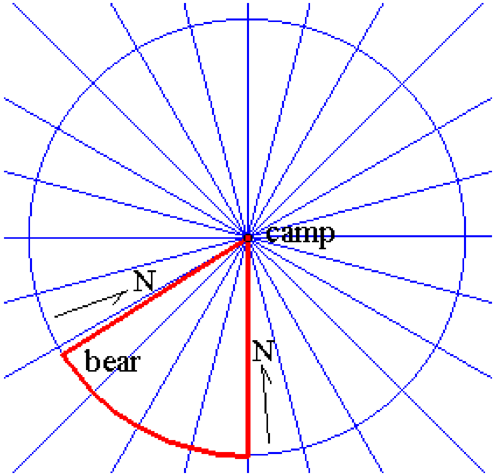


THE GEOMETRY OF EVERYDAY LIFE



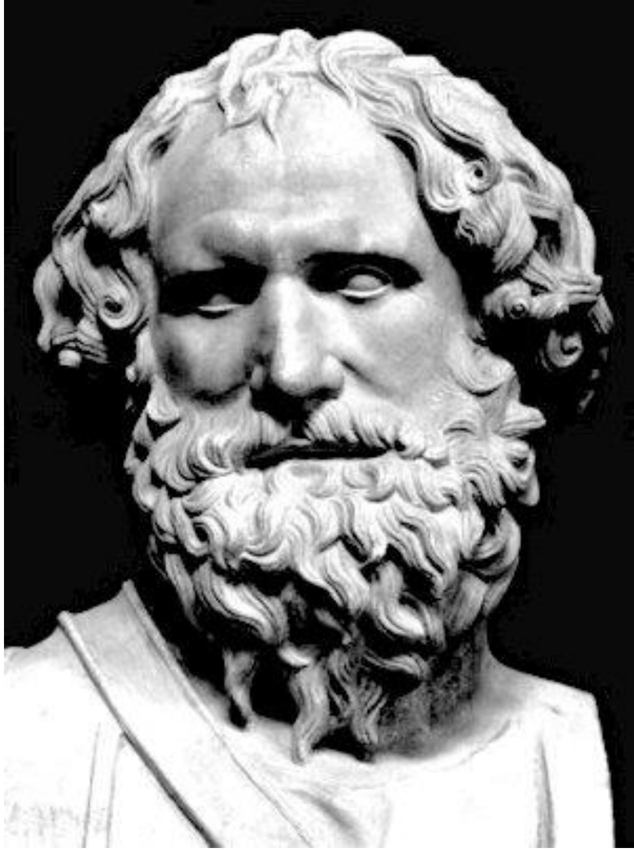
TUNA SANDWICH   SNEAKER   GRANDMA

Problem: A man leaves his house and walks one mile south. He then walks one mile west and sees a Bear. Then he walks one mile north back to his house. What color was the bear?



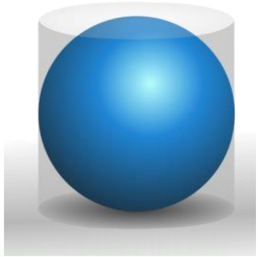
Problem: Is the house location unique?

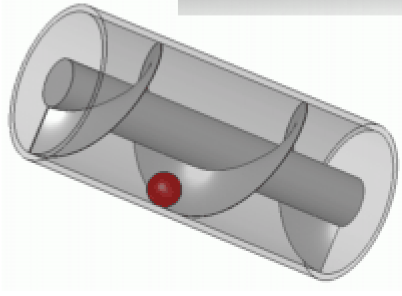
Historical Perspectives

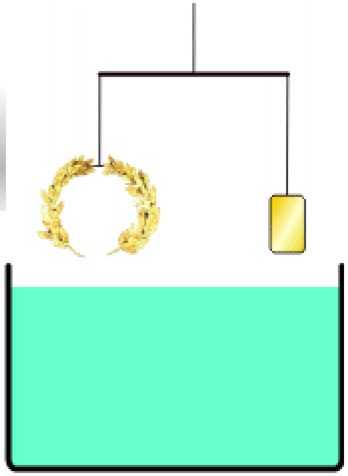


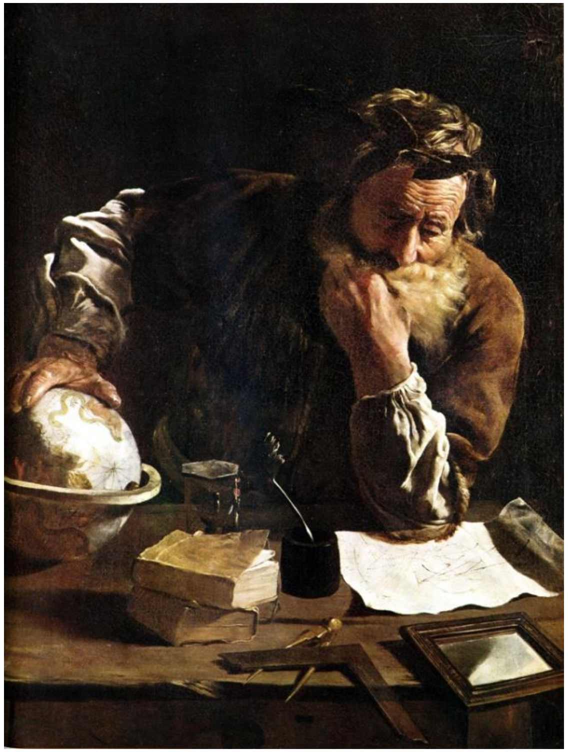
Archimedes of Syracuse (287-212 BC)

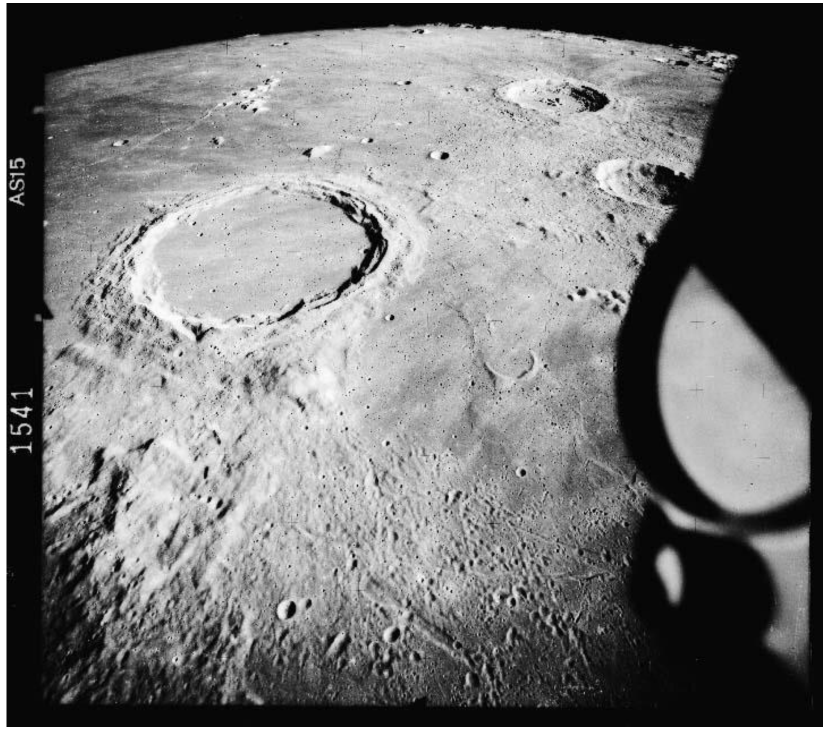
- Mathematician, physicist, engineer, inventor, astronomer
- Leading scientist of classical antiquity
- Originated hydrostatics, mechanics Archimedean screw, spiral, lever
- Discovered Archimedes' principle
- Used infinitesimals, approximated Pi
- Designed siege and naval weapons
- Invented large number notation

