

# Normalization

## 3NF and BCNF

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### CS 4750

### Database Systems

[A. Silberschatz, H. F. Korth, S. Sudarshan, Database System Concepts, Ch.7]

[Ricardo and Urban, Database Illuminated, Ch.6]

[ <https://www.w3schools.in/dbms/database-normalization/> ]

# 3NF and Decomposition

- **Lossless-join**
- Always **dependency preserving**
- Possible to have extra data (there may be redundancy)

Questions:

Is the relation in 3NF?

Is any refinement needed?

To calculate 3NF

- Identify PK of the original table
- Take **Canonical Cover (Fc)**
- Turn **(minimal set of) FDs into tables**

# Canonical Cover ( $F_c$ )

- A **minimal set** of functional dependencies that has the same closure as the original set  $F$
- Extraneous attributes = attribute of FDs that we can removed without changing the closure of FDs
- $F$  logically implies all dependencies in  $F_c$
- $F_c$  logically implies all dependencies in  $F$
- No FD in  $F_c$  contains an extraneous attribute

$F$  and  $F^+$  are  
logically  
equivalent

**Minimal basis for a set of FDs:** For any set of FDs, there is at least one minimal basis, which is a set of FDs **equivalent to the original** (each set implies the other set), with singleton right sides, **no FD that can be eliminated** while preserving equivalence, and **no attribute in a left side that can be eliminated** while preserving equivalence

# Canonical Cover ( $F_c$ )

Compute the canonical cover of a set of functional dependencies  $F$

Always start with  $F$  and use rules to minimize

$F_c = F$

**repeat**

    apply union rule to replace any dependencies  $f: X_1 \rightarrow Y_1$   
    and  $f: X_1 \rightarrow Y_2$  with  $f: X_1 \rightarrow Y_1Y_2$

**for each** functional dependency  $f_i$

**if**  $f_i$  contains an extraneous attribute either in  $X$  or in  $Y$

**then** remove an extraneous attribute

**until**  $F_c$  does not change any further

# Example 1: 3NF and Fc

Given  $R(A,B,C,D,E)$

Let's do this together

FDs =  $\{ A \rightarrow B, AB \rightarrow D, B \rightarrow BDE, C \rightarrow D, D \rightarrow D \}$

Compute Fc and convert the relation into 3NF

Observation: AC is a minimal super key of the given R

(1) write all LHS

A  $\rightarrow$   
B  $\rightarrow$   
AB  $\rightarrow$   
C  $\rightarrow$   
D  $\rightarrow$

(2) copy FDs as is

A  $\rightarrow$  B  
B  $\rightarrow$  B DE  
AB  $\rightarrow$  D  
C  $\rightarrow$  D  
D  $\rightarrow$  D

(3) remove reflexivity

A  $\rightarrow$  B  
B  $\rightarrow$  ~~B~~ DE  
AB  $\rightarrow$  D  
C  $\rightarrow$  D  
~~D  $\rightarrow$  D~~

(4) remove extraneous attr

A  $\rightarrow$  B  
B  $\rightarrow$  DE  
~~AB  $\rightarrow$  D~~  
C  $\rightarrow$  D

A  $\rightarrow$  B and B  $\rightarrow$  D.  
Thus, remove AB  $\rightarrow$  D

# Example 1: 3NF and Fc

Let's do this together

(from previous page)

(4) Remove  
extraneous attr

A	→	B
B	→	DE
C	→	D

Super key

Turn Fc into tables

AC (a minimal super key  
of the original R)

AB

BDE

CD

A relation  $R(A, B, C, D, E)$  is converted into 3NF by putting LHS and RHS of each FD in Fc together in **one** relation.

