# Algorithms Homework 4

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Please make all algorithms as efficient as you can, and state their time and space complexities.

1-13. Solve the following problems from the [Cormen, 2nd Edition] algorithms textbook:

p. 216: 10.4-5
p. 223: 11.1-4
p. 384: 16.2-5
p. 392: 16.3-8
p. 530: 22.1-6
p. 539: 22.2-7, 22.2-8
p. 549: 22.3-11
p. 552: 22.4-2
p. 559: 22-3
p. 567: 23.1-11
p. 574: 23.2-7
p. 600: 24.3-4

14. Give an **optimal** algorithm to determine the diameter of a weighted tree (i.e., longest path between any two nodes). Prove the optimality of your algorithm. What is your time complexity?

15. Give an algorithm to determine whether any two of N segments on a line intersect.

16. Give an algorithm to find the N-way intersection of N rectangles with sides parallel to the axis.

17. Give an algorithm to compute the diameter of a given pointset (i.e., the distance between the farthest pair of points).

18. Give a linear-time algorithm to compute the area of a given convex polygon.

19. Give a linear-time algorithm to compute the area of a given non-convex polygon.

20. What is the worst-case cost of a traveling salesman tour over n points located arbitrarily in the unit square? Prove it. (Express the cost asymptotically as O(f(n)) for some function f.)

21. What is the expected (average) cost of a minimum spanning tree over n points uniformly distributed in the unit square? (Express the cost asymptotically as O(f(n)) for some function f.)

22. Given N **red** points and N **blue** points arbitrarily placed in the plane, we seek to match these 2N points into N pairs, so that no two of the line segments defined by these pairs intersect. (Assume that no three of the 2N distinct points are collinear.) Prove or disprove: there always exists such a non- self-intersecting matching for any arrangement of N red and N blue points arbitrarily located in the plane. Devise an efficient algorithm that given an arbitrary set of 2N red & blue points in the plane, finds such a matching, when one exists. Generalize all this to dimension D.