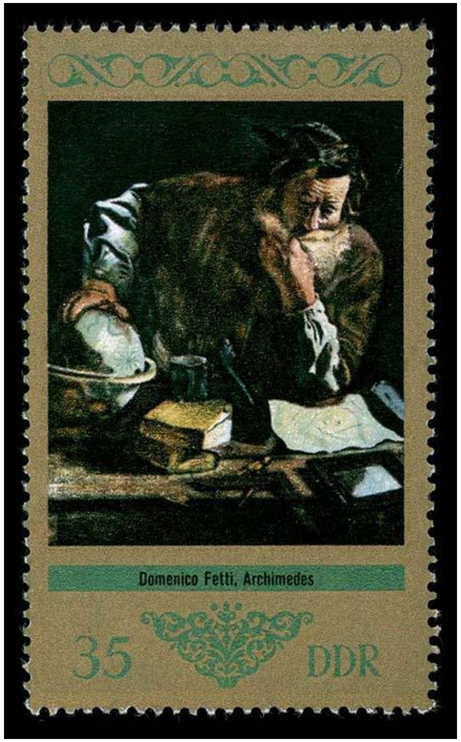


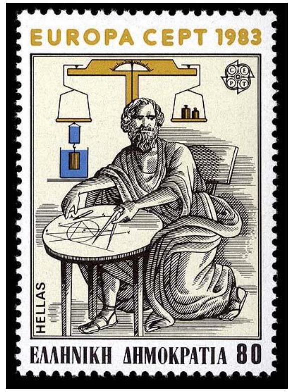


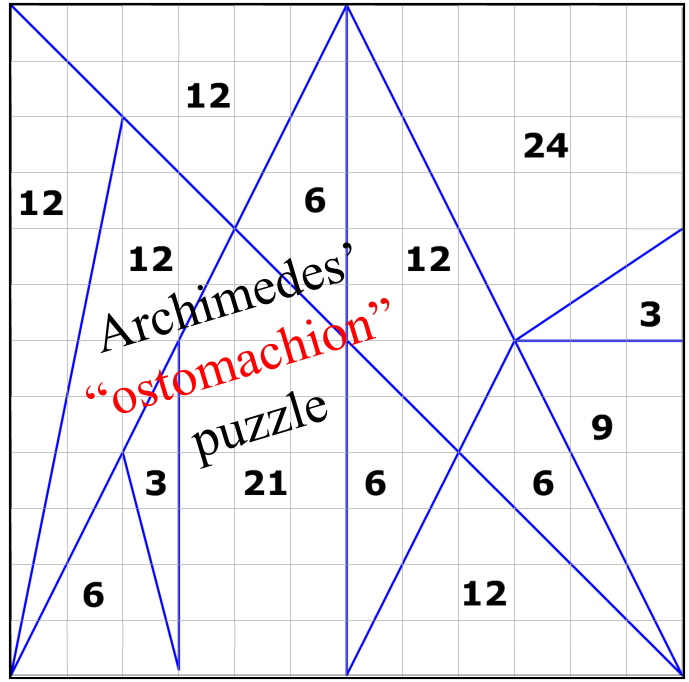


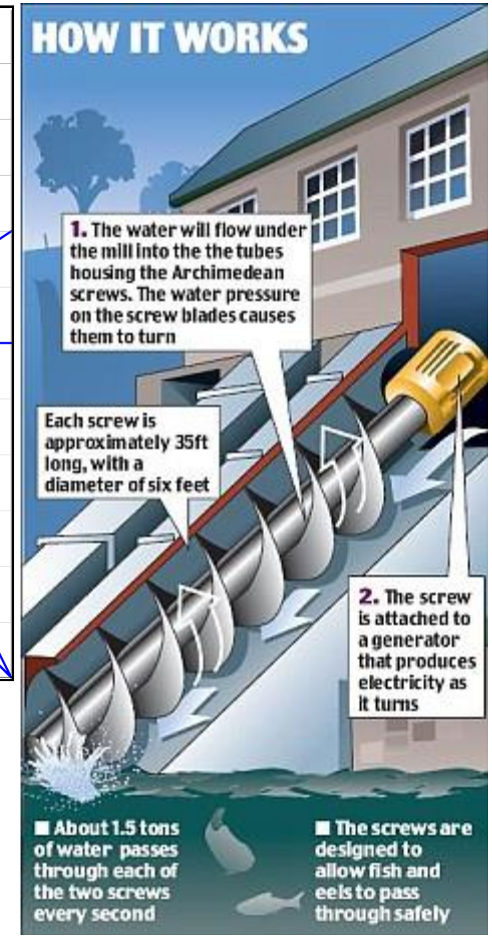


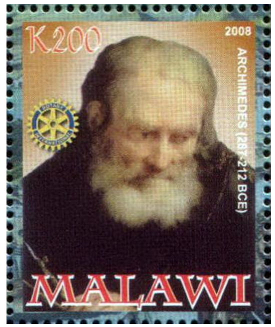

AS15
1541


Domenico Fetti, Archimedes
35
DDR

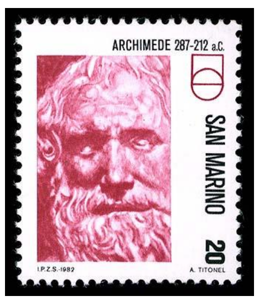

EUROPA CEPT 1983
HELLAS
ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ 80


12
24
12
6
12
12
3
Archimedes'
"ostomachion"
puzzle
9
3
21
6
6
6
12

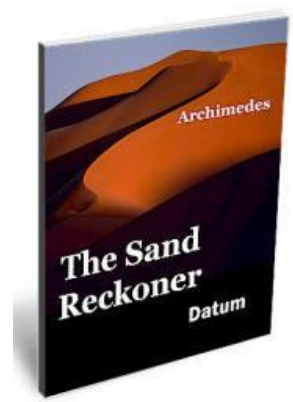

HOW IT WORKS
1. The water will flow under the mill into the the tubes housing the Archimedean screws. The water pressure on the screw blades causes them to turn
Each screw is approximately 35ft long, with a diameter of six feet
2. The screw is attached to a generator that produces electricity as it turns
About 1.5 tons of water passes through each of the two screws every second
The screws are designed to allow fish and eels to pass through safely