Dependency preserving

$R(A, B, C, D, E)$  becomes  $R_1(A, C)$ ,  $R_2(A, B)$ ,  $R_3(B, D, E)$ ,  $R_4(C, D)$

Or write it in another format: AC // AB // BDE // CD

# Example 2: 3NF and Fc

Given  $R(A,B,C)$

Let's do this together

FDs =  $\{ A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C \}$

Compute Fc and convert the relation into 3NF

(1) write all LHS

$A \rightarrow$   
 $B \rightarrow$   
 $AB \rightarrow$

(2) copy FDs as is

$A \rightarrow BC$   
 $B \rightarrow C$   
 $AB \rightarrow C$

Combine the given  
FDs  $A \rightarrow BC$  and  
 $A \rightarrow B$

No reflexivity  
to remove

(3) remove  
extraneous attr

$A \rightarrow BC$   
 $B \rightarrow C$   
 ~~$AB \rightarrow C$~~

Consider  $AB \rightarrow C$  and  
 $B \rightarrow C$ ,  
 $A$  is an extraneous attr,  
remove  $A$  from  
 $AB \rightarrow C$  (resulting in  
 $B \rightarrow C$ )

(4) remove  
extraneous attr

$A \rightarrow \overline{BC}$   
 $B \rightarrow C$

Apply decomposition to  
 $A \rightarrow BC$ , thus,  $A \rightarrow B$   
and  $A \rightarrow C$ .

$A \rightarrow C$  is logically  
equivalent to  $A \rightarrow B$   
and  $B \rightarrow C$  (transitivity).

Thus,  $C$  is an  
extraneous attr, remove  
 $C$  from  $A \rightarrow BC$

# Example 2: 3NF and Fc

Let's do this together

(from previous page)

(4) Remove  
extraneous attr

A	→	B
B	→	C

Super key



Turn Fc into tables

AB  
BC

$R(A, B, C)$  becomes  $R_1(A, B)$  and  $R_2(B, C)$

Or write it in another format: AB // BC

# BCNF and Decomposition

- **Lossless-join**
- Guarantee **redundancy free**
- May involve dependency across relations

Given a relation  $R$ ,

for every **nontrivial** FD  $X \rightarrow Y$  in  $R$ ,  $X$  is a **super key**

- For all FDs, “key  $\rightarrow$  everything”

Questions:

Is the relation in BCNF?

Is any refinement needed?

# BCNF and Decomposition

To calculate BCNF

Compute  $F^+$

**repeat** given a relation  $R$  (or a decomposed  $R$ ) and FDs  $F$

**for each** functional dependency  $f_i$  in a relation  $R$

**if**  $f_i$  violates  $X \rightarrow Y$

**then** decompose  $R$  into two relations:

one with  $X \cup Y$  as its attributes (i.e., everything  $f$ )

one with  $X \cup (\text{attrs}(R) - X - Y)$  as its attributes

**until** no violation

# Example: BCNF and F+

Given  $R(A,B,C,D,E)$

Let's do this together

FDs =  $\{ A \rightarrow B, AB \rightarrow D, B \rightarrow BDE, C \rightarrow D, D \rightarrow D \}$

Compute F+ and convert the relation into BCNF

Compute F+

(1) write all LHS  
& remaining

A  $\rightarrow$   
B  $\rightarrow$   
AB  $\rightarrow$   
C  $\rightarrow$   
D  $\rightarrow$   
E  $\rightarrow$

(2) copy FDs as is

A  $\rightarrow$  B  
B  $\rightarrow$  B DE  
AB  $\rightarrow$  D  
C  $\rightarrow$  D  
D  $\rightarrow$  D  
E  $\rightarrow$

(3) apply reflexivity

A  $\rightarrow$  AB  
B  $\rightarrow$  B DE  
AB  $\rightarrow$  AB D  
C  $\rightarrow$  CD  
D  $\rightarrow$  D  
E  $\rightarrow$  E

(4) apply transitivity

A  $\rightarrow$  AB DE  
B  $\rightarrow$  B DE  
AB  $\rightarrow$  AB DE  
C  $\rightarrow$  CD  
D  $\rightarrow$  D  
E  $\rightarrow$  E

# Example: BCNF and F+

(from previous page)

(4) apply transitivity

Let's do this together

F+	A	→	AB	DE	Not trivial FDs. A, B, C are not super key
	B	→	B	DE	
	AB	→	AB	DE	
	C	→	CD		Trivial FDs (let's abbreviate to TR)
	D	→		D	
	E	→		E	

