Algorithms Homework 5
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Please make all algorithms as efficient as you can, and state their time and space complexities.

1-8. Solve the following problems from the [Cormen, 2nd Edition] algorithms textbook:

p. 939-940: 33.1-4, 33.1-7
p. 946: 33.2-4
p. 956-957: 33.3-2, 33.3-3, 33.3-5
p. 962-963: 33-1, 33-3

9. Give an algorithm to determine if a given graph is 2-colorable (i.e., whether it is possible to assign one of two colors to all the vertices so that no two adjacent vertices share the same color:

10. Give an efficient algorithm to compute the maximum collinear subset of a given pointset.

11. Give a quadratic-time algorithm to find the maximum collinear subset that is also equally-spaced along its containing line.

12. Consider a labeling of the nodes of an n-node tree using the integers 1 through n (where each node is uniquely numbered). Next, label each edge of the tree with the absolute value of the difference between the labels of its two incident nodes. Such an node/edge labeling is called "**graceful**" if all the edge labels are unique (i.e., each of the values 1..(n-1) occurs exactly once over all the edges). Prove or disprove: all infinite trees have a graceful labeling (assume that an infinite tree has finite degree at each node, but contains at least one infinite path from the root.)

13. Prove or disprove: if NP≠co-NP then P≠NP.

14. Prove or disprove: 2-SAT is polynomial-time solvable.

15. Prove or disprove: any group of people can be partitioned into two subgroups such that at least half the friends of each person belong to the subgroup of which that person is not a member? (Assume that each pair of people are either friends or not.)

16. Define a set of integers to be "strange" if all of its members are mutually relatively prime, i.e., the greatest common integer divisor of **all** of the elements is 1. Define a set to be "super-strange" if it is strange, but does not contain any strange **proper** subsets Prove or disprove: for any positive integer K, there exists a super-strange set of size K.

17. Given an integer K>0 and a finite set L of arbitrary lines in $\Re^3$, we seek a maximum-cardinality set S of pairwise-disjoint spheres each of radius K, where the spheres in S are located in $\Re^3$ in such a way that none of them intersects any of the given lines in L (or each other). In other words, we need to find as many pairwise disjoint spheres as possible, all of which avoid intersecting all the given lines. What are the best general lower-bound and upper bound on the cardinality of S, over all possible K and L? Devise an algorithm that for any given K and L determines such a maximum-cardinality set of spheres S.