K200
2008
ARCHIMEDES (287-212 BCE)
MALAWI

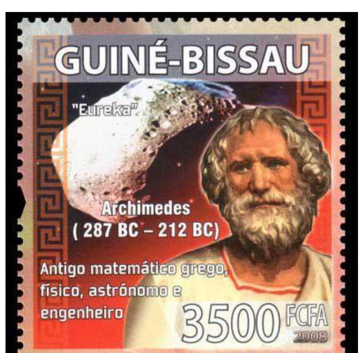

NICARAGUA
LEY DE ARQUIMEDES
AEREO
2
CORDOBAS
DE LA RUE
LAS 10 FORMULAS MATEMATICAS QUE CAMBIARON LA FAZ DE LA TIERRA


ARCHIMEDE 287-212 a.C.
SAN MARINO
20


CORREOS
25
CTS
ESPAÑA


Archimedes
The Sand Reckoner
Datum


EUROPA
ARCHIMEDE
ITALIA
500
I.P.Z.S.-ROMA-1983
E. DONNINI


GUINÉ-BISSAU
"Eureka"
Archimedes
(287 BC – 212 BC)
Antigo matemático grego, físico, astrónomo e engenheiro
3500 FCFA
2008


€ 0,41
800
ITALIA
ANNO MONDIALE DELLA MATEMATICA


ARCHIMEDES
is my homeboy

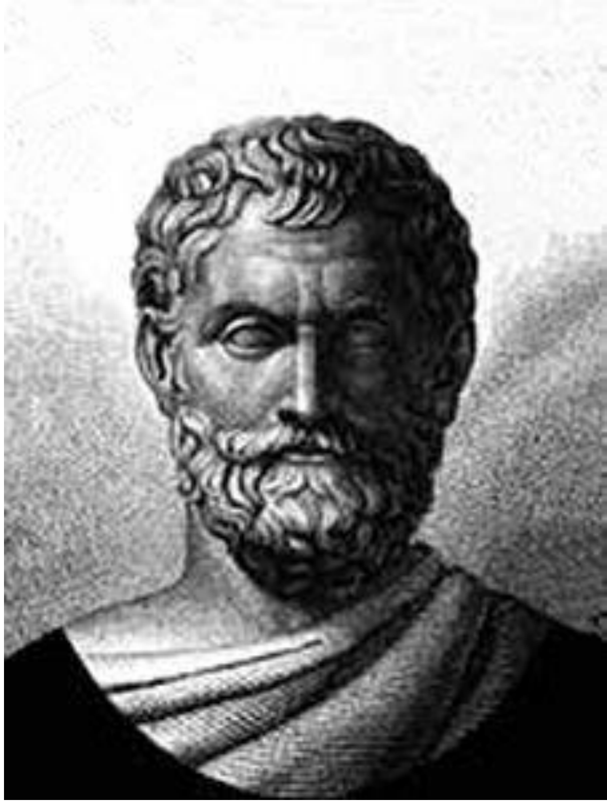
"THERE GOES ARCHIMEDES WITH HIS CONFOUNDED LEVER AGAIN"

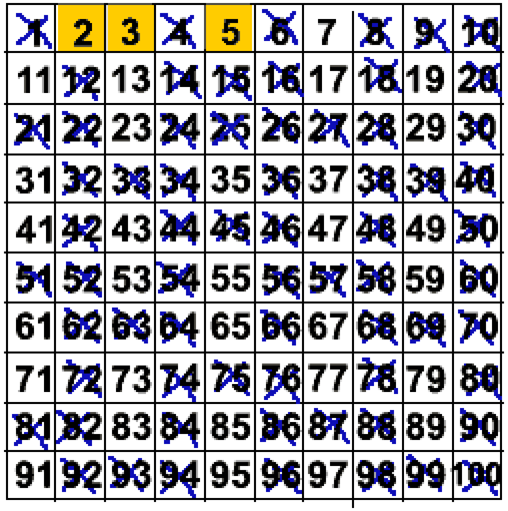
"The periodic table."

Historical Perspectives

Eratosthenes (276BC-194BC)

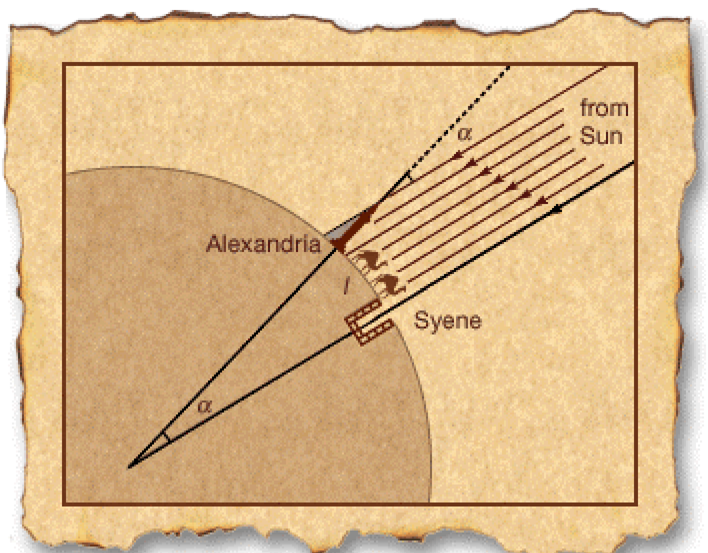
- Chief librarian at Library of Alexandria
- Measured the Earth's size (<1% error!)
- Calculated the Earth-Sun distance
- Invented latitude and longtitude
- Primes - "Sieve of Eratosthenes"
- Chronology of ancient history
- Wrote on astronomy, geography, history, mathematics, philosophy, and literature





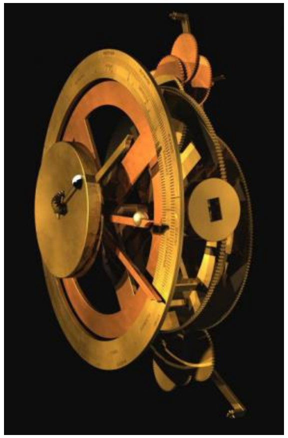
Prime numbers

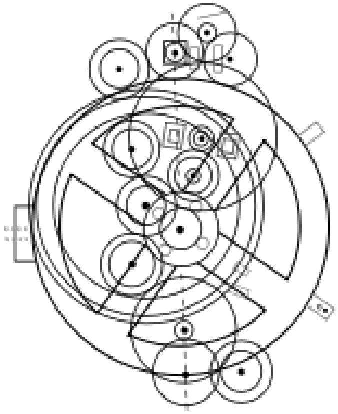
	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

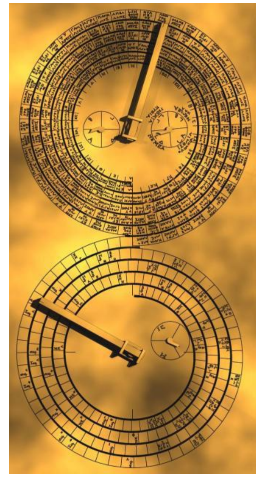


An Ancient Computer: The Antikythera

- Oldest known mechanical computer
- Built around 150-100 BCE !
- Calculates eclipses and astronomical positions of sun, moon, and planets
- Very sophisticated for its era
- Contains dozens of intricate gears
- Comparable to 1700’s Swiss clocks
- Has an attached “instructions manual”
- Still the subject of ongoing research