Based on F+, let's rewrite using the following format to help us calculate

R ( A B C D E )  
! ! ! TR TR

TR – trivial  
SK – super key  
X – neither trivial nor super key  
! – (possibly) need to work on

# Example: BCNF and F+

R ( A B C ~~D~~ ~~E~~ )  
! ! ! TR TR

Let's do this together

F+	A	→	AB	DE
	B	→	B	DE
	AB	→	AB	DE
	C	→		CD
	D	→		D
	E	→		E

To choose which FD to work on, two ways:

- Choose the first FD, or
- Choose the longest FD (yield better solution)

Let's consider A:

A is not a super key, not trivial, thus  $A \rightarrow ABDE$  violates BCNF, break a relation on A

# Example: BCNF and F+

R ( A B C ~~D~~ ~~E~~ )  
! ! ! TR TR

$A \rightarrow ABDE$

Take *RHS*, make a table: ABDE

Take *LHS*, make a table where A is a key

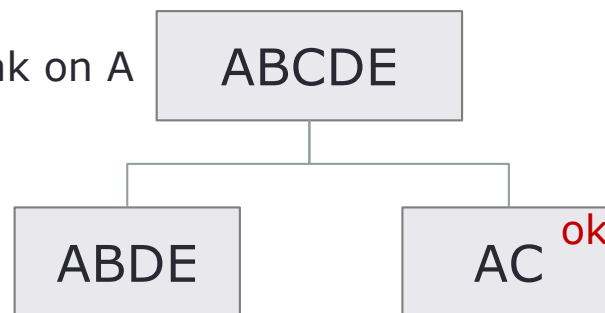
A plus (original – (*RHS*)) --- thus, AC

Let's do this together

F+	A	→	AB	DE
	B	→	B	DE
	AB	→	AB	DE
	C	→		CD
	D	→		D
	E	→		E

There are only 2 attrs,  
this relation is ok

Break on A



Verify table ABDE if  
there is any violation.

A is a super key. Also  
can't break on A twice.  
Still need to work on B.

Restriction: **Cannot** break  
on A 2 times in a row

Next: consider B. B is neither trivial nor super key, break on B

# Example: Calculate BCNF

R ( A B ~~D~~ ~~E~~ )  
X ! TR TR

$B \rightarrow BDE$

Take *RHS*, make a table: BDE

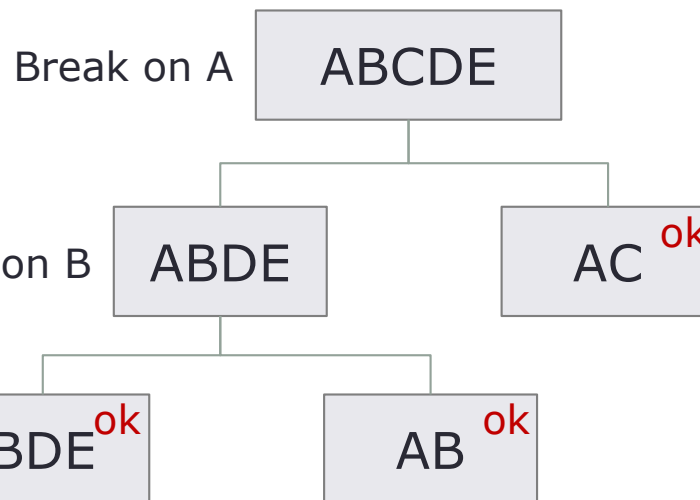
Take *LHS*, make a table where B is a key

B plus (original – (*RHS*)) --- thus, AB

Let's do this together

F+	A	→	AB	DE
	B	→	B	DE
	AB	→	AB	DE
	C	→		CD
	D	→		D
	E	→		E

There are only 2 attrs,  
this relation is ok



Verify that  $B \rightarrow BDE$   
does not violate BCNF.

B is super key. Also,  
can't break on B twice.

D and E are trivial.

R(ABCDE) becomes  
AC // AB // BDE

(notice: results are different 3NF)

# Wrap-Up

- Compute  $F^+$  and  $F_c$
- 3NF and decomposition
- BCNF and decomposition

## What's next?

- SQL