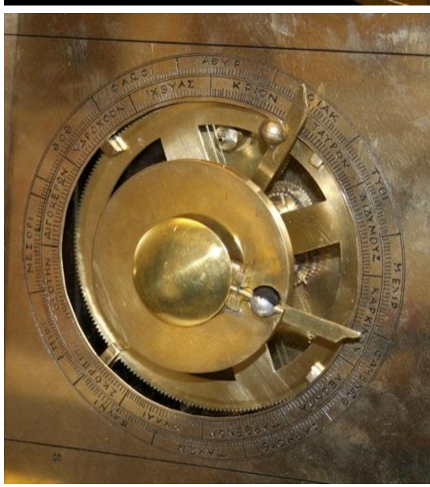
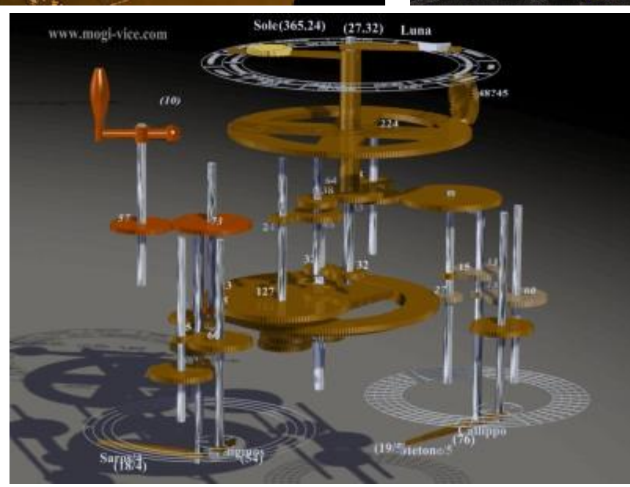


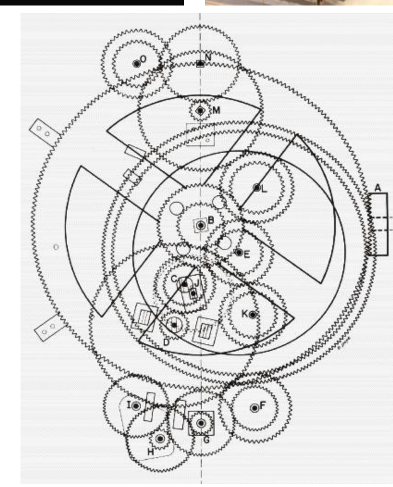


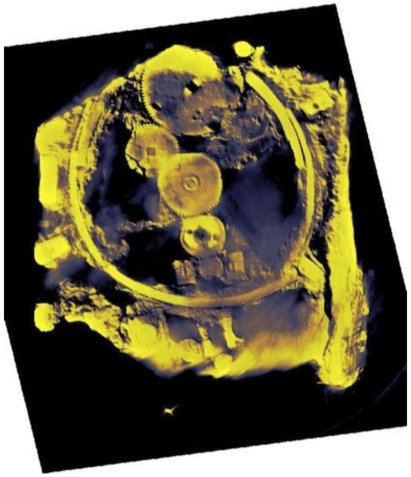





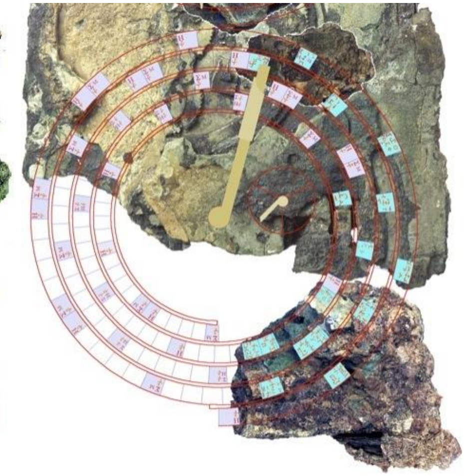


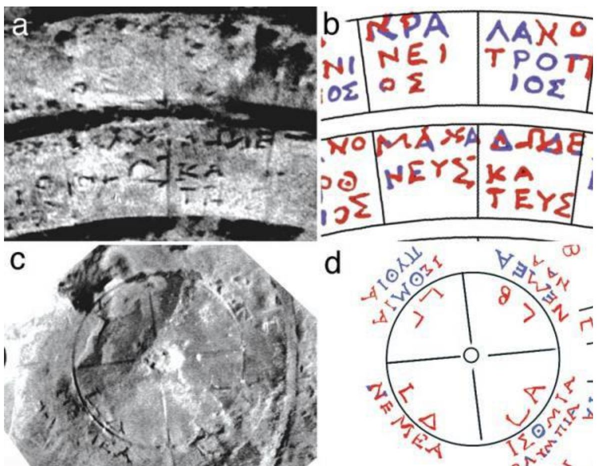



www.mogi-vice.com
Sole(365.24)
(27.32)
Luna

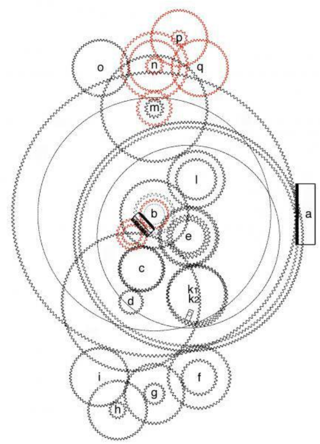


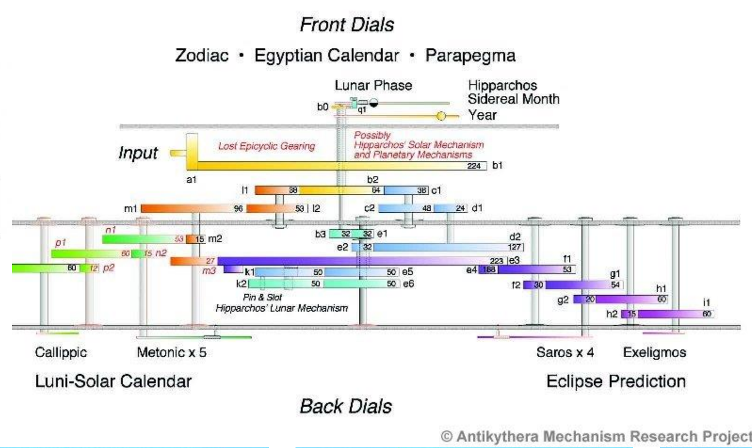


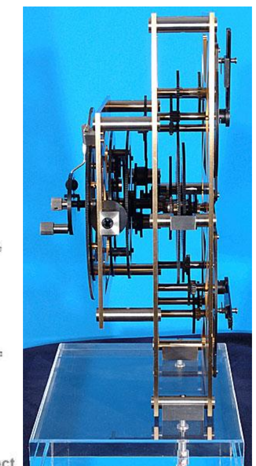

© Antikythera Mechanism Research Project




a
b
c
d




Front Dials
Zodiac · Egyptian Calendar · Parapegma
Lunar Phase
Hipparchos
Sidereal Month
Year
Input
Lost Epicyclic Gearing
Possibly Hipparchos' Solar Mechanism and Planetary Mechanisms
Pin & Slot
Hipparchos' Lunar Mechanism
Callippic
Metonic x 5
Saros x 4
Exeligmos
Luni-Solar Calendar
Eclipse Prediction
Back Dials
© Antikythera Mechanism Research Project



DECODING AN *Ancient Computer*

New explorations have revealed how the Antikythera mechanism modeled lunar motion and predicted eclipses, among other sophisticated tricks

By Tony Freeth

ANCIENT GREEKS knew how to calculate the recurring patterns of lunar eclipses thanks to observations made for centuries by the Babylonians. The Antikythera mechanism would have done those calculations for them—or perhaps for the wealthy Romans who could afford to own it. The depiction here is based on a theoretical reconstruction by the author and his collaborators.

KEY CONCEPTS

- The Antikythera mechanism is a unique mechanical calculator from second-century B.C. Greece. Its sophistication surprised archaeologists when it was discovered in 1901. But no one had anticipated its true power.
- Advanced imaging tools have finally enabled researchers to reconstruct how the device predicted lunar and solar eclipses and the motion of the moon in the sky.
- Inscriptions on the mechanism suggest that it might have been built in the Greek city of Syracuse (now in modern Sicily), perhaps in a tradition that originated with Archimedes.

—*The Editors*

If it had not been for two storms 2,000 years apart in the same area of the Mediterranean, the most important technological artifact from the ancient world could have been lost forever.

The first storm, in the middle of the 1st century B.C., sank a Roman merchant vessel laden with Greek treasures. The second storm, in A.D. 1900, drove a party of sponge divers to shelter off the tiny island of Antikythera, between Crete and the mainland of Greece. When the storm subsided, the divers tried their luck for sponges in the local waters and chanced on the wreck. Months later the divers returned, with backing from the Greek government. Over nine months they recovered a hoard of beautiful ancient Greek objects—rare bronzes, stunning glassware, amphorae, pottery and jewelry—in one of the first major underwater archaeological excavations in history.

One item attracted little attention at first: an undistinguished, heavily calcified lump the size of a phone book. Some months later it fell apart, revealing the remains of corroded bronze gearwheels—all sandwiched together and with teeth just one and a half millimeters long—along with plates covered in scientific scales and Greek inscriptions. The discovery was a shock: until then, the ancients were thought to have made gears only for crude mechanical tasks.

Three of the main fragments of the Antikythera mechanism, as the device has come to be known, are now on display at the Greek National Archaeological Museum in Athens. They look small and fragile, surrounded by imposing bronze statues and other artistic glories of ancient Greece. But their subtle power is even more shocking than anyone had imagined at first.

I first heard about the mechanism in 2000. I was a filmmaker, and astronomer Mike Edmunds of Cardiff University in Wales contacted me because he thought the mechanism would make a great subject for a TV documentary. I learned that over many decades researchers studying the mechanism had made considerable progress, suggesting that it calculated astronomical data, but they still had not been able to fully grasp how it worked. As a former mathematician, I became intensely interested in understanding the mechanism myself.

Edmunds and I gathered an international collaboration that eventually included historians, astronomers and two teams of imaging experts. In the past few years our group has reconstructed how nearly all the surviving parts worked and what functions they performed. The mechanism calculated the dates of lunar and solar eclipses, modeled the moon's subtle apparent motions through the sky to the best of the available knowledge, and kept track of the dates of events of social significance, such as the Olympic Games. Nothing of comparable technological sophistication is known anywhere in the world for at least a millennium afterward. Had this unique specimen not survived, historians would have thought that it could not have existed at that time.

Early Pioneers

German philologist Albert Rehm was the first person to understand, around 1905, that the Antikythera mechanism was an astronomical calculator. Half a century later, when science historian Derek J. de Solla Price, then at the Institute for Advanced Study in Princeton, N.J., described the device in a *Scientific American* article, it still had revealed few of its secrets.

The device, Price suggested, was operated by turning a crank on its side, and it displayed its output by moving pointers on dials located on its front and back. By turning the crank, the user could set the machine on a certain date as indicated on a 365-day calendar dial in the front. (The dial could be rotated to adjust for an extra day every four years, as in today's leap years.) At the same time, the crank powered all the other gears in the mechanism to yield the information corresponding to the set date.

A second front dial, concentric with the calendar, was marked out with 360 degrees and with the 12 signs representing the constellations of the zodiac [*see box on pages 80 and 81*]. These are the constellations crossed by the sun in its apparent motion with respect to the "fixed" stars—"motion" that in fact results from Earth's orbiting the sun—along the path called the ecliptic. Price surmised that the front of the mechanism probably had a pointer showing where along the ecliptic the sun would be at the desired date.

In the surviving fragments, Price identified the remains of a dozen gears that had been part of the mechanism's innards. He also estimated their tooth counts—which is all one can do given that nearly all the gears are damaged and incomplete. Later, in a landmark 1974 study, Price described 27 gears in the main fragment and provided improved tooth counts based on the first x-rays of the mechanism, by Greek radiologist Charalambos Karakalos.

© 2009 SCIENTIFIC AMERICAN, INC.

[THE PLACES]

The Greek and Roman worlds, circa 145 B.C.

Where Was It From?

The Antikythera mechanism was built around the middle of the 2nd century B.C., a time when Rome was expanding at the expense of the Greek-dominated Hellenistic kingdoms (*green*). Divers recovered its corroded remnants (*including fragment at left*) in A.D. 1901 from a shipwreck near the island of Antikythera. The ship sank around 65 B.C. while carrying Greek artistic treasures, perhaps from Pergamon to Rome. Rhodes had one of the major traditions of Greek astronomy, but the latest evidence points to a Corinthian origin. Syracuse, which had been a Corinthian colony in Sicily, is a possibility: the great Greek inventor Archimedes had lived there and may have left behind a technological tradition.

[THE AUTHOR]

Tony Freeth's academic background is in mathematics and mathematical logic (in which he holds a Ph.D.). His award-winning career as a filmmaker culminated in a series of documentaries about increasing crop yields in sub-Saharan Africa, featuring the late Nobel Peace Prize Laureate Norman Borlaug. Since 2000 Freeth has returned to an academic focus with research on the Antikythera mechanism. He is managing director of the film and television production company Images First, and he is now developing a film on the mechanism.

Tooth counts indicate what the mechanism calculated. For example, turning the crank to give a full turn to a primary 64-tooth gear represented the passage of a year, as shown by a pointer on the calendar dial. That primary gear was also paired to two 38-tooth secondary gears, each of which consequently turned by 64/38 times for every year. Similarly, the motion relayed from gear to gear throughout the mechanism; at each step, the ratio of the numbers of gear teeth represents a different fraction. The motion eventually transmitted to the pointers, which thus turned at rates corresponding to different astronomical cycles. Price discovered that the ratios of one of these gear trains embodied an ancient Babylonian cycle of the moon.

Price, like Rehm before him, suggested that the mechanism also contained epicyclic gearing—gears spinning on bearings that are themselves attached to other gears, like the cups on a Mad Hatter teacup ride. Epicyclic gears extend the range of formulas gears can calculate beyond multiplications of fractions to additions and subtractions. No other example of epicyclic gearing is known to have existed in Western technology for another 1,500 years.

Several other researchers studied the mechanism, most notably Michael Wright, a curator at the Science Museum in London, in collaboration with computer scientist Allan Bromley of the University of Sydney. They took the first three-dimensional x-rays of the mechanism and showed that Price's model of the mechanism had to be wrong. Bromley died in 2002, but Wright persisted and made significant advances. For example, he found evidence that the back dials, which at first look like concentric rings, are in fact spirals and discovered an epicyclic mechanism at the front that calculated the phase of the moon.

Wright also adopted one of Price's insights, namely that the dial on the upper back might be a lunar calendar, based on the 19-year, 235-lunar-month cycle called the Metonic cycle. This calendar is named after fifth-century B.C. astronomer Meton of Athens—although it had been discovered earlier by the Babylonians—and is still used today to determine the Jewish festival of Rosh Hashanah and the Christian festival of Easter. Later, we would discover that the pointer was extensible, so that a pin on its end could follow a groove around each successive turn of the spiral.

BladeRunner in Athens

As our group began its efforts, we were hampered by a frustrating lack of data. We had no access to the previous x-ray studies, and we did not even have a good set of still photographs. Two images in a science magazine—x-rays of a goldfish and an enhanced photograph of a Babylonian clay tablet—suggested to me new ways to get better data.

We asked Hewlett-Packard in California to perform state-of-the-art photographic imaging and X-Tek Systems in the U.K. to do three-dimensional x-ray imaging. After four years of careful diplomacy, John Seiradakis of the Aristotle University of Thessaloniki and Xenophon Moussas of the University of Athens obtained the required permissions, and we arranged for the imaging teams to bring their tools to Athens, a necessary step because the Antikythera mechanism is too fragile to travel.

Meanwhile we had a totally unexpected call from Mary Zafeiropoulou at the museum. She had been to the basement storage and found boxes of bits labeled "Antikythera." Might we be interested? Of course we were interested. We now had a total of 82 fragments, up from about 20.

The HP team, led by Tom Malzbender, assembled a mysterious-looking dome about five feet across and covered in electronic flashbulbs that provided lighting from a range of different angles. The team exploited a technique from the computer gaming industry, called polynomial texture mapping, to enhance surface details. Inscriptions Price had found difficult to read were now clearly legible, and fine details could be enhanced on the computer screen by controlling the reflectance of the surface and the angle of the lighting. The inscriptions are essentially an instruction manual written on the outer plates.

A month later local police had to clear the streets in central Athens so that a truck carrying the BladeRunner, X-Tek's eight-ton x-ray machine, could gain access to the museum. The BladeRunner performs computed tomography similar to a hospital's CT scan, but with finer detail. X-Tek's Roger Hadland and his group had specially modified it with enough x-ray power to penetrate the fragments of the Antikythera mechanism. The resulting 3-D reconstruction was wonderful: whereas Price could see only a puzzle of overlapping gears, we could now isolate layers inside the fragment and see all the fine details of the gear teeth.

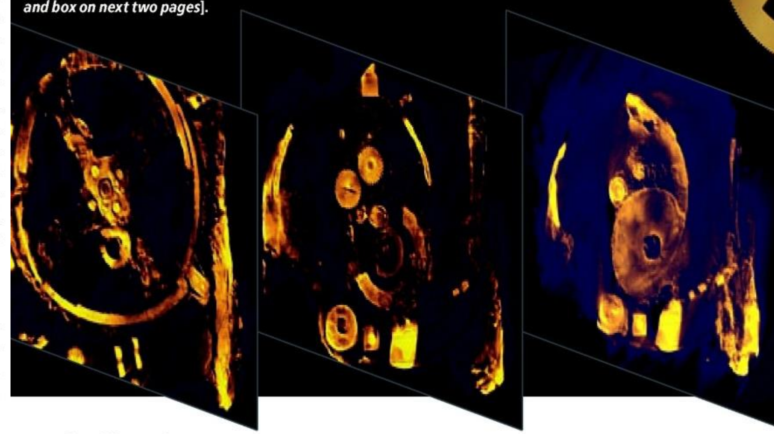
Unexpectedly, the x-rays revealed more than 2,000 new text characters that had been hidden deep inside the fragments. (We have now identified and interpreted a total of 3,000 characters out of perhaps 15,000 that existed originally.) In Athens, Moussas and Yanis Bitsakis, also at the University of Athens, and Agamemnon Tselikas of the Center for History and Palaeography be-

Historians would have thought that SOMETHING SO COMPLEX *could not have existed at the time.*

[THE RECONSTRUCTION]

Anatomy of a Relic

Computed tomography—a 3-D mapping obtained from multiple x-ray shots—enabled the author and his colleagues to get inside views of the Antikythera mechanism's remnants. For example, a CT scan can be used to virtually slice up an object (*below, slices of main fragment*). The information helped the team see how the surviving gears connected and estimate their tooth counts, which determined what calculations they performed. The team could then reconstruct most of the device [*see model at right and box on next two pages*].



gan to discover inscriptions that had been invisible to human eyes for more than 2,000 years. One translated as "... spiral subdivisions 235...," confirming that the upper back dial was a spiral describing the Metonic calendar.

Babylon System

Back at home in London, I began to examine the CT scans as well. Certain fragments were clearly all part of a spiral dial in the lower back. An estimate of the total number of divisions in the dial's four-turn spiral suggested 220 to 225.

The prime number 223 was the obvious contender. The ancient Babylonians had discovered that if a lunar eclipse is observed—something that can happen only during a full moon—usually a similar lunar eclipse will take place 223 full moons later. Similarly, if the Babylonians saw a solar eclipse—which can take place only during a new moon—they could predict that 223 new moons later there would be a similar one (although they could not always see it: solar eclipses are visible only from specific locations, and ancient astronomers could not predict them reliably). Eclipses repeat this way because every 223 lunar months the sun, Earth and the moon return to approximately the same alignment with respect to one another, a periodicity known as the Saros cycle.

Between the scale divisions were blocks of symbols, nearly all containing Σ (*sigma*) or *H* (*eta*), or both. I soon realized that Σ stands for Σεληνη (*selene*), Greek for "moon," indicating a lunar eclipse; *H* stands for Ηλιοσ (*helios*), Greek for "sun," indicating a solar eclipse. The Babylonians also knew that within the 223-month period, eclipses can take place only in particular months, arranged in a predictable pattern and separated by gaps of five or six months; the distribution of symbols around the dial exactly matched that pattern.

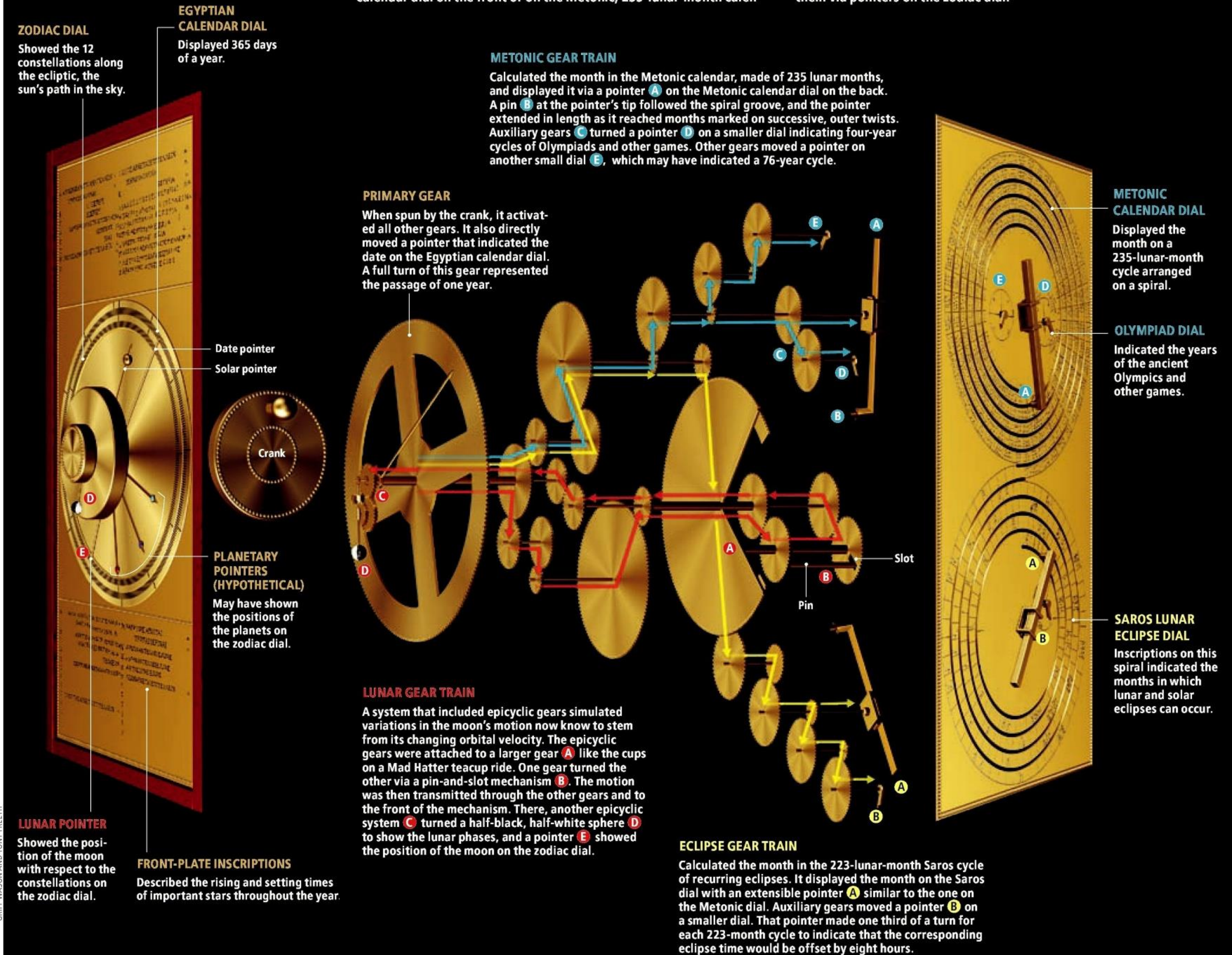
I now needed to follow the trail of clues into the heart of the mechanism to discover where this new insight would lead. The first step was to find a gear with 223 teeth to drive this new Saros dial. Karakalos had estimated that a large gear visible at the back of the main fragment had 222 teeth. But Wright had revised this estimate to 223, and Edmunds confirmed this. With plausible tooth counts for other gears and with the addition of a small, hypothetical gear, this 223-tooth gear could perform the required calculation.

But a huge problem still remained unsolved and proved to be the hardest part of the gearing to crack. In addition to calculating the Saros cy-

[INSIDE THE ANTIKYTHERA MECHANISM]

Astronomical Clockwork

This exploded view of the mechanism shows all but one of the 30 known gears, plus a few that have been hypothesized. Turning a crank on the side activated all the gears in the mechanism and moved pointers on the front and back dials: the arrows colored blue, red and yellow explain how the motion transmitted from one gear to the next. The user would choose a date on the Egyptian, 365-day calendar dial on the front or on the Metonic, 235-lunar-month calendar on the back and then read the astronomical predictions for that time—such as the position and phases of the moon—from the other dials. Alternatively, one could turn the crank to set a particular event on an astronomical dial and then see on what date it would occur. Other gears, now lost, may have calculated the positions of the sun and of some or all of the five planets known in antiquity and displayed them via pointers on the zodiac dial.



GRIFF WASON AND TONY FREETH

cle, the large 223-tooth gear also carried the epicyclic system noticed by Price: a sandwich of two small gears attached to the larger gear in teacup-ride fashion. Each epicyclic gear also connected to another small gear. Confusingly, all four small gears appeared to have the same tooth count—50—which seemed nonsensical because the output would then be the same as the input.

After months of frustration, I remembered that Wright had observed that one of the two epicyclic gears has a pin on its face that engages with a slot on the other. His key idea was that the two gears turned on slightly different axes, separated by about a millimeter. As a consequence, the angle turned by one gear alternated between being slightly wider and being slightly narrower than the angle turned by the other gear. Thus, if one gear turned at a constant rate, the other gear's rate kept varying between slightly faster and slightly slower.

Ask for the Moon

Although Wright rejected his own observation, I realized that the varying rotation rate is precisely what is needed to calculate the moon's motion according to the most advanced astronomical theory of the second century B.C., the one often attributed to Hipparchos of Rhodes. Before Kepler (A.D. 1605), no one understood that orbits are elliptical and that the moon accelerates toward the perigee—its closest point to Earth—and slows down toward the apogee, the opposite point. But the ancients did know that the moon's motion against the zodiac appears to periodically slow down and speed up. In Hipparchos's model, the moon moved at a constant rate around a circle whose center itself moved around a circle at a constant rate—a fairly good approximation of the moon's apparent motion. These circles on circles, themselves called epicycles, dominated astronomical thinking for the next 1,800 years.

There was one further complication: the apogee and perigee are not fixed, because the ellipse of the moon's orbit rotates by a full turn about every nine years. The time it takes for the body to get back to the perigee is thus a bit longer than the time it takes it to come back to the same point in the zodiac. The difference was just 0.112579655 turns a year. With the input gear having 27 teeth, the rotation of the large gear was slightly too big; with 26 teeth, it was slightly too small. The right result seemed to be about halfway in between. So I tried the impossible idea that the input gear had 26 ½ teeth. I pressed the key on my calculator, and it gave 0.112579655—exactly the right answer. It could not be a coincidence to nine places of decimals! But gears cannot have fractional numbers of teeth.

Then I realized that 26 ½ × 2 = 53. In fact, Wright had estimated a crucial gear to have 53 teeth, and I now saw that that count made everything work out. The designer had mounted the pin and slot epicyclically to subtly slow down the period of its variation while keeping the basic rotation the same, a conception of pure genius. Thanks to Edmunds, we also realized that the epicyclic gearing system, which is in the back of the mechanism, moved a shaft that turned inside another, hollow shaft through the rest of the mechanism and to the front, so that the lunar motion could be represented on the zodiac dial and on the lunar phase display. All gear counts were now explained, with the exception of one small gear that remains a mystery to this day.

Further research has caused us to make some modifications to our model. One was about a small subsidiary dial that is positioned in the back, inside the Metonic dial, and is divided into four quadrants. The first clue came when I read the word "*NEMEA*" under one of the quadrants. Alexander Jones, a New York University historian, explained that it refers to the Nemean Games, one of the major athletic events in ancient Greece. Eventually we found, engraved round the four sectors of the dial, most of "*ISTHMIA*," for games at Corinth, "*PYTHIA*," for games at Delphi, "*NAA*," for minor games at Dodona, and "*OLYMPIA*," for the most important games of the Greek world, the Olympics. All games took place every two or four years. Previously we had considered the mechanism to be purely an instrument of mathematical astronomy, but the Olympiad dial—as we named it—gave it an entirely unexpected social function.

Twenty-nine of the 30 surviving gears calculate cycles of the sun and the moon. But our studies of the inscriptions at the front of the mechanism have also yielded a trove of information on the risings and settings of significant stars and of the planets. Moreover, on the "primary" gearwheel at the front of the mechanism remnants of bearings stand witness to a lost epicyclic system that could well have modeled the back-and-forth motions of the planets along the ecliptic (as well as the anomalies in the sun's own motion). All these clues strongly support the inclusion of the sun and of at least some of the five planets known in ancient times—Mercury, Venus, Mars, Jupiter and Saturn.

Wright built a model of the mechanism with epicyclic systems for all five planets. But his ingenious layout does not agree with all the evidence. With its 40 extra gears, it may also be too complex to match the brilliant simplicity of the rest of the mechanism. The ultimate answer may still lie 50 meters down on the ocean floor.

Eureka?

The question of where the mechanism came from and who created it is still open. Most of the cargo in the wrecked ship came from the eastern Greek world, from places such as Pergamon, Kos and Rhodes. It was a natural guess that Hipparchos or another Rhodian astronomer built the mechanism. But text hidden between the 235 monthly scale divisions of the Metonic calendar contradicts this view. Some of the month names were used only in specific locations in the ancient Greek world and suggest a Corinthian origin. If the mechanism was from Corinth itself, it was almost certainly made before Corinth was completely devastated by the Romans in 146 B.C. Perhaps more likely is that it was made to be used in one of the Corinthian colonies in northwestern Greece or Sicily.

Sicily suggests a remarkable possibility. The island's city of Syracuse was home to Archimedes, the greatest scientist of antiquity. In the first century B.C. Roman statesman Cicero tells how in 212 Archimedes was killed at the siege of Syracuse and how the victorious Roman general, Marcellus, took away with him only one piece of plunder—an astronomical instrument made by Archimedes. Was that the Antikythera mechanism? We believe not, because it appears to have been made many decades after Archimedes died. But it could have been constructed in a tradition of instrument making that originated with the eureka man himself.

Many questions about the Antikythera mechanism remain unanswered—perhaps the greatest being why this powerful technology seems to have been so little exploited in its own era and in succeeding centuries.

In *Scientific American*, Price wrote:

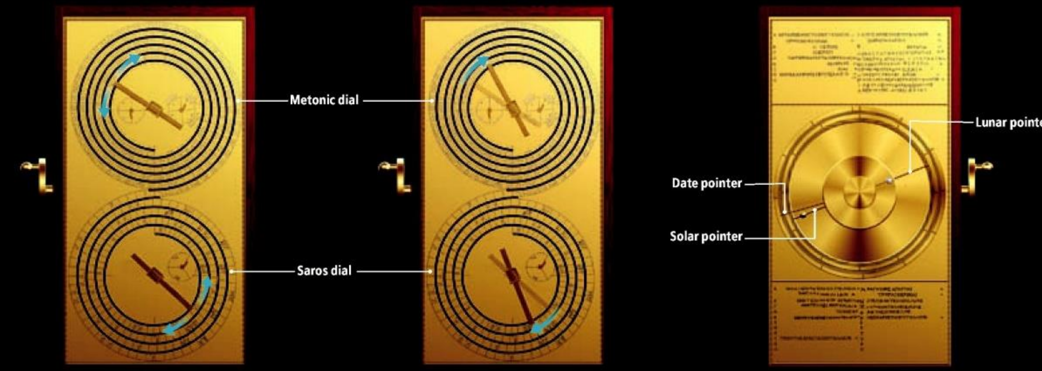
> *It is a bit frightening to know that just before the fall of their great civilization the ancient Greeks had come so close to our age, not only in their thought, but also in their scientific technology.*

Our discoveries have shown that the Antikythera mechanism was even closer to our world than Price had conceived. ■

[A USER'S MANUAL]

How to Predict an Eclipse

Operating the Antikythera mechanism may have required only a small amount of practice and astronomical knowledge. After an initial calibration by an expert, the mechanism could provide fairly accurate predictions of events several decades in the past or future. The inscriptions on the Saros dial, coming at intervals of five or six months, corresponded to months when Earth, the sun and the moon come to a near alignment (and so represented potential solar and lunar eclipses) in a 223-lunar-month cycle. Once the month of an eclipse was known, the actual day could be calculated on the front dials using the fact that solar eclipses always happen during new moons and lunar eclipses during full moons.



RESET DATE
Begin by turning the crank to set the current month and year on the Metonic calendar. The lower pointer will turn to the corresponding month on the Saros (*eclipse*) dial.

FIND ECLIPSE MONTH
Turn the crank to move time forward until the pointer on the Saros dial points to an eclipse inscription. The inscription will indicate month and time of the day (but not the day) of an eclipse and whether it will be solar or lunar.

CALCULATE DAY
Adjust the crank until the lunar and solar pointers are aligned (*for a solar eclipse*) or at 180 degrees (*for a lunar eclipse*). The Egyptian calendar pointer will move correspondingly and indicate the day of the eclipse.

DATA BY X-TEK SYSTEMS, SOFTWARE BY VOLUME GRAPHICS, © 2005 ANTIKYTHERA MECHANISM RESEARCH PROJECT

GRIFF WASON AND TONY FREETH

MORE TO EXPLORE

An Ancient Greek Computer. Derek J. de Solla Price in *Scientific American*, Vol. 200, No. 6, pages 60–67; June 1959.

Gears from the Greeks: The Antikythera Mechanism—A Calendar Computer from ca. 80 B.C. Derek de Solla Price in *Transactions of the American Philosophical Society*, New Series, Vol. 64, No. 7, pages 1–70; 1974.

Decoding the Ancient Greek Astronomical Calculator Known as the Antikythera Mechanism. Tony Freeth et al. in *Nature*, Vol. 444, pages 587–591; November 30, 2006.

Calendars with Olympiad Display and Eclipse Prediction on the Antikythera Mechanism. Tony Freeth, Alexander Jones, John M. Steele and Yanis Bitsakis in *Nature*, Vol. 454, pages 614–617; July 31, 2008.

The Antikythera Mechanism Research Project: **www.antikythera-mechanism.gr**